whose non-degenerate elements are the sets $\tilde{F}_Z$ for all $Z \in \mathscr{Z}_n$. The quotient space K/G_n is homeomorphic to K_n/G'_n where G'_n is the decomposition of K_n whose non-degenerate elements form the good family $\mathscr{Z}_n$. By Lemma 4 K_n/G'_n, and so K/G_n is homeomorphic to the 3-manifold K_n. Since we have supposed that K is a 3-manifold and G_n was shown to consist of sets which are cellular in K, it follows from the theorem of Armentrout (see [1], p. 66, Theorem 2) that K is homeomorphic to K_n for every natural n. This implies that K_1 is homeomorphic to K_2. K_1 is a 3-sphere and K_2 contains the fake 3-cell F, which is impossible (see [5], Theorem 5). This proves that K is not a 3-manifold.

VI. Proof that $\dim K = 3$. $\dim K \leqslant 3$, because $\dim K_n = 3$ for every n and $K = \varprojlim\{K_n, \alpha_{mn}\}$ and $\dim K \geqslant 3$ because $H^3(K, Z) \neq 0$ (see [7], p. 152).

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A theorem on the weak topology of $C(X)$ for compact scattered X

by

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Abstract. We prove that if a function space $C(X)$, where X is a compact scattered space, is K-analytic under the weak topology, then $C(X)$ is a WCG space, i.e. X is an Eberlein compact. This result is related to a recent author's example of a non-WCG space $C(X)$ with X compact scattered, which is Lindelöf in the weak topology, a recent example of Talagrand of a non-WCG space $C(K)$, which is K-analytic in the weak topology, and the recent theorem of Talagrand that every WCG Banach space is K-analytic in the weak topology.

1. Introduction. It was an old problem of Corson [6] whether the WCG Banach spaces (the terminology will be explain in the next section) are exactly the Banach spaces which are Lindelöf in their weak topology. An example of a Banach space which is Lindelöf in the weak topology but not WCG was given by Rosenthal [12] and, on the other hand, Talagrand [15] proved that a WCG Banach space is $\mathscr{K}$-analytic (which is much more than the Lindelöf property) in the weak topology. It was still open after these works if the Corson's problem has an affirmative solution in the class of function spaces [9], Problem 6, 6′, [4], Problem 7. Recently, the author [11] and independently, about the same time, Talagrand [16] constructed the appropriate counterexamples. The content of these examples is however quite different. The function space $C(X)$ in the author's example is not $\mathscr{K}$-analytic in the weak topology, while the compact X is scattered, whereas the Talagrand's space $C(K)$ is $\mathscr{K}$-analytic in the weak topology, but the compact K is not scattered.

The aim of this paper is to show that if a function space $C(X)$ is $\mathscr{K}$-analytic in the weak, or pointwise topology and the compact X is scattered, then $C(X)$ is a WCG-space, or equivalently — X is an Eberlein compact.

It is worth while to mention that one can exploit the Talagrand's example to show (1) that in fact there is no topological property which is invariant under continuous mappings, closed hereditarily and characterizes the Eberlein compacts as the compacts whose function space in the weak (or pointwise) topology has this

(1) By means, for example, of the reasonings given in [11], the proof of Lemma 1; cf. also [2].

property. Our result shows that, when restrict ourselves to the class of all scattered compacts, the $\mathcal{K}$-analyticity is such a property.

The author would like to thank to K. Alster for some valuable conversations about the subject of this paper.

2. Terminology and notation. Our terminology follows [7], [9] and [13]. The symbol N stands for the set of natural numbers; $|A|$ denotes the cardinality of a set A and $|\lambda|$ is the cardinality of an ordinal λ; all cardinals are assumed to be infinite. In Section 5 we shall use some basic facts about the *stationary sets* of ordinals; all needed material one can find in [8], Chap. X, § 1.

For a compact space K we denote by $C(K)$ the Banach space of all continuous real-valued functions on K endowed with the sup-norm; the same set equipped with the pointwise topology is denoted by $C_p(K)$.

A Banach space is *weakly compactly generated* (shortly WCG) if it is the closed linear span of a weakly compact subspace; a function space $C(K)$ is WCG iff K is an *Eberlein compact*, i.e. if K can be embedded in a Banach space with the weak topology, or else — if K can be embedded in some $C_p(T)$ with compact T [9]. In the sequel we exploit the following characterization of Eberlein compacts [12] (cf. also [3], [4] and [1]): K is an Eberlein compact iff there exists a σ-point-finite family $\mathcal{U}$ of open F_σ-sets in K which separates the points of K (²); if K is zero dimensional one can assume that $\mathcal{U}$ consists of clopen sets.

A completely regular space A is $\mathcal{K}$-analytic [5] iff it is the projection of some closed subset of the product $\tilde{A}\times N^N$, where $\tilde{A}$ is a compactification of A (³).

3. Auxiliary facts. The following recent result of Alster [1] will play a very important role in the sequel.

ALSTER'S THEOREM. *Let K be a compact scattered space and let $\mathcal{F}$ be a family of closed subsets of K such that every point of K belongs to less than $\mathfrak{t}$ members of $\mathcal{F}$, where $\mathfrak{t}$ is a regular cardinal. Then there is a decomposition $\mathcal{F} = \bigcup_{s\in S}\mathcal{F}_s$, such that $|\mathcal{F}_s|<\mathfrak{t}$ for every $s\in S$ and the family $\{\bigcup\mathcal{F}_s\colon s\in S\}$ is point-finite.*

We need also the following two simple (and certainly well-known) facts about $\mathcal{K}$-analytic spaces.

LEMMA A ([16], Cor. 12). *If K is a compact space with $C_p(K)$ $\mathcal{K}$-analytic, then the weight of K is equal to its density* (⁴).

LEMMA B. *If a space A is $\mathcal{K}$-analytic then there exist sets $A_{i_1\ldots i_k}$, where $(i_1,\ldots,i_k)$ runs over all finite sequences of natural numbers, such that*

(²) We say that a family $\mathcal{A}$ of subsets of a set A separates the points of a set $B\subset A$ if for every pair of distinct points from B there is a member of $\mathcal{A}$ which contains exactly one of them.

(³) The choice of $\tilde{A}$ is inessential. The $\mathcal{K}$-analycity is a substitute of the classical notion of analycity for non-metrizable spaces.

(⁴) It follows from the fact that $C_p(K)$ admits a continuous injection into a space of weight $\mathfrak{t}$ equal to the density of K and the well known fact (cf. [10], 3.15) that if a $\mathcal{K}$-analytic space has such an injection, then it has a net of cardinality $\leqslant\mathfrak{t}$.

(i) $A = \bigcup_i A_i$, $A_{i_1\ldots i_k} = \bigcup_i A_{i_1\ldots i_k i}$,

(ii) *if $(i_k)\in N^N$ and $a_k\in A_{i_1\ldots i_k}$, then the sequence (a_k) has an accumulation point in A.*

Proof. Let $\tilde{A}$ be a compactification of A and let F be a closed subset of the product $\tilde{A}\times N^N$ which projects onto A. Given a finite sequence of natural numbers $(i_1,\ldots,i_k)$ put

$$N_{i_1\ldots i_k} = \{(j_m)\in N^N\colon j_m = i_m \text{ for } m\leqslant k\}$$

and let $A_{i_1\ldots i_k}$ be the projection of the set $F\cap(\tilde{A}\times N_{i_1\ldots i_k})$ parallel to N^N-axis.

4. THEOREM. *If X is a compact scattered space whose function space $C(X)$ is $\mathcal{K}$-analytic in the weak, or equivalently — in the pointwise topology, then X is an Eberlein compact.*

Remark. Recall, that for a compact scattered space X the weak and the pointwise topology coincide on every norm-bounded subset of $C(X)$ ([13], Cor. 19.7.7). We shall consider in the sequel just the pointwise topology, i.e., the space $C_p(X)$ (see also [16], Théorème 4).

5. Proof of the theorem. The proof goes by induction with respect to the cardinality of X (cf. the proof of the proposition in [1]) (⁵). So, let us assume that

(∗) *if K is a space of cardinality less than $\mathfrak{m}$ satisfying the assumptions of the theorem, then K is an Eberlein compact,*

and let X be a space of cardinality $\mathfrak{m}$ which satisfies the assumptions of the theorem. We shall consider separately the case of singular $\mathfrak{m}$ and the case of regular $\mathfrak{m}$.

A. The case of singular $\mathfrak{m}$. We begin with the following

LEMMA 1. *Let $\mathcal{U}$ be a family of clopen subsets of X of cardinality $|\mathcal{U}|<\mathfrak{m}$. Then there exists a σ-point-finite family $\mathcal{V}$ of clopen sets in X such that $\bigcup\mathcal{V}\subset\bigcup\mathcal{U}$ and if $\mathcal{U}$ separates the points $a,b\in X$ then so does $\mathcal{V}$.*

Proof. Let φ_U be the characteristic function of a set $U\in\mathcal{U}$. The diagonal mapping $\varphi = \Delta\{\varphi_U\colon U\in\mathcal{U}\}$ maps X continuously onto a compact subspace K of the Cantor cube of weight $|\mathcal{U}|$. Thus K is a compact, scattered space ([13], Proposition 8.5.3) of cardinality $<\mathfrak{m}$ (see footnote (⁵)) and $C_p(K)$ is $\mathcal{K}$-analytic (as the mapping adjoint to φ embedds $C_p(K)$ onto a closed subspace of $C_p(X)$); by the inductive assumption (∗) we infer that K is an Eberlein compact. Observe that $X\setminus\bigcup\mathcal{U} = \varphi^{-1}(p)$, where p is the point of the Cantor cube with all coordinates equal to 0. Let $\mathcal{W}$ be a σ-point-finite family of clopen subsets of K which separates the points of K (see Sec. 2); one can assume moreover that $p\notin\bigcup\mathcal{W}$. We take $\mathcal{V} = \{\varphi^{-1}(W)\colon W\in\mathcal{W}\}$. If $\mathcal{U}$ separates points $x,y\in X$, then $\varphi(x)\neq\varphi(y)$ hence $\mathcal{W}$ separates the points $\varphi(x),\varphi(y)\in K$ and thus $\mathcal{V}$ separates the points x, y.

We pass to the proof of the case A. There exists an increasing sequence

(⁵) Recall, that the weight and the cardinality of a scattered space coincide.

$X_1 \subset \ldots \subset X_\xi \subset \ldots \subset X$, $\xi < \lambda$ of sets such that $X = \bigcup_{\xi<\lambda} X_\xi$, $|X_\xi| < \mathfrak{m}$ and $|\lambda| = \mathfrak{n} < \mathfrak{m}$. For every $\xi < \lambda$ choose a family $\mathscr{U}_\xi$ of clopen subsets of X fo cardinality $|\mathscr{U}_\xi| < \mathfrak{m}$ which separates the points of X_ξ. Let us replace each $\mathscr{U}_\xi$ by a family $\mathscr{V}_\xi$, as in Lemma 1. The family $\mathscr{W} = \bigcup_{\xi<\lambda} \mathscr{V}_\xi$ consists of clopen sets, separates the points of X (as $\bigcup_{\xi<\lambda} \mathscr{U}_\xi$ does) and each point of X belongs to at most $\mathfrak{n}$ members of $\mathscr{W}$. Using Alster's Theorem (where $\mathfrak{t}$ is the succesor of $\mathfrak{n}$) we obtain a decomposition $\mathscr{W} = \bigcup_{s\in S} \mathscr{W}_s$, where $|\mathscr{W}_s| \leqslant \mathfrak{n}$ for $s \in S$ and the family $\{W_s\colon W_s = \bigcup \mathscr{W}_s, s \in S\}$ is point-finite. Using once again Lemma 1 we take for every $s \in S$ a σ-point-finite clopen family $\mathscr{G}_s$ which separates the same points as $\mathscr{W}_s$ and $\bigcup \mathscr{G}_s \subset W_s$. The family $\mathscr{G} = \bigcup_{s\in S} \mathscr{G}_s$ is σ-point-finite, consists of clopen sets and separates the points of X. Thus X is an Eberlein compact.

B. The case of regular $\mathfrak{m}$. Let λ be the initial ordinal of cardinality $\mathfrak{m}$; in the sequel Ω stands for the set of all ordinals less than λ and $\Lambda \subset \Omega$ is the set of all limit ordinals from Ω.

LEMMA 2. *Let $X_1 \subset \ldots \subset X_\xi \subset \ldots \subset X$, $\xi < \lambda$ be an increasing sequence of closed subsets of X of cardinality $< \mathfrak{m}$ such that $X = \bigcup_{\xi<\lambda} X_\xi$. If the set*

$$L = \{\xi \in \Lambda\colon X_\xi \setminus \bigcup_{\alpha<\xi} X_\alpha \neq \emptyset\}$$

is not stationary in Ω, then X is an Eberlein compact.

Proof. Let $f\colon L \to \Omega$ be a regressive function such that $|f^{-1}(\alpha)| < \mathfrak{m}$ for every $\alpha \in \Omega$; extend f over Ω letting $f(\alpha+1) = \alpha$ and $f(\alpha) = \alpha$ for $\alpha \in \Lambda \setminus L$. For every $\xi < \lambda$ we choose a family $\mathscr{W}_\xi$ of clopen subsets of X of cardinality $< \mathfrak{m}$, disjoint from $X_{f(\xi)}$ such that the family $\{W \cap (X_\xi \setminus X_{f(\xi)})\colon W \in \mathscr{W}_\xi\}$ is a base of the space $X_\xi \setminus X_{f(\xi)}$. Put $\mathscr{U} = \bigcup_{\xi<\lambda} \mathscr{W}_\xi$. Given a point $x \in X$ we have $x \in X_\xi$ for some $\xi < \lambda$ and thus x can belong only to the members of the family $\bigcup \{\mathscr{W}_\alpha\colon f(\alpha) < \xi\}$ of cardinality $< \mathfrak{m}$. Let x_1, x_2 be distinct points of X and let ξ_i be the first ordinal ξ with $x_i \in X_\xi$; assume that $\xi_1 \leqslant \xi_2 = \eta$. Observe that η is either non-limit, or $\eta \in L$; in both cases $f(\eta) < \eta$ and hence $x_2 \in X_\eta \setminus X_{f(\eta)}$. Since $x_1 \in X_\eta$ there exists $W \in \mathscr{W}_\eta$ with $x_2 \in W$ and $x_1 \notin W$. Thus $\mathscr{U}$ separates also the points of X. By Alster's Theorem (where $\mathfrak{t} = \mathfrak{m}$) we have a decomposition $\mathscr{U} = \bigcup_{s\in S} \mathscr{U}_s$, where $|\mathscr{U}_s| < \mathfrak{m}$ and the family $\{\bigcup \mathscr{U}_s\colon s \in S\}$ is point-finite. Now replace each family $\mathscr{U}_s$ by a family $\mathscr{V}_s$, as in Lemma 1. The family $\mathscr{V} = \bigcup_{s\in S} \mathscr{V}_s$ is clopen, σ-point-finite and separates the points of X, which completes the proof.

We are ready now for the proof of the case B. Let $A_{i_1 \ldots i_k} \subset C_p(X)$ satisfy the condition (i) and (ii) of Lemma B, Sec. 3. Let $X = \{x_\alpha\colon \alpha < \lambda\}$ and put $X_\xi = \overline{\{x_\alpha\colon \alpha < \xi\}}$ for $\xi < \lambda$. The sequence $X_1 \subset \ldots \subset X_\xi \subset \ldots \subset X$ is as in Lemma 2, by Lemma A, Sec. 3 (cf. also footnote (5)). Put

$$P_\xi = X_\xi \setminus \bigcup_{\alpha<\xi} X_\alpha \quad \text{and} \quad L = \{\xi \in \Lambda\colon P_\xi \neq \emptyset\}.$$

A property of the sequence (X_ξ) which will be important in the sequel is that

$$X_\xi = \overline{\bigcup_{\alpha<\xi} X_\alpha} \quad \text{for every limit } \xi < \lambda.$$

Suppose that X is not an Eberlein compact; then, by Lemma 2, the set L is stationary. Our goal is to derive a contradiction from this statement.

We shall construct inductively: a) natural numbers $i_1, i_2, \ldots$, b) points $a_1, a_2, \ldots \in X$, c) functions $f_1, f_2, \ldots \in C_p(X)$ such that

$$(c_1) \qquad f_k \in A_{i_1 \ldots i_k},$$

$$(c_2) \qquad f_i(a_k) = 1 \quad \text{if} \quad i \geqslant k,$$

$$(c_3) \qquad f_i(a_k) = 0 \quad \text{if} \quad i < k.$$

Assume that we have done it. Let $a \in X$ be an accumulation point of the sequence (a_k) and let $f \in C_p(X)$ be an accumulation point of the sequence (f_k), which exists by the condition (ii) of Lemma B, Sec. 3. By the condition (c_2) we have $f(a_k) = 1$ for every $k \in N$ and, by continuity of f, also $f(a) = 1$. On the other hand, the condition (c_3) gives $f_i(a) = 0$ for every $i \in N$, and thus $f(a) = 0$. This is the contradiction we were looking for (6).

It remains to construct the desired objects. This requires some preliminary reasoning. Let us put for arbitrary $A \subset X$

$$L(A) = \{\xi \in L\colon P_\xi \cap A \neq \emptyset\},$$

and let

$$F = X \setminus \bigcup \{U \subset X\colon U \text{ is open and } L(U) \text{ is not stationary}\}.$$

The space F contains an isolated point p ($F \neq \emptyset$, as X is compact and L stationary); let Z be a clopen neighbourhood of p in X such that $Z \cap F = \{p\}$ and let us put $E = L(Z) \setminus \{\alpha\colon \alpha \leqslant \tau\}$ where $p \in X_\tau$; of course the set E is stationary. For every $\xi \in E$ choose a point $p_\xi \in P_\xi \cap Z$ and a clopen neighbourhood V_ξ of p_ξ contained in $Z \setminus X_\tau$; since V_ξ is compact and disjoint from F, the set $L(V_\xi)$ is not stationary. Let $f_\xi \in C_p(X)$ be the characteristic function of the set V_ξ.

We pass to the construction. We shall define inductively:

(a) natural numbers $i_1, i_2, \ldots$ and points $a_1, a_2, \ldots$ from Z,

(b) stationary subsets $E_1 \supset E_2 \supset \ldots$ of the set E,

(c) ordinal numbers $\lambda_k \in E_k$,

such that for $k \geqslant 1$ the following conditions hold:

$$(\alpha_k) \qquad E_k \subset \{\xi\colon f_\xi \in A_{i_1 \ldots i_k}\},$$

$$(\beta_k) \qquad f_\xi(a_k) = 1 \quad \text{for} \quad \xi \in E_k,$$

$$(\gamma_k) \qquad f_{\lambda_i}(a_k) = 0 \quad \text{for} \quad i < k.$$

(6) Similar reasonings appear in the standard proofs of a classical theorem on countable compactness in function spaces (cf. [14], the proof of 11.1 in Chap. IV).

To begin with put $E_0 = E$, $A_\emptyset = C_p(X)$ and assume that the construction is done for $k = n$.

By the condition (i) of Lemma B, Sec. 3 there exists i_{k+1} such that the set $G = \{\xi \in E_k\colon f_\xi \in A_{i_1 \ldots i_{k+1}}\}$ is stationary; the set $H = G \setminus (L(V_{\lambda_1}) \cup \ldots \cup L(V_{\lambda_k}))$ is also stationary and for every $\xi \in H$ the set $V_\xi \setminus (V_{\lambda_1} \cup \ldots \cup V_{\lambda_k}) = W_\xi$ is a neighbourhood of the point p_ξ. Since $p_\xi \in \overline{\bigcup_{\alpha<\xi} X_\alpha}$ for $\xi \in H$, there exists $s(\xi) < \xi$ with $W_\xi \cap X_{s(\xi)} \neq \emptyset$. The function $s\colon H \to \Omega$ is a regressive function with the stationary domain H and therefore there exist a stationary set $I \subset H$ and an ordinal $\alpha \in \Omega$ such that $I \subset s^{-1}(\alpha)$. Thus $W_\xi \cap X_\alpha \neq \emptyset$ for every $\xi \in I$ and, since $|X_\alpha| < \mathfrak{m}$, there exist a stationary set $E_{k+1} \subset I$ and a point $a_{k+1} \in X_\alpha$ such that $a_{k+1} \in W_\xi$ whenever $\xi \in E_{k+1}$. Finally, let λ_{k+1} be an arbitrary ordinal from E_{k+1}. We have $f_\xi(a_{k+1}) = 1$ for $\xi \in E_{k+1}$, as $W_\xi \subset V_\xi$ and also $f_{\lambda_i}(a_{k+1}) = 0$ for $i \leqslant k$, as $a_{k+1} \notin V_{\lambda_1} \cup \ldots \cup V_{\lambda_k}$. This completes the construction.

It is now easily verified that the conditions (c_1), (c_2), (c_3) are fullfield if we put $f_i = f_{\lambda_i}$ (indeed, if $i \geqslant k$ then $\lambda_i \in E_i \subset E_k$ and therefore $f_{\lambda_i}(a_k) = 1$).

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Kan fibrations in the category of simplicial spaces

by

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Abstract. The notion of Kan fibration of simplicial spaces is defined as a direct generalization of Kan fibration of simplicial sets. The covering homotopy property for these fibrations is proved.

Introduction. The category of simplicial spaces (see § 1) has two homotopy theories, which might be called, respectively, topological and algebraic. The topological homotopy theory is based on the simplicial space I_*, whose space of n-simplices is the unit interval I for each $n \geqslant 0$, and whose face and degeneracy operators are all identity maps. Thus, in this theory, two morphisms of simplicial spaces, $f_i\colon X \to Y$, $i = 0, 1$, are homotopic if there is a morphism $P\colon I_* \times X \to Y$, such that, for each $n \geqslant 0$, $F_n|\{i\} \times X = f_{in}$ for $i = 0, 1$. On the other hand, the algebraic theory is the natural extension of the usual homotopy theory in the category of simplicial sets, and is based on the simplicial unit interval, $\Delta[1]$, regarded as a discrete simplicial space.

It is proved in [3], 11.9 and 11.10, that geometric realization of simplicial spaces preserves both kinds of homotopies. Thus, in using techniques in homotopy theory which obtain results about spaces by first working with simplicial spaces and then realizing (techniques which have been much in vogue in recent years, largely in connection with infinite loop spaces), it is possible to work with either of the simplicial homotopy theories, whichever is the more convenient. The use of what we have called the topological theory, has been fairly widespread, for example in [3]. However, by analogy with the category of simplicial sets, it seems to the author that the algebraic homotopy theory should prove much richer.

The purpose of the present paper is to extend some of the basic notions which have been developed for simplicial sets, to simplicial spaces, in particular the notion of Kan fibration. The corresponding notion of fibration with respect to the topological simplicial homotopy theory has been developed in [3], § 12, and is there