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Inclusion operators in Bergman spaces on bounded symmetric domains in C^n

by

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Abstract. Let Z be a sequence of points of a bounded symmetric domain D in C^n and $T_Z^{p,\alpha,D}$ the "inclusion" operator appearing in the problem of interpolation in Bergman spaces. In this paper we give a characterization of sequences $Z \subset D$ for which the operator $T_Z^{p,\alpha,D}$ acting from the Bergman space $A_\alpha^p(D)$ to l^q is bounded.

1. Introduction. Let D be a domain in C^n, K_D its Bergman function and $A_\alpha^p(D)$ the Bergman space consisting of all functions analytic in D and such that

$$\int_D |f(z)|^p K_D(z)^{-\alpha} dv(z) < \infty,$$

where v is the $2n$-dimensional Lebesgue measure on C^n.

For $f \in A_\alpha^p(D)$ and $Z = \{z_m\} \subset D$ define the "inclusion" operator as

$$T_Z^{p,\alpha,D}(f) = \{f(z_m) K_D(z_m)^{-(\alpha+1)/p}\}_m.$$

In this paper we assume that D is biholomorphic to a bounded symmetric domain in C^n and characterize sequences $Z \subset D$ for which the operator $T_Z^{p,\alpha,D}$ is bounded from $A_\alpha^p(D)$ to l^q. Our theorem extends the results of P. J. McKenna [2] and A. Zabulionis [5] obtained for the case of the unit disc, $\alpha = 0$ and $q = p$. We also show that the operator $T_Z^{p,\alpha,D}$ cannot be compact and as a result give a motivation for the choice of the exponent in the definition of the operator.

The proof of the main result uses a modification of a technique originating from McKenna and later used by Zabulionis.

Finally I would like to thank Prof. P. Wojtaszczyk for many stimulating discussions.

2. Notation and definitions. Everywhere in the sequel D will stand for a domain in C^n.

We say that D is *circular* if $tz \in D$ whenever $z \in D$ and $t \in C$, $|t| = 1$, and completely circular if all $t \in C$, $|t| \leq 1$ can be admitted.

If $\Phi\colon D \to C^n$ is a holomorphic mapping, then by Φ' we will denote its complex Jacobi matrix and by $J\Phi = |\det \Phi'|^2$ its real Jacobian.

A mapping $\Phi\colon D \to D_1$ will be called *biholomorphic* if Φ is 1-1 and onto and both Φ and Φ^{-1} are holomorphic. In particular, for a biholomorphic mapping we have $J\Phi(z) \neq 0$ $(z \in D)$. An automorphism of D is a biholomorphic mapping from D onto itself. The group of all such mappings will be denoted by $\mathrm{Aut}(D)$.

A domain D is called *symmetric* if it is homogeneous (i.e., $\mathrm{Aut}(D)$ acts transitively on D) and each point of D is a fixed point of an involution from $\mathrm{Aut}(D)$. (The homogeneity is in fact implied by the second assumption.) Everywhere in the sequel we will additionally assume that a *symmetric domain* means a *domain biholomorphic to a bounded symmetric one.*

$K_D(z, w)$ and $K_D(z) = K_D(z, z)$ will stand for the Bergman kernel and the Bergman function for D. In symmetric domains $K_D(z, w)$ does not vanish (see Lemma 2), and so we can define the weight functions

$$\mu_\alpha^D(z) = K_D(z)^{-\alpha}, \quad s_\alpha^D(z) = K_D(z)^{-(\alpha+1)} \qquad (z \in D)$$

and a function measuring, in a way, the distance between points in D

$$\varrho_D(z, w) = K_D(z) K_D(w) |K_D(z, w)|^{-2} \qquad (z, w \in D).$$

The Bergman norm of a function f is

$$\|f\|_{p,\alpha,D} = \Big(\int_D |f|^p \mu_\alpha^D \, dv\Big)^{1/p}$$

for $p \in (0, \infty)$ and $\alpha \in (\alpha_D, \infty)$, where α_D is the infimum of those α for which the space $A_\alpha^p(D)$ is non-trivial. α_D does not depend on p (cf. Corollary 2).

The inclusion operator becomes

$$T_Z^{p,\alpha,D}(f) = \{f(z_m) s_\alpha^D(z_m)^{1/p}\}_m.$$

We will say that the domain D_1 is *model* for D if

1° D_1 is biholomorphic to D,

2° D_1 is bounded, symmetric and complete circular,

3° $v(D_1) = 1$.

It is well known that every domain which is symmetric (in our sense) is biholomorphic to a Cartesian product of domains called *irreducible Cartan domains*, which are bounded and completely circular. Thus for every symmetric domain there exists a model one. For an exhaustive discussion of homogeneous and symmetric domains see [4].

A distance d on D will be called *invariant* if $\varphi \in \mathrm{Aut}D$ and $z, w \in D$ imply $d(z, w) = d(\varphi(z), \varphi(w))$ and the topology induced by d coincides with the usual topology of D. There are many examples of such distances, e.g., Bergman, Kobayashi or Gleason distances.

For an invariant distance d we put

$$M(d) = \sup\{r\colon \bar{B}_d(z, r) \text{ is compact}\},$$

$M(d)$ obviously does not depend on z and is always positive.

If d is a distance on D and $Z = \{z_m\} \subset D$, then we say that Z is (δ, d)-*separated* if

$$\inf\{d(z_k, z_m)\colon k, m \in N, k \neq m\} \geq \delta.$$

As can be observed there will be many indices to be dealt with; so in situations where there is no danger of confusion some of them will be omitted.

In the sequel, powers of non-vanishing holomorphic functions will often be taken. They should be understood as any holomorphic branch of the power.

3. Main result.

THEOREM. *Let D be a symmetric domain with an invariant distance d. Then the following conditions are equivalent*:

(a) $T_Z^{p,\alpha,D}(A_\alpha^p(D)) \subset l^p$,

(b) *there exists a* $q \in (0, \infty)$ *such that* $T_Z^{p,\alpha,D}(A_\alpha^p(D)) \subset l^q$,

(c) Z *satisfies the following condition*:

$$\exists_{\eta > 0} \sup_k \sum_m \varrho_D(z_k, z_m)^{-\eta} < \infty,$$

(d) *for every* $\delta \in (0, M(d))$, Z *is a finite union of* (δ, d)-*separated subsequences*,

(e) *there exists a* $\delta > 0$ *such that* Z *is a finite union of* (δ, d)-*separated subsequences.*

By the closed graph theorem conditions (a) and (b) of the theorem imply the boundedness of the operators in question.

4. Preliminary lemmas.

LEMMA 1. *Assume that D is a complete circular domain and $G \subset D$ is circular. Then for every function $f\colon D \to R$ plurisubharmonic in D we have*

(1) $$f(0) \leq v(G)^{-1} \int_G f(z)\, dv(z).$$

The proof is done by a simple use of the Fubini theorem. As a corollary we obtain that if $f\colon D \to C$ is pluriharmonic, then we have equality in (1).

Below we list the main properties of the Bergman kernel K_D.

LEMMA 2.

(i) $K(z, w) = \overline{K(w, z)}$ *and* $K(z, w)$ *is holomorphic in* z *in* D;

(ii) *for every* $f \in A_0^2(D)$ *and* $z \in D$ *we have*

$$f(z) = \int_D f(w) K(z, w) \, dv(w);$$

(iii) *if* $\Phi\colon D \to D_1$ *is biholomorphic, then for all* $z, w \in D$

$$K_D(z, w) = K_{D_1}(\Phi(z), \Phi(w)) \det \Phi'(z) \overline{\det \Phi'(w)},$$

and in particular

$$K_D(z) = K_{D_1}(\Phi(z)) J\Phi(z);$$

(iv) *if* D *is model, then for every* $z \in D$ *we have* $K(z, 0) = 1$, $K(z) \geq 1$ *and* $K(z) = 1$ *iff* $z = 0$;

(v) *if* D *is symmetric, then for* $z, w \in D$, $K(z, w) \neq 0$.

Proof. (i)–(iii) are standard and can be found in most texts on several complex variables. In (iv), to show that $K(z, w) = 1$ we use the remark following Lemma 1 and (ii). $K(z) \geq 1$ follows by the extremal property of K, according to which $|f(z)|^2 \leq K(z) \|f\|_{2,0}$. Substitution $f = \boldsymbol{I}$ shows what we need. $K(z)$ is strictly plurisubharmonic, and so if it attains its minimum, then it is at exactly one point, and we have shown that 0 is such a point. (v) follows from (iv) by (iii) and the homogeneity of D.

COROLLARY 1. *If* D *is symmetric and* $\Phi\colon D \to D_1$ — *biholomorphic, then for* $z, w \in D$

$$\varrho_D(z, w) = \varrho_{D_1}(\Phi(z), \Phi(w)),$$

and if D *is model, then*

$$\varrho_D(z, 0) = K_D(z).$$

LEMMA 3. *Let* $\Phi\colon D \to D_1$ *be a biholomorphic mapping. Then for every* $p \in (0, \infty)$, $\alpha \in (\alpha_{D_1}, \infty)$ *there exist operators* $U_\Phi^{p,\alpha}\colon A_\alpha^p(D_1) \to A_\alpha^p(D)$ *and* $V_\Phi^{p,\alpha}\colon \boldsymbol{R}^N \to \boldsymbol{R}^N$ *such that* $U_\Phi^{p,\alpha}$ *is an isometry of* $A_\alpha^p(D_1)$ *onto* $A_\alpha^p(D)$, $V_\Phi^{p,\alpha}|_{l^q}$ — *an isometry of* l^q *onto itself for all* $q > 0$ *and if* $Z \subset D$, *then*

$$T_{\Phi(Z)}^{p,\alpha,D_1} = V_\Phi^{p,\alpha} \circ T_Z^{p,\alpha,D} \circ U_\Phi^{p,\alpha}.$$

Proof. Put

$$U_\Phi^{p,\alpha}(f) = f \circ \Phi (\det \Phi')^{2(\alpha+1)/p}.$$

It is easily checked that $U_\Phi^{p,\alpha}$ is an isometry and

$$(U_\Phi^{p,\alpha}(f))(z) s_\alpha^D(z) = f(\Phi(z)) s_\alpha^{D_1}(\Phi(z)) \left(\frac{\det \Phi'(z)}{|\det \Phi'(z)|}\right)^{2(\alpha+1)/p}.$$

Hence we only have to define

$$V_\Phi^{p,\alpha}(\{x_m\}) = \left\{\left(\frac{\overline{\det \Phi'(z_m)}}{\det \Phi'(z_m)}\right)^{2(\alpha+1)/p} x_m\right\}_m.$$

COROLLARY 2.

(i) α_D *is a biholomorphic invariant,*

(ii) $\alpha_D \leq 0$,

(iii) *if* D *is model, then for* $\alpha > \alpha_D$ *the space* $A_\alpha^p(D)$ *contains the constants.*

Proof. (i) and (ii) are evident, and so we prove only (iii). Suppose $A_\alpha^p(D)$ does not contain the constants and $f \in A_\alpha^p(D)$. Then

$$\|f\|_{p,\alpha,D}^p = \int_D |f(z)|^p K(z)^{-\alpha} dv(z) = 1/2\pi \int_D \int_0^{2\pi} |f(e^{it} z)|^p K(e^{it} z)^{-\alpha} dt \, dv(z)$$

$$= 1/2\pi \int_D \int_0^{2\pi} |f(e^{it} z)|^p K(z)^{-\alpha} dt \, dv(z) \geq |f(0)|^p \cdot \int_D K(z)^{-\alpha} dv(z).$$

The integral in the last expression is equal to the p-th power of the norm of the function $\boldsymbol{I}$ in $A_\alpha^p(D)$ and thus is infinite. Hence we must have $f(0) = 0$ for every $f \in A_\alpha^p(D)$. But if we take any $\varphi \in \mathrm{Aut}(D)$, then $(U_\varphi f)(0) = 0$ and so $f(\varphi(0)) = 0$. By homogeneity it follows that $f = 0$.

We will now define auxiliary functions $\psi_{w,p,\alpha}^D$ for a model domain D. We put

$$\psi_{w,p,\alpha}^D(z) = \left(\frac{K(z, w)^2}{K(w)}\right)^{(\alpha+1)/p}.$$

Then we have

LEMMA 4.

(i) $|\psi_w(z)| s_\alpha(z)^{1/p} = \varrho(z, w)^{-(\alpha+1)/p}$,

(ii) $\|\psi_w\|_{p,\alpha,D} = \|\boldsymbol{I}\|_{1,\alpha,D}^{1/p}$.

Proof. (i) is just an easy verification. For (ii) observe that if $\varphi \in \mathrm{Aut}(D)$ and $\varphi(w) = 0$, then

$$|\psi_w(z)|^p \mu_\alpha(z) = J\varphi(z) \mu_\alpha(\varphi(z)),$$

which gives the desired equality. By Corollary 2 (iii) the right-hand side is finite.

LEMMA 5. *Let* $H\colon \boldsymbol{N} \times \boldsymbol{N} \to \boldsymbol{R}_+$ *be a function satisfying*

$$\sup_k \sum_m H(k, m) = M < \infty.$$

Then for every $\beta > 0$ *there exist subsets* $N_i \subset \boldsymbol{N}$ $(i = 1, \ldots, L)$ *such that* $\boldsymbol{N} = \bigcup_{i=1}^L N_i$ *and*

$$(k, m \in N_i, \; k \neq m) \Rightarrow H(k, m) < \beta. \tag{2}$$

Proof. Choose $L \in \boldsymbol{N}$ so that $L\beta > M$. We will assign natural numbers to the sets N_i by induction. For $m \leq L$ we put $m \in N_m$.

Now assume that we have made the assignment up to k, so that (2) holds. Then, by the choice of L, there exists an i_0 such that for $m \leqslant k$, $m \in N_{i_0}$, we have $H(k+1, m) < \beta$. Thus we assign $k+1$ to N_{i_0}.

5. Proof of the main result. First we will reduce the problem to the case of model domains.

If D is symmetric and d an invariant distance on D, then we choose a model domain D_1 for D and $\Phi\colon D \to D_1$ biholomorphic. For z, $w \in D_1$ we define

$$d_1(z, w) = d(\Phi^{-1}(z), \Phi^{-1}(w)).$$

Then d_1 is an invariant distance on D_1 and $M(d_1) = M(d)$. We have also shown that α_D, ϱ_D are invariant under biholomorphic mappings. If we put the above remarks together with Lemma 3, we see that the theorem for (D, d) can be derived from the theorem for (D_1, d_1).

Let us assume that D is model.

(a $\Rightarrow$ b) and (d $\Rightarrow$ e) are evident.

(b $\Rightarrow$ c) As we have already remarked, (b) implies the boundedness of $T_Z^{p,\alpha,D}\colon A_\alpha^p \to l^q$. Then, by Lemma 4 (ii), we have

$$\|T_Z^{p,\alpha}(\psi_w)\|_q^q \leqslant C \|I\|_{1,\alpha,D}^{q/p}.$$

On the other hand, by Lemma 4 (i),

$$\|T_Z^{p,\alpha}(\psi_w)\|_q^q = \sum_m \varrho(w, z_m)^{-(\alpha+1)q/p}.$$

Thus, after substituting $w = z_k$, we obtain the desired estimate.

(c $\Rightarrow$ d) Fix $z_0 \in D$ and let $\delta < M(d)$. Then $\bar{B}_d(z_0, \delta)$ is compact. Since $\varrho(z, z_0)$ is a continuous function in D, there exists a constant C such that if $z \in \bar{B}_d(z_0, \delta)$, then $\varrho(z, z_0) \leqslant C$. Because of the invariance of d and ϱ we get

$$\varrho(z, w) > C \Rightarrow d(z, w) > \delta. \tag{3}$$

Now take $H(k, m) = \varrho(z_k, z_m)^{-\eta}$. By Lemma 5, we split Z into subsequences Z_i such that

$$(z_k, z_m \in Z_i,\ k \neq m) \Rightarrow \varrho(z_k, z_m)^{-\eta} < C^{-\eta};$$

thus, by (3), $d(z_k, z_m) > \delta$.

(e $\Rightarrow$ a) Let B_r stand for $B_d(0, r)$. It is easily seen that B_r is circular. Fix $w \in D$ and take $\varphi \in \operatorname{Aut}(D)$ such that $\varphi(0) = w$. Then

$$|f(w)|^p s_\alpha(w) = |f(w)|^p K(w, w)^{-(\alpha+1)} = |f \circ \varphi|^p(0)\, |K(\cdot, w)^{-2} \circ \varphi|^{\alpha+1}(0)\, K(w)^{\alpha+1}.$$

Since the function on the right-hand side above is plurisubharmonic, we get by Lemma 1

$$|f(w)|^p s_\alpha(w) \leqslant \nu(B_r)^{-1} \int_{B_r} |f|^p \circ \varphi(z) \left|K(\varphi(z), w)\right|^{-2(\alpha+1)} K(w)^{\alpha+1}\, d\nu(z).$$

Further, observe that

$$\begin{aligned}\left|K(\varphi(z), w)\right|^{-2(\alpha+1)} K(w)^{\alpha+1} &= \left|\frac{K(w)\, K(\varphi(z))}{K(\varphi(z), w)^2}\right|^{\alpha+1} K(\varphi(z))^{-(\alpha+1)}\\ &= K(z)^{\alpha+1} K(\varphi(z))^{-1} \mu_\alpha(\varphi(z))\\ &= \frac{K(z)}{K(\varphi(z))} K(z)^\alpha \mu_\alpha(\varphi(z)) = J\varphi(z)\, \mu_\alpha(\varphi(z))\, K(z)^\alpha.\end{aligned}$$

Thus we have

$$|f(w)|^p s_\alpha(w) \leqslant \nu(B_r)^{-1} \sup\{K(z)^\alpha\colon z \in B_r\} \int_{B_d(w,r)} |f|^p \mu_\alpha\, d\nu. \tag{4}$$

Clearly it is enough to prove (a) under the assumption that Z is (δ, d)-separated. In this case, taking $r \leqslant \min\{\delta/2, M(d)\}$, we ensure that the balls $B_d(z_m, r)$ are pairwise disjoint and the supremum apearing in the right-hand side of (4) is finite. Hence, substituting $w = z_m$ in (4) and summing over m, we get a bound on the l^p-norm of $T_Z^{p,\alpha,D}(f)$.

6. Final remarks. R. Rochberg proved in [3] that if Z is a (δ, d_0)-separated sequence in a symmetric domain D, d_0 being the Bergman distance in D, and δ is large enough, then the operator $T_Z^{p,\alpha,D}\colon A_\alpha^p(D) \to l^p$ has a continuous right inverse for all $p \in (0, \infty)$. Looking a little more carefully at the proof of our theorem, we can prove Rochberg's result in the case $p \leqslant 1$ and with any invariant distance in place of d_0.

Using both Rochberg's theorem and our own, we can easily obtain the following result, which, for $\alpha = 0$ and D equal to either the unit ball or the polydisc, was proved by E. Amar in [1].

Corollary 3. *Suppose $Z \subset D$ is a sequence separated with respect to some invariant distance. Then Z can be split into a finite number of subsequences Z_i with the property that $T_{Z_i}^{p,\alpha,D}$ is right invertible.*

Proof. If Z is separated with respect to some invariant distance then for every $\delta < M(d_0)$, Z is a finite union of (δ, d_0)-separated subsequences. Since $M(d_0) = \infty$, we have no restriction on δ and so may use the theorem of Rochberg.

As another application of our theorem we get a result on the boundary behaviour of the Bergman function.

Corollary 4. *If D is a bounded symmetric domain and $Z = \{z_m\} \subset D$ a sequence with no cluster points in D, then*

$$\lim_{m \to \infty} K_D(z_m) = \infty. \tag{5}$$

This result is known, but since it will be needed in the sequel, we sketch the proof.

Take any invariant distance d in D. It is easily seen that d is complete. Thus every infinite subsequence of Z contains an infinite d-separated subsequence. For such a subsubsequence Z' we have, by the theorem, $T_{Z'}^{1,0,D}(I) \in l^1$ or $\sum_m K_D(z'_m)^{-1} < \infty$, which proves (5).

Remark 1. The operator $T_Z^{p,\alpha,D}\colon A_\alpha^p(D) \to l^q$ is compact if and only if Z is finite.

Proof. First observe that, again, Lemma 3 allows us to reduce the proof to the case of a model domain.

By Lemma 4 (i), we have

$$|\psi_w(w)s_\alpha(w)^{1/p} - \psi_z(w)s_\alpha(w)^{1/p}| \geqslant 1 - \varrho(z, w)^{-(\alpha+1)/p}. \tag{6}$$

Suppose Z is infinite and $T_Z^{p,\alpha,D}(A_\alpha^p) \subset l^q$. Then (c) of the theorem holds. As in the proof of (c $\Rightarrow$ d), we can find an infinite subsequence Z' of Z such that if $z_k, z_m \in Z'$ and $k \neq m$, then $\varrho(z_k, z_m)^{-\eta} < \beta < 1$. And thus

$$1 - \varrho(z_k, z_m)^{-(\alpha+1)/p} > \gamma > 0. \tag{7}$$

From (6) and (7) we infer that if z_k, z_m are as before then

$$\|T_Z^{p,\alpha,D}(\psi_{z_k} - \psi_{z_m})\|_q > \gamma$$

and, since the norms of ψ_{z_k} are all equal, $T_Z^{p,\alpha,D}$ cannot be compact.

Remark 2. As an application of the results obtained we would like to give a motivation for the choice of the exponent in the definition of the operator $T_Z^{p,\alpha,D}$. For this purpose define

$$W_Z^{\beta,D}(f) = \{f(z_m)K(z_m)^{-\beta}\}_m.$$

Suppose Z is an infinite sequence in D; then the following hold:

(i) if $W_Z^\beta(A_\alpha^p) \subset l^q$ for some $q \in (0, \infty)$, then $\beta \geqslant (\alpha+1)/p$,

(ii) if $W_Z^\beta(A_\alpha^p) \supset l^q$ for some $q \in (0, \infty)$, then $\beta \leqslant (\alpha+1)/p$.

Thus if we want an operator $W_Z^{\beta,D}$ whose image could be exactly equal to some l^q, and this is what we are usually looking for, then $T_Z^{p,\alpha,D}$ is the only choice, and in that case q must be equal to p.

For the proof we may assume as usual that D is model.

Let $\gamma = (\alpha+1)/p - \beta$ and define an operator $S_Z^\gamma\colon \boldsymbol{R}^N \to \boldsymbol{R}^N$ as

$$S_Z^\gamma(\{x_m\}) = \{K(z_m)^{-\gamma}x_m\}_m.$$

Then

$$T_Z^{p,\alpha} = S_Z^\gamma \circ W_Z^\beta.$$

To prove (i) suppose the contrary, i.e., $\gamma > 0$. It is evident that Z cannot have any cluster points in D; thus, by Corollary 4, $K(z_m) \to \infty$ and so S_Z considered as acting from l^q to l^q is compact. But since, by Remark 1, $T_Z^{p,\alpha}$ is not compact, we get a contradiction.

Now suppose $\gamma < 0$. If $W_Z^\beta(A_\alpha^p) \supset l^q$ and $Z' \subset Z$, then $W_{Z'}^\beta(A_\alpha^p) \supset l^q$ as well. As before, one can see that Z has no cluster points in D, and so, as in the proof of Corollary 4, we can find an infinite subsequence Z' of Z separated with respect to some invariant distance. Then, by the theorem, $T_{Z'}^{p,\alpha}(A_\alpha^p) \subset l^p$, but this means that $S_{Z'}^{-\gamma}(l^p) \supset l^q$, which is impossible and this contradiction proves (ii).

Finally I would like to remark that one might regard the Bergman spaces $A_\alpha^p(D)$, D as a Cartesian product of domains $D_i \subset C^{n_i}$ and $\boldsymbol{\alpha}$ as a multiindex, with the norm

$$\|f\|_{p,\alpha,D} = \Big(\int_D |f(z_1, \ldots, z_l)|^p \prod_{i=1}^{l} K_{D_i}(z_i)^{-\alpha_i}\, dv(z_1, \ldots, z_l)\Big)^{1/p}.$$

One also gets a natural generalization of the "inclusion" operator.

All the results presented, except Corollary 4, can be extended without much effort to this setting. Corollary 4 can also be extended under the assumption $p \leqslant 1$, where we can use our proof of Rochberg's theorem. It seems to be certain that the whole of Rochberg's result can be extended as well, and so Corollary 4 should hold in full generality.

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