[14] S. Mardešić and J. Segal, *Shapes of compacta and* ANR-*systems*, Fund. Math. 72 (1971), pp. 61–68.
[15] K. Morita, *Čech cohomology and covering dimension for topological spaces*, Fund Math. 87 (1975), pp. 251–259.
[16] — *On shapes of topological spaces*, Fund. Math. 86 (1975), pp. 251–259.
[17] M. Moszyńska, *On shape and fundamental deformation retracts* II, Fund. Math. 77 (1972), pp. 235–240.
[18] — *Uniformly movable compact spaces and their algebraic properties*, Fund. Math. 77 (1972), pp. 125–144.
[19] — *Various approaches to the fundamental groups*, Fund. Math. 78 (1973), pp. 107–118.
[20] — *The Whitehead Theorem in the theory of shapes*, Fund. Math. 80 (1973), pp. 221–263.
[21] J. Nagata, *Modern Dimension Theory*, New York 1965.
[22] T. Porter, *Čech homotopy* I, J. London Math. Soc. 6 (1973), pp. 429–436.
[23] — *Borsuk's theory of shape and Čech homotopy*, Math. Scand. 33 (1973), pp. 83–99.
[24] H. Schubert, *Kategorien* I, Berlin 1970.
[25] E. Spanier, *Algebraic Topology*, New York 1966.

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Invariant uniformization

by

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Abstract. We show that the invariant version of the Kondo-Addison Uniformization Theorem fails. Several counterexamples of algebraic interest are presented.

Can one pick a point from each countable linear order? To make this problem model-theoretically interesting we identify isomorphic structures and to make it nontrivial in ZFC, set theory with choice, we require that the picking be done in a countable-ordinal-sequence-definable way. Roughly speaking, a set is definable from a countable sequence of ordinals iff for some ZF formula φ and some countable sequence α of ordinals, it is the unique solution of $\varphi(\underline{x}, \underline{\alpha})$. A set definable in any mathematically accepted way will be countable-ordinal-sequence-definable. Henceforth we shall interpret "one can pick a point (proper substructure, proper extension, etc.) from each linear order" as meaning that there is a countable-ordinal-sequence-definable function which assigns to each isomorphism type of a countable linear order the isomorphism type of a point (proper substructure, proper extension etc.) of the linear order, i.e., to the isomorphism type of $\langle A, < \rangle$ it assigns the isomorphism type of some structure $\langle A, <, a \rangle$ where $a \in A$ ($\langle A, B, < \rangle$ where $B \subseteq A$, $\langle B, A, < \rangle$ where $A \subseteq B$, etc.). "One cannot always pick..." shall be interpreted as meaning that it is relatively consistent with ZFC that there is no countable-ordinal-sequence-definable function which picks... All of the results below of the form "one cannot always pick a ..." have as consequences "it is relatively consistent with ZF and the principle of dependent choices that there is no function (definable or not) which selects a ...".

We first show that one cannot always pick points from certain structures called bireals and then show that: One cannot always pick a point from each countable linear order or from each countable semigroup and one cannot always pick a proper substructure for each countable algebra which has such. Although one can always pick a proper extension by adding a new point, it is relatively consistent with the existence of an inaccessible cardinal that one cannot pick a countable nonisomorphic extension of each countable structure which has such. It is also relatively consistent

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with the existence of an inaccessible that one cannot pick a cofinal subset of order type $\leqslant \omega$ from each countable well-order. In the following paper Hanf shows the unsolvability of the long open problem of picking an ordered basis for each countable Boolean algebra.

We now put these problems in the setting of descriptive set theory. The spaces considered below will be cartesian products of countably many copies of ω, $2^{(\omega^n)}$, and $\omega^{(\omega^n)}$, $n = 1, 2, \ldots$ Given a subset X of such a space consisting of, say, triples $\langle x, y, z\rangle$, X is *invariant* iff $\langle x, y, z\rangle \in X$ and $\langle \omega, x, y, z\rangle \cong \langle \omega, x', y', z'\rangle$ implies $\langle x', y', z'\rangle \in X$. A subset Y of X is an *invariant uniformization* of X with respect to z iff Y is invariant, $\exists z(\langle x, y, z\rangle \in X)$ iff $\exists z(\langle x, y, z\rangle \in Y)$, and $\langle x, y, z\rangle$ and $\langle x, y, z'\rangle \in Y$ implies $\langle \omega, x, y, z\rangle \cong \langle \omega, x, y, z'\rangle$. Invariant uniformization with respect to y and z is defined similarly. Clearly, one can pick a point from each countable linear order iff there is an invariant countable-ordinal-sequence-definable uniformization with respect to p of $\{\langle <, p\rangle \in 2^{\omega\times\omega} \times \omega \colon \langle \omega, <\rangle$ is a linear order$\}$ and one can pick a proper substructure for each countable 1-unary algebra iff there is an invariant countable-ordinal-sequence-definable uniformization of $\{\langle f, U\rangle \in \omega^\omega \times 2^\omega \colon U$ is closed under $f\}$. Similarly, the other problems of the previous paragraph are equivalent to invariant uniformization problems.

An *enumeration* of a structure is an ordering of its universe of type ω. Most algebraic and model-theoretic constructions on countable structures are either deterministic, i.e., involve no arbitrary choices (for example, the construction of the subalgebra generated by a given set), or are deterministic modulo an enumeration of the structure's universe as are most constructions which use Zorn's Lemma (for example, the extension of an ideal to a prime ideal). In the section on enumerations we show that a wide class of invariant uniformization problems reduce to the problem of picking an enumeration of each countable structure.

If one cannot pick a point from each linear order, then clearly $2^{\omega\times\omega} \times \omega$ has no invariant countable-ordinal-sequence-definable uniformization with respect to the last coordinate. Hence

THEOREM 1. *It is relatively consistent with* ZFC *that there is an invariant* Δ_1^0 *set with no invariant countable-ordinal-sequence-definable uniformization.*

The classical analogue [5, p. 130] requires a Π_2^1 set.

A consequence of this corollary and Shoenfield's Absoluteness Lemma is the solution of a problem of Vaught that inspired this paper: Is there an invariant Π_1^1 set with no invariant Π_1^1 uniformization? There is. Thus in contrast to the Strong Separation [6] and Reduction [12, 13, 14] Principles, the Kondo-Addison Uniformization Theorem fails for invariant sets. In the last section we prove there is an invariant Δ_1^0 set with no invariant uniformization in $\mathscr{A}$, the smallest family containing the open sets and closed under complementation, countable union, and the operation (A), i.e., fusion, see [3].

One cannot go much further in ZF since in ZF+V = L, every invariant Π_1^1 set has an invariant Δ_2^1 uniformization. But using results of Solovay we can show

that if there is a measurable cardinal or even if $\omega_1^{L[r]} < \omega_1$ for every real r, then there is an invariant Δ_1^0 set with no invariant uniformization by a Borel combination of Π_2^1 sets.

Bireals. A *bireal* is a structure $\langle A, <, U\rangle$ for which $\langle A <\rangle$ has order type $\omega^* + \omega$ and $U \subseteq A$. One can think of a bireal as an $\omega^* + \omega$ sequence of 0's and 1's.

THEOREM 2. *It is relatively consistent with* ZFC *that there is no countable-ordinal-sequence-definable choice function for the family of (unordered) pairs of isomorphism types of bireals.*

Proof. The proof is a routine Cohen forcing argument using undefinability methods of Lévy and Solovay. Jech [2] or Scott [8] may be used for notational reference.

Let I be the set of integers. Let B be the complete Boolean algebra of regular open sets of the space $2^{\omega_1 \times 2 \times I}$ under the product topology. Let $P = \{f \in {}^{\operatorname{dom}(f)}2 \colon \operatorname{dom}(f)$ is a finite subset of $\omega_1 \times 2 \times I\}$ be identified with the set of Baire intervals of $2^{\omega_1 \times 2 \times I}$. For $\alpha \in \omega_1$, let $P_\alpha = \{f \in P \colon \operatorname{dom}(f) \subseteq \alpha \times 2 \times I\}$, let B_α be the complete subalgebra generated by P_α and let H_α be the group of automorphisms of B which leave elements of B_α fixed. Note: $B = \bigcup_{\alpha\in\omega_1} B_\alpha$. We shall suppose that in V^B, the B-valued extension of the universe V, $\|\underline{x} = \underline{y}\| = 1$ implies $x = y$. Each $h \in H_\alpha$ determines a unique automorphism $h \colon V^B \to V^B$ which leaves elements of V^{B_α} fixed.

LEMMA 3. *If $t \in V^B$ is definable in V^B from a countable ordinal sequence $s \in V^B$, then for some $\alpha \in \omega_1$, $h(t) = t$ for all $h \in H_\alpha$.*

Proof. Suppose s is a countable ordinal sequence in V^B. Then, since B satisfies the countable chain condition, $\{\|\underline{s}(\underline{n}) = \underline{\beta}\| \colon n \in \omega$ and β is an ordinal of $V\}$ is countable and hence $s \in V^{B_\alpha}$ for some $\alpha \in \omega_1$. If t is the unique solution of $\|\psi(\underline{t}, \underline{s})\| = 1$ and if $h \in H_\alpha$, then $\|\psi(\underline{t}, \underline{s})\| = 1 = h(1) = h\|\psi(\underline{t}, \underline{s})\| = \|\psi(\underline{h(t)}, \underline{h(s)})\| = \|\psi(\underline{h(t)}, \underline{s})\|$ and so by uniqueness $h(t) = t$.

For $\alpha \in \omega_1$ and $\sigma \in \{0, 1\}$, let $r_{\alpha\sigma}$ be the bireal $\langle I, <, U\rangle$ in V^B for which $<$ is the usual ordering of the integers and U is the B-valued subset of I such that for $i \in I$, $\|\underline{i} \in \underline{U}\| = \{f \in 2^{\omega_1 \times 2 \times I} \colon f(\alpha, \sigma, i) = 1\} \in B$. Note: $\|\underline{r_{\alpha 0}} \cong \underline{r_{\alpha 1}}\| = 0$. For $\alpha \in \omega_1$ and $j \in I$ let $h_{\alpha j}$ be the permutation of $\omega_1 \times 2 \times I$ whose value at $\langle \beta, \sigma, i\rangle$ is $\langle \beta, \sigma, i\rangle$ if $\beta \neq \alpha$, $\langle \beta, 1, i+j\rangle$ if $\beta = \alpha$ and $\sigma = 0$, and $\langle \beta, 0, i-j\rangle$ if $\beta = \alpha$ and $\sigma = 1$. Extend $h_{\alpha j}$ to B and V^B in the usual way. Note: $h_{\alpha j} \in H_\alpha$, $h_{\alpha j}(r_{\alpha\sigma}) \cong r_{\alpha(1-\sigma)}$, and for any $p \in P$ and $\alpha \in \omega_1$ there is a $j \in I$ such that $p \wedge h_{\alpha j}(p) \neq 0$, just pick j so that $\operatorname{dom}(p) \cap \operatorname{dom}(h_{\alpha j}(p)) \cap (\{\alpha\} \times 2 \times I) = \emptyset$.

Now suppose there is a countable-ordinal-sequence-definable choice function for pairs of isomorphism types of bireals in V^B. Then there is a countable-ordinal-sequence-definable choice function f for the family of pairs of bireals such that $r \cong r'$ and $s \cong s'$ implies $f(\{r, s\}) \cong f(\{r', s'\})$. By the lemma we may pick an $\alpha \in \omega_1$, so that $h(f) = f$ for $h \in H_\alpha$. Now $\|f(\{r_{\alpha 0}, r_{\alpha 1}\}) = r_{\alpha 0}\| \vee \|f(\{r_{\alpha 0}, r_{\alpha 1}\}) = r_{\alpha 1}\| = 1$ so one of the disjuncts, suppose it is the first, is nonzero. Then for some $p \in P$,

$$p \leqslant \|f(\{r_{\alpha 0}, r_{\alpha 1}\}) = r_{\alpha 0}\|.$$

Pick j so that $p \wedge h_{\alpha j}(p) \neq 0$. Since

$$h_{\alpha j}(p) \leqslant h_{\alpha j}(\|f(\{r_{\alpha 0}, r_{\alpha 1}\}) = r_{\alpha 0}\|) = \|f(\{h_{\alpha j}(r_{\alpha 0}), h_{\alpha j}(r_{\alpha 1})\}) = h_{\alpha j}(r_{\alpha 0})\|$$
$$\leqslant \|f(\{r_{\alpha 0}, r_{\alpha 1}\}) \cong r_{\alpha 1}\|,$$
$$p \wedge h_{\alpha j}(p) \leqslant \|f(\{r_{\alpha 0}, r_{\alpha 1}\}) = r_{\alpha 0} \wedge f(\{r_{\alpha 0}, r_{\alpha 1}\}) \cong r_{\alpha 1}\| = 0,$$

a contradiction.

COROLLARY 4. *One cannot always pick a point from each bireal.*

Proof. Each point of a bireal divides it into a pair of reals in the obvious way. Hence if one could pick a point from each bireal, the problem of choosing one of a pair of isomorphism types of bireals would be reduced to the solvable problem of choosing one of a pair of ordered pairs of reals.

COROLLARY 5. *It is relatively consistent with* ZFC *that there is no countable-ordinal-sequence-definable function which selects a point from each countable set of reals.*

Proof. One can pick points from eventually periodic bireals, hence, by the above, one cannot always pick points from bireals which are not eventually periodic. For such bireals $\langle A, <, U\rangle$, each point $a \in A$ is uniquely determined by its cut $\langle \{b \in A\colon b \geqslant a\}, <, U\rangle$. Hence one cannot always pick a cut from the set of all cuts of each such bireal. But then one cannot always pick a real from each countable set of reals.

Some unsolvable problems.

THEOREM 6. *One cannot always pick:*

(1) *a point from each countable linear order,*

(2) *a point from each countable semigroup,*

(3) *a proper substructure from each countable structure which has such, and, if the existence of an inaccessible cardinal is consistent,*

(4) *a countable nonisomorphic extension of each countable structure.*

Proof. (1) For any bireal ... 0 1 1 0 0 ... the choice of a point or even Dedekind cut of a linear order of type $\ldots 2+\eta+3+\eta+3+\eta+2+\eta+2 \ldots$, where η is the order type of the rationals, definably determines a point in the bireal which, by the previous section, is not always possible.

(2) For any linear order $\langle L, <\rangle$, each point in the semigroup $\langle L, \max_{<}\{\,,\,\}\rangle$ determines a point in the linear order.

(3) For any bireal $\langle I, <, U\rangle$, each proper substructure of $\langle I$, the $<$-immediate successor operation, $U\rangle$ is generated by a unique point and thus determines a point of the bireal.

(4) Each element a of a structure $\langle A, U_1, U_2, \ldots\rangle$, where $U_n \subseteq A$, determines a real $r \subseteq \omega$ by $n \in r$ iff $x \in U_n$. Given a countable set X of reals, let $\langle Y, U_1, U_2, \ldots\rangle$ be a structure such that each of its elements determines a real in X and each real in X is determined by infinitely many of its elements. Then every countable nonisomorphic extension of $\langle Y, U_1, U_2, \ldots\rangle$ determines a proper countable extension of X. Hence if one could pick such extensions in a countable-ordinal-sequence-definable way, one could, by iterating this process of extension ω_1 times, construct a countable-ordinal-sequence-definable set of reals of cardinality $\aleph_1$. But it is relatively consistent with the existence of an inaccessible that $\aleph_1 \neq 2^{\aleph_0}$ and every countable-ordinal-sequence-definable set of reals has cardinality $\aleph_0$ or $2^{\aleph_0}$.

Among other things, one cannot always pick a nontrivial subset, equivalence relation, or automorphism for each countable structure which has such and one cannot always pick a fixed point ($f(x) = x$) from each 1-unary algebra with exactly two fixed points. One cannot always pick for each countable tree a branch running through it. One cannot always pick for each countable partial order a linear order extending it. One cannot always pick a maximal ideal for each countable cylindric algebra with constants. Hence, given an appropriate definition of "theory" (one must not require a well-ordering of the similarity type), one cannot always pick a completion of each consistent countable theory.

In the model constructed in the previous section $V = L[a]$ for some $a \subseteq \omega_1$. Hence GCH and $\omega_1^L = \omega_1$ held. A consequence of the latter is that one can pick an enumeration of each countable well-order. We show in the next section that this is not always possible.

Well-orders.

THEOREM 7. *It is relatively consistent with* ZFC+$\exists$ *an inaccessible cardinal that one cannot pick an enumeration of each countable well-order.*

Proof. (The author acknowledges Gerald E. Sack's gracious assistance with this proof.) Assume that every countable-ordinal-sequence-definable set of reals contains a perfect set. This assumption is relatively consistent with the existence of an inaccessible by [11]. Suppose E is an invariant countable-ordinal-sequence-definable uniformization of $\{\langle <, <_\omega\rangle\colon \langle \omega, <\rangle$ is a well-order and $\langle \omega, <_\omega\rangle$ has order type $\omega\}$. Then $W = \{R \subseteq \omega \times \omega\colon \langle \omega, \in, R\rangle \cong \langle \omega, <_\omega, <\rangle$ for some $\langle <, <_\omega\rangle \in E\}$ is an uncountable countable-ordinal-sequence-definable set containing exactly one representative of each countable well-order type. Let W^* be a perfect subset of W. Hence W^* is Π_1^0 and cofinal in W. Hence R is a well-ordering iff for some $R' \in W^*$, $\langle \omega, R\rangle$ is isomorphic to an initial segment of $\langle \omega, R'\rangle$. But this is impossible since "R is a well-ordering" is not Σ_1^1.

By the same argument and [10] we have that if there is a measurable cardinal or even if $\omega_1^{L[r]} < \omega_1$ for every real r, then there is no invariant Σ_2^1 uniformization of $\{\langle <, <_\omega\rangle\colon \langle \omega, <\rangle$ is a well-order and $\langle \omega, <_\omega\rangle$ has order type $\omega\}$.

COROLLARY 8. *It is relatively consistent with* ZFC+$\exists$ *an inaccessible cardinal that one cannot always pick a cofinal subset of type $\leqslant\omega$ from each countable well-order.*

Proof. Suppose it is possible to pick a cofinal subset of type $\leqslant\omega$ from each countable well-order. We show that one can then pick an enumeration of each well-order. Let $\langle A, <\rangle$ be a well-order. Let $\{a_0, a_1, \ldots\}$ be the assigned cofinal subset listed in order. Let $\{a_{i0}, a_{i1}, \ldots\}$ be, in order, the assigned cofinal subset of $\langle [a_i, a_{i+1}), <\rangle$. Define a_{ijk}, a_{ijkn}, etc. likewise. Let $a \in A$. Let $s(0)$ be the least $i \in \omega$ such that $a \leqslant a_i, \ldots$, let $s(n+1)$ be the least $i \in \omega$ such that $a \leqslant a_{s(0)s(1)\ldots s(n)i}, \ldots$ Since $a_{s(0)} \geqslant a_{s(0)s(1)} \geqslant \ldots$ cannot be an infinite strictly decreasing sequence, there must be at least m such that $a = a_{s(0)s(1)\ldots s(m)}$. The map $a \to \langle s(0), s(1), \ldots, s(m)\rangle$ is 1-1 from A into the set of finite sequences of natural numbers. Hence the usual enumeration on the latter set definably determines an enumeration of $\langle A, <\rangle$.

Enumeration: a universal $\underset{\sim}{\Sigma}_2^1$ problem.

THEOREM 9. *If $\Sigma \supseteq \underset{\sim}{\Sigma}_2^1$ is a collection of sets closed under finite intersection and second-order existential quantification and if $E = \{\langle R, <\rangle : R \subseteq \omega \times \omega$ and $\langle \omega, <\rangle$ has order type $\omega\}$ has an invariant uniformization in Σ, then so does every invariant $\underset{\sim}{\Sigma}_2^1$ class.*

Proof. Let Σ be as hypothesized and let F be an invariant uniformization of E. It suffices to prove that every invariant $\underset{\sim}{\Pi}_1^1$ set X has an invariant Σ uniformization. Suppose X consists of pairs $\langle R, S\rangle$ and is to be uniformized with respect to S. For any R, let $<_R$ be a well-ordering of $L[R]$ such that $(\forall x <_R y)(\varphi(x, y))$ is $\underset{\sim}{\Delta}_2^0$ in R and y if φ is $\underset{\sim}{\Delta}_2^0$, see [1]. Let $Y = \{\langle R, S\rangle \in X : (\exists <, R', S')(\langle R, <\rangle \in F \wedge \langle \omega, \in, R', S'\rangle \cong \langle \omega, <, R, S\rangle \wedge \langle R', S'\rangle \in X \cap L[R'] \wedge (\forall S'' <_{R'} S')(\langle R', S''\rangle \notin X \cap \cap L[R']))\}$. Clearly Y is invariant and uniform. If $(\exists S)(\langle R, S\rangle \in X)$, then by invariance there is an S' such that $\langle R', S'\rangle \in X$ where $\langle R, <\rangle \in F$ and $\langle \omega, \in, R'\rangle \cong \langle \omega, <, R\rangle$ and by Shoenfield's Absoluteness Lemma [9] there is such an S' in $L[R']$ and so $(\exists S)(\langle R, S\rangle \in Y)$. Thus Y is an invariant uniformization of X. Finally, Y is Σ since the first conjunct of the matrix of its definition is Σ and the others are $\underset{\sim}{\Sigma}_2^1$.

Invariant descriptive set theory. Let $X = \{\langle S, U, p\rangle \in 2^{\omega \times \omega} \times 2^{\omega} \times \omega : S$ is the graph of the immediate successor operation of a nonperiodic bireal $\langle \omega, <, U\rangle\}$. The product topology on X is the topology it inherits as a subspace of $2^{\omega \times \omega} \times 2^{\omega} \times \omega$. With this topology X is separable and, by [4, p. 207, K(b)], metrically topologically complete.

THEOREM 10. *The set X is not invariantly uniformized with respect to p by any set with the Baire property in the product topology.*

Proof. The proof is similar to the forcing proof in the section on bireals. We shall identify elements $\langle S, U, p\rangle$ of X with the structures $\langle \omega, S, U, p\rangle$ of a first-order language with symbols for S, U, and p. The UC_Δ topology on X is the topology whose closed sets are generated by the sets $\mathrm{Mod}_X(\sigma)$ where σ is a finite universal sentence of the language and $\mathrm{Mod}_X(\sigma)$ is the class of its models in X. For $i \in I$, let $\varphi(p_i)$ be the formula $(\forall x)(x$ the ith successor (negative successors are predecessors) of p implies $\varphi(x))$. Then in the theory of X, $\forall x \varphi(x) \Leftrightarrow \bigwedge_{i \in I} \varphi(p^i)$ and the sentences $S(p^i, p^j)$ and $p^i = p^j$ are either true or false and so may be eliminated. Thus in the theory of X all universal sentences are equivalent to conjunctions of finite Boolean combinations of the sentences Up^i with $i \in I$. Thus the open sets of the UC_Δ topology are generated by finite conjunctions of the basis sets $\mathrm{Mod}_X(Up^i)$ and $\mathrm{Mod}_X(\neg Up^i)$ with $i \in I$.

Suppose $P \subseteq X$ invariantly uniformizes X with respect to p and has the Baire property in the product topology. Then P is equivalent modulo a meager set of the product topology to an open set Q of the UC_Δ topology (see, e.g., the proof of Theorem 3.4 in [2]). Let $s\colon X \underset{\text{onto}}{\overset{1-1}{\to}} X$ by $s(\langle \omega, <, U, p\rangle) = \langle \omega, <, U, p^1\rangle$ and let s^i be the ith power of s under composition, $i \in I$. Then X is the disjoint union of the sets $(s^i)^*(P)$, $i \in I$. Since X is not meager in itself under the product topology, P is not meager in X. Hence Q is nonempty and must include a nonempty intersection $O_1 \cap \ldots \cap O_n$ of UC_Δ basis sets. But clearly for some, in fact almost all, $i \in I$, $(O_1 \cap \ldots \cap O_n) \cap (s^i)^*(O_1 \cap \ldots \cap O_n) \neq \emptyset$. But then $Q \cap (s^i)^*(Q)$ includes a set which is open in the UC_Δ and hence open in the finer product topology. This is impossible since $P \cap (s^i)^*(P) = \emptyset$ implies $Q \cap (s^i)^*(Q)$ is a meager set of the product topology.

If X has no invariant uniformization with the Baire property, then X and hence the whole space $2^{\omega \times \omega} \times 2^{\omega} \times \omega$ has no invariant uniformization in $\mathscr{A}$, the smallest family containing the open sets and closed under complementation, countable union, and the operation (A), i.e., fusion, see [3]. Hence

COROLLARY 11. *There is an invariant Δ_1^0 set which is not invariantly uniformized by any set in $\mathscr{A}$.*

References

[1] J. W. Addison, *Some consequences of the axiom of constructibility*, Fund. Math. 46 (1959), pp. 337–357.
[2] T. J. Jech, *Lectures in set theory with particular emphasis on the method of forcing*, Lecture Notes in Mathematics 217, Berlin 1971.
[3] C. Kuratowski and A. Mostowski, *Set Theory*, Amsterdam 1967.
[4] J. L. Kelley, *General Topology*, Princeton, New Jersey, 1955.
[5] A. Lévy, *Definability in axiomatic set theory* II, Mathematical Logic and Foundations of Set Theory (Yehoshua Bar-Hillel, editor), Amsterdam 1970, pp. 129–145.
[6] E. G. K. López-Escobar, *An interpolation theorem for denumerably long formulas*, Fund. Math. 57 (1965), pp. 253–272.
[7] D. Myers, *The failure of the $L_{\omega_1\omega}$ analogue of $\underset{\sim}{\Pi}_1^1$ uniformization*, Notices of the American Mathematical Society 19 (1972), p. A-330, Abstract 72T-E13.
[8] D. Scott and R. Solovay, *Boolean-valued models for set theory*, to appear.
[9] J. R. Shoenfield, *The problem of predicativity*, Essays on the Foundations of Mathematics, Jerusalem 1961, pp. 132–139.

[10] R. M. Solovay, *On the cardinality of $\underset{\sim}{\Sigma}_2^1$ sets of reals*, Foundations of Mathematics, Berlin 1969, pp. 58–73.
[11] — *A model of set theory in which every set of reals is Lebesgue measurable*, Ann. of Math. 92 (1970), pp. 1–56.
[12] R. Vaught, *Descriptive set theory in $L_{\omega_1\omega}$*, Cambridge Summer School in Mathematical Logic, Lecture Notes in Mathematics 337, Berlin 1973, pp. 574–598.
[13] — *A Borel invariantization*, Bulletin of the American Mathematical Society 79 (1973), pp. 1292–1295.
[14] — *Invariant sets in topology and logic*, Fund. Math. 82 (1974), pp. 269–294.

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