Théorème 2. *Avec les notations précédentes, si $\Pi_1(X_0) \neq \{0\}$, alors la loi de réciprocité n'est pas valable pour $k(X)$.*

On en déduit que la loi de réciprocité pour $k(X)$ ne peut-être valable que si $\Pi_1(X_0) = \{0\}$; or:

$$\big(\Pi_1(X_0) = \{0\}\big) \Leftrightarrow \big(T^q(J_0) = \{0\} \text{ pour tout } q\big)$$

où $T^q(J_0)$ est le q-groupe de Tate de la Jacobienne J_0 de X_0.

En particulier on sait que la réduite J_0 est de type additif si et seulement si $T^q(J_0) = \{0\}$ pour tout q; ceci montre que le seul cas où on peut obtenir la loi de réciprocité est celui envisagé dans le théorème 1.

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On a problem of Ryškov concerning lattice coverings

by

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1. In small dimensions the most efficient covering of n-dimensional space $\boldsymbol{R}^n$ by spheres is given by the so-called Voronoi form of the first type,

$$(n+1)\sum_{i=1}^{n} x_i^2 - \Big(\sum_{i=1}^{n} x_i\Big)^2;$$

i.e. by the dual lattice A_n^* to the root lattice A_n. Ryškov and Baranovskiĭ [7] have shown that this is the most efficient lattice covering for $n \leqslant 5$, extending earlier work of Bambah [1], [2], Delone and Ryškov [5] and others. In [6] Ryškov shows that A_n^* is not the most efficient lattice covering for all even $n \geqslant 114$ and all odd $n \geqslant 201$, and raises the question of finding the first dimension n for which there is a better lattice. In the present note we construct lattice coverings which are more efficient than A_n^* in all dimensions $n \geqslant 24$. This is made possible by the recent proof in [4] that the covering radius of the 24-dimensional Leech lattice Λ is $\sqrt{2}$ times the packing radius.

2. The covering density $\theta(L)$ of a lattice L having covering radius R and determinant d is $V_n R^n/d$. For A_n^* we have $R = \sqrt{n(n+2)/12(n+1)}$, $d = (n+1)^{-1/2}$ and

$$\theta(A_n^*) = V_n\sqrt{n+1}\left(\frac{n+2}{n+1}\right)^{n/2}\left(\frac{n}{12}\right)^{n/2} \tag{1}$$

(see for example Bleicher [3]); and in particular

$$\theta(A_{24}^*) = V_{25}\cdot 5\left(\frac{52}{25}\right)^{12} = 63.269\ldots$$

On the other hand, for the Leech lattice Λ, $R = \sqrt{2}$ and $d = 1$ (see [4]), therefore

$$\theta(\Lambda) = V_{24} \cdot 2^{12} = 7.9035\ldots,$$

which proves that Λ_n^* is not the most economical covering in $\boldsymbol{R}^{24}$.

For dimensions $n = 24k+m$ with $k \geqslant 0$, $0 \leqslant m \leqslant 23$, we define the lattice

$$\Lambda_n = \underbrace{\Lambda \oplus \ldots \oplus \Lambda}_{k \text{ copies}} \oplus \sqrt{\frac{m+1}{m+2}}\, A_m^*.$$

It is easily seen that Λ_n has $R = \sqrt{n/12}$ and

$$d = (m+1)^{-1/2}\{(m+1)/(m+2)\}^{m/2},$$

therefore

$$\theta(\Lambda_n) = V_n \sqrt{m+1}\left(\frac{m+2}{m+1}\right)^{m/2}\left(\frac{n}{12}\right)^{n/2}. \tag{2}$$

Comparison of (1) and (2) shows that, since $(x+1)\{(x+2)/(x+1)\}^{x/2}$ is a monotonically increasing function of x, Λ_n is more economical than A_n^* for all $n \geqslant 24$. Of course for $n \leqslant 23$ Λ_n and A_n^* coincide, and asymptotically they grow at the same rate.

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