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DEPARTMENT OF MATHEMATICS
FACULTY OF EDUCATION
YAMAGUCHI UNIVERSITY
Yamaguchi, 753
Japan

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On the relationships between shape properties of subcompacta of S^n and homotopy properties of their complements

by

Sławomir Nowak (Warszawa)

Abstract. Taking for the set of morphisms from X to Y the direct limit of the sets of all homotopy classes or all weak homotopy classes or all shapping between the n-fold suspension of X and Y we obtain (respectively) the stable homotopy category $\mathscr{S}$ or the stable weak homotopy category $\mathscr{S}w\mathcal{O}_n$ of open subsets of S^n or the stable shape category $\mathscr{S}\mathscr{S}h_n$ of subcompacta of S^n.

We prove that there exists an isomorphism $\mathscr{D}_n\colon \mathscr{S}\mathscr{S}h_n \to \mathscr{S}w\mathcal{O}_n$ such that $\mathscr{D}_n(X) = S^n\setminus X$. If we limit ourselves to movable compacta, then $\mathscr{S}w\mathcal{O}_n$ can be replaced by a suitable full subcategory of $\mathscr{S}$. These facts generalize the classical Spanier–Whitehead duality.

Applications to the ordinary shape theory are also given. In particular, if $1<k\leqslant n$ and $X\subset S^n$ is an approximatively 1-connected continuum, then $\mathrm{Sh}(X) = \mathrm{Sh}(S^k)$ iff $S^n\setminus X$ and $S^n\setminus S^k$ are isomorphic in $\mathscr{S}$.

The relationships between shape properties of closed subsets of S^n and properties of their complements have been studied by many mathematicians ([Sh]). If $X, Y\subset S^n$ are compacta with sufficiently large codimension and X, Y satisfy some conditions concerning the way in which they are embedded in S^n, then $\mathrm{Sh}(X) = \mathrm{Sh}(Y)$ iff $S^n\setminus X$ and $S^n\setminus Y$ are homeomorphic ([Sh]). In the case when $1\neq k\leqslant n\geqslant 5$ and $Y = S^k$ (see [R]), or more generally, Y is an S^k-like continuum (see [V]), the assumption concerning the codimension of X and Y may be eliminated.

We begin with examples. They will illustrate and motivate some of the problems which will be discussed here.

The Alexander duality theorem states that the Čech cohomology groups (which belong to the most important invariants of the shape theory) of a closed subset X of S^n are uniquely determinated by the topological (homotopical) type of $S^n\setminus X$. Since the second suspension $\Sigma^2(X)$ of a compactum $X\neq\emptyset$ is an approximatively 1-connected continuum, $\mathrm{Fd}(\Sigma^2(X)) = \mathrm{Fd}(\Sigma^2(Y))$ if X and Y are subcontinua of S^n with homotopically equivalent complements ([N], p. 35).

Similarly, if $\Sigma^2(X)\in\mathrm{FANR}$ and $S^n\setminus X$ is homotopically equivalent to $S^n\setminus Y$ [G–L], then $\Sigma^2(Y)\in\mathrm{FANR}$.

On the other hand, if $S^3\setminus K$ is a 3-cube with a knotted channel joining two opposite 2-dimensional faces, then $\mathrm{Fd}(K) = 2$ and $S^3 \setminus K$ is homeomorphic to $S^3 \setminus S^1$.

In the last section we construct also a movable continuum $X \subset S^3$ and a non-movable continuum $Y \subset S^3$ with homeomorphic complements.

With these facts in mind we are naturally facing the problem of finding appropriate apparatus for distinguishing shape properties (α) such that $X \in (\alpha)$ iff $S^n \setminus X$ is homeomorphic to $S^n \setminus Y$ (or $S^n \setminus X$ is homotopy equivalent to $S^n \setminus Y$) and $Y \in (\alpha)$, where X and Y are arbitrary compacta lying in S^n.

We consider the categories $\mathcal{S}\mathcal{S}h_n$ and $\mathcal{S}w\mathcal{O}_n$ of all compacta lying in S^n and their complements with morphisms which are (respectively) the direct limit of sequences

$$\mathcal{S}\mathcal{S}h_n(X, Y) = \underrightarrow{\lim}\{\mathrm{Sh}(X, Y) \to \mathrm{Sh}(\Sigma(X), \Sigma(Y)) \to \mathrm{Sh}(\Sigma^2(X), \Sigma^2(Y)) \to \ldots\}$$

and

$$\mathcal{S}w\mathcal{O}_n(U, V) = \{U, V\}_w = \underrightarrow{\lim}\{[U, V]_w \to \\ \to [\Sigma(U), \Sigma(V)]_w \to [\Sigma^2(U), \Sigma^2(V)]_w \to \ldots\}$$

under iterated suspensions, where $[X, Y]_w$ denotes the set of all weak homotopy classes from X to Y ([Sw] p. 162).

Replacing in the second formula the sets of weak homotopy classes by the sets of homotopy classes, one gets the *stable homotopy category*.

We say that compacta $X, Y \subset S^n$ have the same *stable shape* and we write $\mathrm{SSh}(X) = \mathrm{SSh}(Y)$ iff they are isomorphic in the *stable shape category* $\mathcal{S}\mathcal{S}h_n$. This relation is weaker than the relation of the equality of shapes. All invariants of the shape theory are invariants of the stable shape theory.

Analogously, the relation of stable weak homotopy equivalence is weaker than the relation of homotopy equivalence and all invariants of the weak stable homotopy theory are invariants of the homotopy theory.

We prove that there is an isomorphism $\mathcal{D}_n\colon \mathcal{S}\mathcal{S}h_n \to \mathcal{S}w\mathcal{O}_n$ such that $\mathcal{D}_n(X) = S^n \setminus X$.

We also prove that the full subcategory of $\mathcal{S}\mathcal{S}h_n$ whose objects are movable compacta is isomorphic to the stable homotopy category of their complements.

These results generalize the classical Spanier–Whitehead duality theorem ([S–W]) and correspond to the second Chapman theorem ([B$_2$] p. 314, [M–S] p. 262, [Sh] p. 164) concerning the existence of a category isomorphism of the shape category of compact Z-sets in Q onto the category of their complements.

From the main theorems it follows that $\mathrm{SSh}(X) = \mathrm{SSh}(Y)$ iff $S^n \setminus X$ and $S^n \setminus Y$ are stably homotopically equivalent. Clearly, if shape property (α) is an invariant of the stable shape theory and $X, Y \subset S^n$ are compacta with homotopically equivalent complements, then $X \in (\alpha)$ iff $Y \in (\alpha)$.

For example, movability is not an invariant of the stable shape theory, but having a movable k-fold suspension for almost all k is an invariant of this theory.

The paper [H] and the last chapter of [D–P] are devoted to the generalization of the Spanier–Whitehead duality to the shape theory, but the authors fix their attention on the other problems.

Using our machinery, we also prove that $\mathrm{Sh}(X) = \mathrm{Sh}(Y)$ iff $S^n \setminus X$ is stable homotopy equivalent to $S^n \setminus Y$, where X is an arbitrary approximatively 1-connected continuum and Y is homeomorphic to the inverse limit of the inverse sequence of the wedges of k-spheres with $k \neq 1$.

1. Preliminaries. The Hilbert space l^2 consists of all real sequences $(x_1, x_2, \ldots)$ with $\Sigma x_i^2 < \infty$ and E^n consists of all points $(x_1, x_2, \ldots)$ of l^2 such that $x_k = 0$ for $k > n$. It follows that $E^n \subset E^m$ for $m \geqslant n$. The point $(x_1, x_2, \ldots, x_n, 0, \ldots)$ of E^n will be denoted by $(x_1, x_2, \ldots, x_n)$.

The geometric n-sphere in E^{n+1} consisting of all points $(x_1, x_2, \ldots, x_{n+1}) \in E^{n+1}$ with $x_1^2 + x_2^2 + \ldots + x_{n+1}^2 = 1$, is homeomorphic to the set

$$S^n = \{(x_1, x_2, \ldots, x_{n+1}) \in E^{n+1}\colon |x_1| + |x_2| + \ldots + |x_{n+1}| = 1\}.$$

By the *suspension* $\Sigma(X)$ of a subset X of S^n we understand the union of al segments joining points of X with the poles $(0, 0, \ldots, 0, 1) = a$ and $(0, 0, \ldots, 0, -1) = b$ of S^{n+1} and the set $\{a, b\}$. In particular, $\Sigma(\emptyset) = \{a, b\}$.

If X is a compactum then this definition is equivalent to the standard one. If $X \subset S^n$ is an absolute neighborhood retract for metric spaces and $x_0 \in X$, then $(\Sigma(X), x_0)$ is homotopically equivalent to the reduced suspension of (X, x_0).

One can also define $\Sigma^k(X)$, $\Sigma(f)$ and $\Sigma^k(f)$ for every $k \geqslant 1$ and every map $f\colon X \to Y$ in a way analogous to the classical case.

Suppose that $A \subset S^n$ and $0 < r < 1$. Then we denote by $\Sigma(A, r)$ the union of $\Sigma(A)$ and the set consisting of all points $(x_1, x_2, \ldots, x_{n+2}) \in S^{n+1} \setminus \Sigma(A)$ with $|x_{n+2}| > r$. By $S(A, r)$ we denote the closure of $\Sigma(A, r)$.

We shall need a lemma.

(1.1) LEMMA. *Suppose that $X \subset Y \subset S^n$ are compacta, $x_0 \in X$ and $y_0 \in S^n \setminus Y$. Then for $0 < r \leqslant s < 1$ we have commutative diagrams*

$$\begin{array}{ccc} (S^n \setminus Y, y_0) & \to & (S^{n+1} \setminus \Sigma(Y), y_0) \\ \downarrow & & \downarrow \\ (S^n \setminus X, y_0) & \to & (S^{n+1} \setminus \Sigma(X), y_0) \end{array} \qquad \begin{array}{ccc} (\Sigma(X), x_0) & \to & (S(X, S), x_0) \\ \downarrow & & \downarrow \\ (\Sigma(Y), x_0) & \to & (S(Y, r), x_0) \end{array}$$

$$\begin{array}{ccccc} (\Sigma(S^n \setminus Y), y_0) & \to & (\Sigma(S^n \setminus Y, s), y_0) & \to & (S^{n+1} \setminus Y, y_0) \\ \downarrow & & \downarrow & & \downarrow \\ (\Sigma(S^n \setminus X), y_0) & \to & (\Sigma(S^n \setminus Y, r), y_0) & \to & (S^{n+1} \setminus X, y_0) \end{array}$$

where all arrows are homotopy equivalences induced by the inclusions.

Proof. In order to prove Lemma (1.1) it suffices to observe that inclusions induce isomorphisms of homotopy groups.

Suppose that $X, Y \subset S^n$ are absolute neighborhood retracts for metric spaces and $f, g\colon (\Sigma^k(X), x_0) \to (\Sigma^k(Y), y_0)$, where $k \geqslant 2$, $x_0 \in X$ and $y_0 \in Y$.

Consider the map $h\colon (\Sigma^k(X), x_0) \to (\Sigma^k(Y), y_0)$ defined by the formula

$$h(x_1, x_2, \ldots, x_{n+k+1}) =$$

$$= \begin{cases} \alpha f(2x_1, \ldots, 2x_{n+k}, 2x_{n+k+1}+1) & \text{for } x \in \Sigma^k(X) \text{ with } -1 \leqslant x_{n+k+1} \leqslant \frac{1}{2} \\ \alpha f\left(\dfrac{-2x_1 x_{n+k+1}}{1+x_{n+k+1}}, \ldots, \dfrac{-2x_{n+k}x_{n+k+1}}{1+x_{n+k+1}},\ 2x_{n+k+1}+1\right) & \\ & \text{for } x \in \Sigma^k(X) \text{ with } -\frac{1}{2} \leqslant x_{n+k+1} \leqslant 0 \\ \alpha g\left(\dfrac{2x_1 x_{n+k+1}}{1-x_{n+k+1}}, \ldots, \dfrac{2x_{n+k}x_{n+k+1}}{1-x_{n+k+1}},\ 2x_{n+k+1}+1\right) & \\ & \text{for } x \in \Sigma^k(X) \text{ with } 0 \leqslant x_{n+k+1} \leqslant \frac{1}{2} \\ \alpha g(2x_1, \ldots, 2x_{n+k}, 2x_{n+k+1}-1) & \text{for } x \in \Sigma^k(X) \text{ with } \frac{1}{2} \leqslant x_{n+k+1} \leqslant 1 \end{cases}$$

where $\alpha\colon (\Sigma^k(Y), y_0) \to (\Sigma^k(Y), y_0)$ is a map such that $\alpha \simeq \mathrm{id}_Y$ and $\alpha(\Sigma^k(\{y_0\})) = \{y_0\}$.

One can prove that the homotopy class of h depends only on the homotopy classes of f and g. The addition defined by $[f]+[g] = [h]$ makes the set $[(\Sigma^k(X), x_0)(\Sigma^k(Y), y_0)]$ an abelian group.

If $X, Y \in \mathrm{ARN}(\mathfrak{M})$ are simply connected, then the forgetful functor obtained by suppressing base points induces an isomorphism of the set of all pointed homotopy classes $[(X, x_0), (Y, y_0)]$ onto the set $[X, Y]$ of all free homotopy classes ([Sp] p. 383).

Therefore we can identify $[(\Sigma^k(X), x_0), (\Sigma^k(Y), y_0)]$ with $[\Sigma^k(X), \Sigma^k(Y)]$ for $k \geqslant 2$.

The operation Σ induces a function of $[X, Y]$ into $[\Sigma(X), \Sigma(Y)]$. It will also be denoted by Σ. If $k \geqslant 2$, then $\Sigma\colon [\Sigma^k(X), \Sigma^k(Y)] \to [\Sigma^{k+1}(X), \Sigma^{k+1}(Y)]$ is a homomorphism.

The *suspension category* $\mathscr{S}$ was introduced by E. H. Spanier and J. H. C. Whitehead in 1953. We consider the full subcategory $\mathscr{S}_n$ of $\mathscr{S}$ whose objects are $X \subset S^n$ with $X \in \mathrm{ANR}(\mathfrak{M})$. The set of morphisms $\mathscr{S}_n(X, Y) = \{X, Y\}$ is the direct limit of the sequence

$$[X, Y] \xrightarrow{\Sigma} [\Sigma(X), \Sigma(Y)] \xrightarrow{\Sigma} [\Sigma^2(X), \Sigma^2(Y)] \xrightarrow{\Sigma} \ldots$$

$\mathscr{S}_n$ is a preadditive category (this means that $\{X, Y\}$ is an abelian group, the operation $\{Y, Z\} \times \{X, Y\} \to \{X, Z\}$ of composition is biadditive and $\mathscr{S}_n$ has zero objects). If $f\colon \Sigma^k(X) \to \Sigma^k(Y)$ is a map, then $\{f\}$ will denote the corresponding element of $\{X, Y\}$. Elements of $\{X, Y\}$ are called *stable homotopy classes*.

The following categories are also used in this paper:

$\mathscr{HP}ol_n$ — the category of subpolyhedra of S^n and homotopy classes,

$\mathscr{SP}ol_n$ — the category of subpolyhedra of S^n and stable homotopy classes,

$\mathscr{HCP}ol_n$ — the category of complements of subpolyhedra of S^n and homotopy classes,

$\mathscr{SCP}ol_n$ — the category of complements of subpolyhedra of S^n and stable homotopy classes.

2. Stable categories of direct and inverse systems. If $\mathscr{C}$ is a category, we denote by $\mathrm{Inj}\mathscr{C}$ ([E–H] p. 8) the category $(\mathrm{Pro}^\circ)^\circ$ of direct systems over $\mathscr{C}$. If $\underline{X} = \{X_\alpha, p_\alpha^{\alpha'}, A\}$ and $\underline{Y} = \{Y_\beta, q_\beta^{\beta'}, B\}$ are objects of $\mathrm{Inj}\mathscr{C}$, then

$$\mathrm{Inj}\mathscr{C}(\underline{X}, \underline{Y}) = \varprojlim_{\alpha} \varinjlim_{\beta} \mathscr{C}(X_\alpha, Y_\beta).$$

Consider for every compactum $X \subset S^n$ (or respectively, for every open subset G of S^n, the inverse systems $\mathscr{H}_n(X) = \{X_\alpha, [i_\alpha^{\alpha'}], A\} \in \mathrm{ObPro}\mathscr{HCP}ol_n$ and $\mathscr{S}_n(X) = \{X_\alpha, \{i_\alpha^{\alpha'}\}, A\} \in \mathrm{ObPro}\mathscr{SCP}ol_n$ (the direct systems $\mathscr{H}_n(G) = \{G_\alpha, [j_\alpha^{\alpha'}], A\} \in \mathrm{ObInj}\mathscr{HP}ol_n$ and $\mathscr{S}_n(G) = \{G_\alpha, \{j_\alpha^{\alpha'}\}, A\} \in \mathrm{ObInj}\mathscr{SP}ol_n$), where A is the family of all neighborhoods $X_\alpha \in \mathrm{Ob}\mathscr{HCP}ol_n = \mathrm{Ob}\mathscr{SCP}ol_n$ of X (A is the family of all compact polyhedra $G_\alpha \subset G$) ordered by inclusion and $i_\alpha^{\alpha'}\colon X_{\alpha'} \to X_\alpha$ ($j_\alpha^{\alpha'}\colon G_\alpha \to G_{\alpha'}$) are inclusions for all α, α' with $\alpha \leqslant \alpha'$).

If $X, Y \subset S^n$ are compacta then there is a canonical bijection of the set of all shape morphisms $\mathscr{S}h(X, Y)$ onto the set $\mathrm{Pro}\mathscr{HCP}ol_n(\mathscr{H}_n(X), \mathscr{H}_n(Y))$.

A. Dold and D. Puppe ([D–P] p. 100) and H. W. Henn ([He] p. 331) have introduced the *stable shape category* $\mathscr{SS}h$. $\mathscr{SS}h$ is a preabelian category. If $X, Y \subset S^n$ are compacta then the group

$$\mathrm{Pro}\mathscr{SCP}ol_n(\mathscr{S}_n(X), \mathscr{S}_n(Y)) = \varprojlim_{\alpha} \varinjlim_{\beta} \{X_\alpha, Y_\beta^{\bullet}\} \cong \mathscr{SS}h(X, Y).$$

Yu. T. Lisica has introduced ([L] p. 300) the *coshape category* $\mathscr{C}o\mathscr{S}h$ and the *coshape functor* from the homotopy category to the coshape category. If U and V are open subsets of S^n then there is a canonical bijection of the set $\mathscr{C}o\mathscr{S}h(U, V)$ onto the set $\mathrm{Inj}\mathscr{HP}ol_n(\mathscr{H}_n(U), \mathscr{H}_n(V))$.

We can define in the same way as before the *stable coshape category* $\mathscr{SC}o\mathscr{S}h$, which is a preabelian category. If U and V are open subsets of S^n, then the abelian group $\mathscr{SC}o\mathscr{S}h(U, V)$ is canonically isomorphic to the group

$$\mathrm{Inj}\mathscr{SP}ol_n(\mathscr{S}_n(U), \mathscr{S}_n(V)).$$

Using the fact that cofinal systems in $\mathrm{Pro}\mathscr{C}$ and $\mathrm{Inj}\mathscr{C}$ are isomorphic, Lemma (1.1), the suspension theorem ([Sp] p. 458) and the theorem which states that $\mathrm{Fd}(X) \leqslant n-1$ for every closed subset of S^n, one can prove the following propositions.

(2.1) Proposition. *Suppose that X and Y are subcompacta of S^n. Then $\mathscr{S}h(\Sigma^k(X), \Sigma^k(Y))$ is an abelian group for $k \geqslant 2$ and $\mathscr{SS}h(X, Y)$ is canonically isomorphic to $\mathscr{S}h(\Sigma^k(X), \Sigma^k(Y))$, where $k \geqslant \max(2, n)$ for nonempty X and Y and $k \geqslant \max(3, n)$ in the general case.*

(2.2) PROPOSITION. *Suppose that U and V are open subsets of S^n. Then $\mathscr{C}o\mathscr{S}h(\Sigma^k(U), \Sigma^k(V))$ is an abelian group for $k \geq 2$ and $\mathscr{S}\mathscr{C}o\mathscr{S}h(U, V)$ is canonically isomorphic to $\mathscr{C}o\mathscr{S}h(\Sigma^k(U), \Sigma^k(V))$ for $k \geq \max(2, n)$ for nonempty U and V and $k \geq \max(3, n)$ in the general case.*

3. Stable weak category of open subsets of S^n. Two maps $f, g\colon X \to Y$ are called *weakly homotopic* (notation: $f \underset{w}{\simeq} g$) iff for any finite CW complex Z and any map $h\colon Z \to X$ we have $fh \simeq gh$. The relation "$\simeq$" is an equivalence relation and we denote the set of all weak homotopy classes from X to Y by $[X, Y]_w$. The class represented by $f\colon X \to Y$ is denoted by $\{f\}_w$.

$\mathscr{H}w\mathscr{O}_n$ denotes the category whose objects are open subsets of S^n and whose morphisms are weak homotopy classes.

Using Lemma (1.1), one can prove that if U is an open subset of S^n then there exists an open subset $V \supset \Sigma(U)$ of S^{n+1} such that the inclusion $i\colon \Sigma(U) \to V$ is a homotopy equivalence. In view of the last fact, for every map $f\colon P \to \Sigma^k(U)$ where P is a finite CW complex, there exist a finite polyhedron $Q \subset U$ and a map $g\colon P \to \Sigma^k(U)$ with $g(P) \subset \Sigma(Q)$ and $f \simeq g$. It follows that Σ induces a function of $[X, Y]_w$ into $[\Sigma(X), \Sigma(Y)]_w$, if $X = \Sigma^k(U)$ and $Y = \Sigma^l(V)$, where U is an open subset of S^m and V is an open subset of S^n. Using the formula (1.1) one can also equip $[\Sigma^k(X), \Sigma^k(Y)]$ with a natural structure of an abelian group for $k \geq 2$. The direct limit $\{X, Y\}_w$ of the sequence

$$[X, Y]_w \to [\Sigma(X), \Sigma(Y)]_w \to [\Sigma^2(X), \Sigma^2(Y)]_w \to \ldots$$

is an abelian group. Elements of $\{X, Y\}_w$ are called *stable weak homotopy classes*. If $f\colon \Sigma^k(X) \to \Sigma^k(Y)$ is a map, $\{f\}_w$ will denote the corresponding element of $\{X, Y\}_w$, $\mathscr{S}w\mathscr{O}_n$ denotes the category whose objects are open subsets of S^n and whose morphisms are stable weak homotopy classes. $\mathscr{S}w\mathscr{O}_n$ is a preadditive category.

We now can establish the main result of this section.

(3.1) PROPOSITION. *There are a category isomorphism $\mathscr{A}_n$ of the coshape category of open subsets of S^n onto the weak homotopy category of open subsets of S^n and an additive category isomorphism $\mathscr{B}_n$ of the stable coshape category of open subsets of S^n onto the stable weak homotopy category of open subsets of S^n such that $\mathscr{A}_n(U) = \mathscr{B}_n(U) = U$ for every open subset U of S^n, the value of the coshape morphism induced by a map f under $\mathscr{A}_n$ is equal to $[f]_w$ and the value of the stable coshape morphism induced by f under $\mathscr{B}_n$ is equal to $\{f\}_w$.*

We shall use the following lemma.

(3.2) LEMMA. *Suppose that X is a compact polyhedron and $Y_1 \subset Y_2 \subset Y_3 \subset \ldots$ is a sequence of compact subpolyhedra of S^n such that $\bigcup_{k=1}^{\infty} Y_k = U$ is an open subset of S^n. Then $[X, U]$ and $\{X, U\}$ are respectively the direct limits of the sequences $\{[X, Y_1] \to [X, Y_2] \to \ldots\}$ and $\{\{X, Y_1\} \to \{X, Y_2\} \to \ldots\}$.*

We omit the proof of Lemma (3.2), which is straightforward.

Proof of Proposition (3.1). Let us show how $\mathscr{A}_n$ and $\mathscr{B}_n$ assign morphisms. Suppose that U and V are open subsets of S^n, $X_1 \subset X_2 \subset X_3 \subset \ldots$ and $Y_1 \subset Y_2 \subset Y_3 \subset \ldots$ are sequences of compact polyhedra in S^n such that $\bigcup X_n = U$ and $\bigcup Y_n = V$. Since cofinal systems are isomorphic in Inj$\mathscr{C}$, we may identify $\mathscr{C}o\mathscr{S}h(U, V)$ (or respectively $\mathscr{S}\mathscr{C}o\mathscr{S}h(U, V)$) with $\varprojlim_m \varinjlim_n [X_m, Y_n]$ (or with $\varprojlim_m \varinjlim_n \{X_m, Y_n\}$).

It follows from lemma (3.2) that every coshape morphism $\underline{f}$ from U to V is represented by a sequence $f_m\colon X_m \to V$ satisfying

$$f_m \simeq f_k | X_m \quad \text{for } m = 1, 2, \ldots \text{ and } k \geq m.$$

Let $g_1 = f_1\colon X_1 \to V$. Suppose that for every $m = 1, 2, \ldots, k$ we have a map $g_m\colon X_m \to V$ such that

$$g_{m-1} = g_m | X_{m-1} \quad \text{and} \quad f_m \simeq g_m \quad \text{for } m \leq k. \tag{3.3}$$

Using the homotopy extension theorem we can construct a map $g_{k+1}\colon X_{k+1} \to V$ which satisfies (3.3) for $m \leq k+1$.

It follows that every coshape morphism $\underline{f}$ from U to V is represented by a sequence $g_m\colon X_m \to V$ which satisfies (3.3) for every $m = 1, 2, \ldots$

Setting

$$g(x) = g_m(x) \quad \text{for } x \in X_m,$$

we get a map $g\colon U \to V$. The weak homotopy class of g is the value of $\mathscr{A}_n$ on $\underline{f}$.

Using (2.2), one can define $\mathscr{B}_n(\underline{f})$ for every $\underline{f} \in \mathscr{S}\mathscr{C}o\mathscr{S}h(U, V)$.

(3.4) COROLLARY. *Suppose that U and V are open subsets of S^n. Then $\{U, V\}_w$ is canonically isomorphic with $[\Sigma^k(U), \Sigma^k(V)]_w$ for $k \geq \max(2, n)$ for nonempty U and V and $k \geq \max(3, n)$ in the general case.*

4. The main theorems. We have the following theorem.

(4.1) THEOREM. *There is a contravariant additive category isomorphism $\mathscr{D}_n\colon \mathscr{S}\mathscr{S}h_n \to \mathscr{S}w\mathscr{O}_n$ of the stable shape category of compacta lying in S^n onto the stable weak homotopy category $\mathscr{S}w\mathscr{O}_n$ of all open subsets of S^n such that $\mathscr{D}_n(X) = S^n \setminus X$. Moreover, if $X, Y \subset X$ are subcompacta of S^n, then $\mathscr{D}_n(\underline{i}) = \{j\}_w$, where $\underline{i}\colon X \to Y$ and $j\colon S^n \setminus Y \to S^n \setminus X$ are respectively the stable shape morphism induced by the inclusion $i\colon X \to Y$ and the inclusion $j\colon S^n \setminus Y \to S^n \setminus X$.*

Proof. E. H. Spanier and J. H. C. Whitehead ([S–W] p. 63) have proved that there exists an additive category isomorphism $d_n\colon \mathscr{S}\mathscr{P}ol_n \to \mathscr{S}\mathscr{C}\mathscr{P}ol_n$ which satisfies the following conditions

(4.2) $d_n(X) = S^n \setminus X$,

(4.3) $d_n(\{i\}) = \{j\}$, where $i\colon X \to Y$ and $j\colon S^n \setminus Y \to S^n \setminus X$ are inclusions.

Suppose that X and Y are subcompacta of S^n and $\mathscr{S}_n(X) = \{X_\alpha, \{i_\alpha^{\alpha'}\}, A\}$, $\mathscr{S}_n(Y) = \{Y_\beta, \{i_\beta^{\beta'}\}, B\} \in \mathrm{ObPro}\mathscr{S}\mathscr{C}\mathscr{P}ol_n$ (we use the notation of Section 2).

Then $\mathcal{S}_n(S^n\setminus X) = \{U_\alpha, \{j_\alpha^{\alpha'}\}, A\}$, $\mathcal{S}_n(S^n\setminus Y) = \{V_\beta, \{j_\beta^{\beta'}\}, B\}$, where $j_\alpha^{\alpha'}\colon U_\alpha = S^n\setminus X_\alpha \to S^n\setminus X_{\alpha'} = U_{\alpha'}$ and $j_\beta^{\beta'}\colon V_\beta = S^n\setminus Y_\beta \to S^n\setminus Y_{\beta'} = V_{\beta'}$ are inclusions.

Since $d_n\colon \{X_\alpha, Y_\beta\} \to \{V_\beta, U_\alpha\}$ is an isomorphism, using (4.2) and (4.3) we can extend d_n to an isomorphism $\mathcal{D}_n'$ of the abelian group $\varinjlim_\beta \varprojlim_\alpha \{X_\alpha, Y_\beta\} = \mathrm{Pro}\mathcal{S}\mathcal{C}\mathcal{P}ol_n(\mathcal{S}_n(X), \mathcal{S}_n(Y))$ onto the abelian group

$$\varinjlim_\beta \varprojlim_\alpha \{V_\beta, U_\alpha\} = \mathrm{Inj}\mathcal{S}\mathcal{P}ol_n(\mathcal{S}_n\setminus Y), \mathcal{S}_n(S^n\setminus X)).$$

Setting

$$\mathcal{D}_n'(X) = S^n\setminus X,$$

we can also extend d_n to an additive isomorphism of the stable shape category of compact subsets of S^n onto the stable coshape category of their complements. The composition $\mathcal{D}_n = \mathcal{B}_n\mathcal{D}_n'\colon \mathcal{S}\mathcal{S}h_n \to \mathcal{S}w\mathcal{O}_n$ is an additive functor, which satisfies the required conditions (see Proposition (3.1)).

The proof is finished.

(4.4) Remark. If X and Y are closed subsets of S^n and $\underline{\alpha}\colon X \to Y$ is a stable shape morphism, then $\mathcal{D}_n(\underline{\alpha})$ is uniquely determined by the properties of $\mathcal{D}_n$ and (1.1).

Indeed, for every $k \geqslant \max(2, n)$ and for sufficiently large $m \geqslant n+k$ the shape morphism $\underline{f}\colon \Sigma^k(X) \to \Sigma^k(Y)$ which represents $\underline{\alpha}$ is the composition $\underline{g}_1 \underline{g}_2 \ldots \underline{g}_r$ of shape morphism $\underline{g}_i\colon X_i \to X_{i+1} \subset S^m$, $i = 1, 2, \ldots, r$, such that $X_1 = \Sigma^k(X)$, $X_{r+1} = \Sigma^k(Y)$ and $\underline{g}_i$ is induced by inclusion map or $\underline{g}_i$ is a shape equivalence and $\underline{g}_i^{-1}$ is induced by inclusion for $i = 1, 2, \ldots, r$ (see [D–S] p. 54).

(4.5) Corollary. *Suppose that X and Y are compact subsets of S^n. Then the following conditions are equivalent*

(a) $\mathcal{S}\mathcal{S}h(X) = \mathcal{S}\mathcal{S}h(Y)$;

(b) $\mathcal{S}h(\Sigma^k(X)) = \mathcal{S}h(\Sigma^k(Y))$ *for some* $k \geqslant \max(2, n)$;

(c) $\Sigma^k(S^n\setminus X)$ *and* $\Sigma^k(S^n\setminus Y)$ *are homotopy equivalent for some* $k \geqslant \max(2, n)$;

(d) $S^n\setminus X$ *and* $S^n\setminus Y$ *have the same stable homotopy type.*

Proof. From the suspension theorem it follows that (c) and (d) are equivalent.

From (4.1) it follows that (a) is satisfied iff $S^n\setminus X$ and $S^n\setminus Y$ have the same stable weak homotopy type.

The space $S^n\setminus X$ has the same stable weak homotopy type as $S^n\setminus Y$ iff $\Sigma^k(S^n\setminus X)$ and $\Sigma^k(S^n\setminus Y)$ have the same weak homotopy type for every $k \geqslant \max(2, n)$.

The Whitehead theorem implies that $\Sigma^k(S^n\setminus X) \cong \Sigma^k(S^n\setminus Y)$ if and only if they have the same weak homotopy type.

From (2.1) we deduce that (b) and (a) are equivalent. The proof is finished.

5. The main theorems in the movable case. Let us prove

(5.1) Proposition. *Suppose that U and V are open subsets of S^n and $X_1 \subset X_2 \subset \ldots$ is a sequence of compact polyhedra with $\bigcup_{n=1}^{\infty} X_n = V$. Then we have a short exact sequence*

$$0 \to \varprojlim{}^1 \{\Sigma(X_k), U\} \to \{V, U\} \to \{V, U\}_w \to 0.$$

Proof. Let us choose $a_0 \in U$. For every pointed CW complex X and every $n = 0, \pm 1, \pm 2, \pm 3, \ldots$ we define

(5.2) $h^n(X) = \varinjlim\{[X, S^n(U)] \to [S(X), S^{n+1}(U)] \to [S^2(X), S^{n+2}(U)] \to \ldots\}$

where S is the reduced suspension.

If $f\colon X \to Y$ is a map which preserves the base points, then $h^n(f)\colon h^n(Y) \to h^n(X)$ is a homomorphism defined by the condition

(5.3) $h^n(f)(\alpha)$ is represented by a map $gS^k(f)$, where $g\colon S^k(Y) \to S^{n+k}(U)$ represents $\alpha \in h^n(Y)$.

It is clear that for every $n = 0, \pm 1, \pm 2, \ldots$ we have a natural transformation $E^n\colon h^{n+1}S \to h^n$ such that $E^n(X)\colon h^{n+1}(S(X)) \to h^n(X)$ is an isomorphism.

If $f\colon X \to Y$ is a map which preserves the base points and $j\colon Y \to T_f$ is an inclusion, where T_f is the reduced mapping cone of f, then the sequence

(5.4) $[S^{4k}(T_f), S^{4k+n}(U)] \to [S^{4k}(Y), S^{4k+n}(U)] \to [S^{4k}(X), S^{4k+n}(U)]$ where $k \geqslant \dim X, \dim Y$, is exact ([W] p. 134).

Since (5.4) is isomorphic with the sequence $h^n(T_f) \to h^n(Y) \to h^n(X)$, we infer that the family $\{h^n\}$ of functors defined by (5.2) and (5.3) and the family $\{E^n\}$ of natural transformations form an extraordinary cohomology theory on the category of finite dimensional CW complexes.

Since the injections $h^n(X) \cong [S^{4k}(X), S^{n+4k}(U)] \to [S^{4k}(X_\alpha), S^{4k+n}(U)] \cong h^n(X_\alpha)$ represent the group $h^n(X)$ as a direct product for every finite dimensional CW complex $X = \bigvee_{\alpha\in A} X_\alpha$ with $\dim X \leqslant k$ ([Sp] p. 407 and [Sw] p. 152), our cohomology theory is additive.

For every $n = 0, \pm 1, \pm 2, \ldots$ we have a short exact sequence ([W] p. 605, [Sw] p. 127)

$$0 \to \varprojlim_k{}^1 h^{n-1}(Y_k) \to h^n(Y) \to \varprojlim h^n(Y_k) \to 0$$

if Y is a finite dimensional CW complex and $Y_1 \subset Y_2 \subset \ldots$ are subcomplexes of Y such that $Y = \bigcup_{n=1}^{\infty} Y_n$.

In particular, the sequence

$$0 \to \varprojlim{}^1 h^{-1}(X_k) \to h^0(V) \to \varprojlim h^0(X_k) \to 0$$

is exact.

We known that $h^0(V) \cong \{V, U\}$, $h^{-1}(X_k) \cong \{\Sigma(X_k), U\}$. Using (3.2), we get $\{V, U\}_w \cong \varprojlim\{X_k, U\}$.

The proof is finished.

The next example shows that there are open subsets U and V of S^3 such that $\{U, V\}_w$ and $\{U, V\}$ are not isomorphic.

(5.5) Example. *Suppose that $p = (p_1, p_2, \ldots)$ is a sequence of natural numbers such that $p_n \geqslant 2$ for $n = 1, 2, \ldots$ Then $\{S^3\setminus S(p), S^3\setminus S^0\}_w = 0$ and*

$$\{S^3\setminus S(p), S^3\setminus S^0\} \neq 0,$$

where $S(p)$ is a solenoid generated by the sequence p.

Let $A_1 \supset A_2 \supset \ldots$ be a sequence of open subsets of S^3 satisfying the conditions

(5.6) $A_n \cong S^1$, $B_n = S^3 \setminus A_n$ is a finite subpolyhedron of S^3 and $S(p) = \bigcap_{n=1}^{\infty} A_n$;

(5.7) for every $n = 1, 2, \ldots$ we can select a generator e_n of the group $H_1(A_n; Z)$ such that $(i_{n+1})_*(e_{n+1}) = p_n e_n$, where $i_{n+1}\colon A_{n+1} \to A_n$ is an inclusion.

Then B_n has the stable homotopy type of S^1 and $\Sigma(B_n)$ has the stable homotopy type of S^2.

The group $\{\Sigma(B_k), S^3 \setminus S_0\} \cong \{S^2, S^2\}$ and the group Z are isomorphic. The sequence $\{\Sigma(B_k), S^3 \setminus S^0\}$ is isomorphic to the sequence (see (5.6) and (5.7))

$$\{Z \xleftarrow{f_1} Z \xleftarrow{f_2} Z \leftarrow \ldots\} = \underline{A}$$

where $f_n(z) = p_n z$ for every $z \in Z$ and $n = 1, 2, \ldots$

Then ([M–S] p. 173 and p. 174) $\lim^1 \underline{A} \neq 0$.

Since $\{B_k\, S^3 \setminus S_1\} \cong \{S^1, S^2\} \cong 0$ and $\{S^3 \setminus S(p), S^3 \setminus S^0\}_w \cong \underline{\lim}\{B_k, S^3 \setminus S^0\}$ (see (3.2)), we infer that $0 = \{S^3 \setminus S(p), S^3 \setminus S^0\} \cong \lim^1 A$.

The next theorem is a consequence of Proposition (5.1).

(5.8) Theorem. *Suppose that X is a subcompactum of S^n and $\Sigma^k(X)$ is movable for some k. Then $\{S^n \setminus X, U\} \cong \{S^n \setminus X, U\}_w$ for every open subset U of S^n.*

Proof. If $\Sigma^k(X)$ is movable, then there exists a sequence $X_1 \subset X_2 \subset \ldots$ of finite subpolyhedra of S^n such that the inverse sequence of the complements of its members is movable in $\mathscr{S}$ and $S^n \setminus X = \bigcup_{n=1}^{\infty} X_n$. Applying the Spanier–Whitehead functor ([S–W] p. 63) and using the properties of the functor Hom(Hom$(X) = \{X, U\}$ for every X), we get that the sequence $\{\Sigma(X_n), U\}$ is movable and

$$\lim{}^1 \{\Sigma(X_n), U\} = 0\,.$$

It follows (see Proposition (5.1)) that $\{S^n \setminus X, U\} \cong \{S^n \setminus X, U\}_w$. The proof is finished.

6. Applications. The following proposition will be very useful.

(6.1) Proposition. *Suppose that $X, Y \subset S^n$ are approximatively m-connected continua, where $m \geqslant 1$ and $\mathrm{Fd}(X) \leqslant 2m$. Then the following conditions are equivalent*

(a) $\mathrm{Sh}(X) = \mathrm{Sh}(Y)$;

(b) *$S^n \setminus X$ and $S^n \setminus Y$ are stable homotopy equivalent.*

The proof of Proposition (6.1) is based on the lemma:

(6.2) Lemma. *Suppose that X, Y are approximatively m-connected continua with $\mathrm{Fd}(X)$, $\mathrm{Fd}(Y) \leqslant 2m$, where $m \geqslant 1$. Then the following conditions are equivalent*:

(c) $\mathrm{SSh}(X) = \mathrm{SSh}(Y)$;

(d) $\mathrm{Sh}(X) = \mathrm{Sh}(Y)$.

Proof. It is clear that (d) implies (c).

Suppose that (c) is satisfied. There exist inverse sequences $\underline{X} = \{X_k, p_k^{k+1}\}$ and $\underline{Y} = \{Y_k, q_k^{k+1}\}$ of m-connected polyhedra with dimension $\leqslant 2m$ ([M–S] p. 137) such that $\mathrm{Sh}(X) = \mathrm{Sh}(\underline{\lim}\,\underline{X})$ and $\mathrm{Sh}(Y) = \mathrm{Sh}(\underline{\lim}\,\underline{Y})$.

From the suspension theorem it follows that

$$[X_k, Y_l] \cong \{Y_k, Y_l\}, \quad [Y_k, X_l] \cong \{Y_k, X_l\}, \quad [X_k, X_l] \cong \{X_k, X_l\}$$

and

$$[Y_k, Y_l] \cong \{Y_k, Y_l\} \quad \text{for } k = 1, 2, \ldots \text{ and } l = 1, 2, \ldots$$

Hence $\mathrm{Sh}(X) = \mathrm{Sh}(Y)$.

Proof of Proposition (6.1). It follows from the Alexander duality theorem that $H^n(X; Z) \cong H^n(Y; Z)$ for $n \geqslant 1$. Therefore ([N] p. 35) $\mathrm{Fd}(X) = \mathrm{Fd}(Y) \leqslant 2m$. The proof is finished.

By $\mathscr{BS}(m)$ we denote the family of all wedges of m-dimensional spheres. We say that a continuum X is $\mathscr{BS}(m)$-like iff X is homeomorphic to the inverse limit of members of $\mathscr{BS}(m)$.

(6.3) Proposition. *Let $2 \leqslant m \leqslant n$ and $S^n \supset Y$ be a $\mathscr{BS}(m)$-like continuum. If $X \subset S^n$ is an approximatively 1-connected continuum, then the following conditions are equivalent*:

(a) $\mathrm{Sh}(X) = \mathrm{Sh}(Y)$;

(b) $H_r(S^n \setminus X) \cong H_r(S^n \setminus Y)$ *for* $r = 0, 1, 2, \ldots, n$.

We need the lemma.

(6.4) Lemma. *If X is an approximatively 1-connected continuum and $H^r(X; G) = 0$ for every abelian group G and for every $1 \leqslant r \leqslant m$, then X is approximatively m-connected.*

Proof. Let $x_0 \in X$ and $(X, x_0) = \underline{\lim}\{(X_k, x_k), p_k^{k+1}\}$, where X_k is a simply connected polyhedron. Using Proposition (1.5) of [N–S] we infer that for every k there exists $l > k$ and a map $q \simeq p_k^l$ such that $q(X_l^{(m)}) = \{x_n\}$.

This completes the proof of Lemma (6.4).

Proof of Proposition (6.3). It follows from the Alexander duality theorem that $H^r(X; G) \cong H^r(Y; G)$ for every $r \geqslant 1$, $H^r(X; G) = 0 = H^r(Y; G)$ for $1 \leqslant r \neq m$ and $H_r(S^n \setminus X; G) = 0 = H_r(S^n \setminus Y; G)$ for $1 \leqslant r \neq n-m-1$ in the case when (a) or (b) is satisfied.

By Lemma (6.4) X is approximatively $(m-1)$-connected if (a) or (b) is satisfied.

(a) $\Rightarrow$ (b) is obvious.

(b) $\Rightarrow$ (a). The spaces $\Sigma^2(S^n \setminus X) = P$ and $Q = \Sigma^2(S^n \setminus Y)$ are simply connected absolute neighborhood retracts for metric spaces. We know ([Sw] p. 241) that $H^k(P; G) \cong \mathrm{Hom}(H_k(P; Z); G)$, $H^k(Q; G) \cong \mathrm{Hom}(H_k(Q; Z); G)$ for $k > 1$ and $H^k(P; G) = 0 = H^k(Q; G)$ for $1 \leqslant k \neq n-m+1 = l$.

Hence

$$[P; Q] \cong H^l(P; \pi_l(Q)) \cong \mathrm{Hom}(H_l(P); \pi_l(Q)) \cong \mathrm{Hom}(H_l(P; Z); H_l(Q; Z))\,.$$

Since $H_l(P;Z) \cong H_l(Q;Z)$, we infer that there is a map $f\colon P \to Q$ which induces an isomorphism $f_*\colon H_l(P;Z) \to H_l(Q;Z)$.

The Whitehead theorem implies that f is a homotopy equivalence.

It follows that $S^n \setminus X$ and $S^n \setminus Y$ are stable homotopy equivalent and (see Proposition (6.1)) $\operatorname{Sh}(X) = \operatorname{Sh}(Y)$. The proof of Proposition (6.3) is finished.

7. Some examples. Let us prove the following

(7.1) THEOREM. *There are a movable continuum $X \subset S^3$ and a nonmovable continuum $Y \subset S^3$ such that $S^3 \setminus X$ and $S^3 \setminus Y$ are homeomorphic.*

Proof. It suffices to construct a movable continuum $X \subset E^3$ and a nonmovable continuum $Y \subset E^3$ such that $E^3 \setminus X$ and $E^3 \setminus Y$ are homeomorphic.

Let us set

$$a_{kn} = \begin{cases} \dfrac{1}{4n(n+1)}\left(1 - \dfrac{1}{5^k}\right) & \text{for } k = 1, 2, \ldots \text{ and } n = 1, 2, \ldots \\[2ex] \dfrac{1}{4n(n+1)} & \text{for } k = 0 \text{ and } n = 1, 2, \ldots \end{cases}$$

Then $\dfrac{1}{5n(n+1)} = a_{1n} < a_{2n} < a_{3n} < \ldots < \lim a_{kn} = a_{0n}$ for $n = 1, 2, \ldots$

We denote by B_{nk}, where $n = 1, 2, \ldots$ and $k = 0, 1, 2, \ldots$, the open disc lying in E^2 and bounded by the circle $K_{n,k} \subset E^2$ with the centre $\left(\dfrac{1}{n}, 0\right)$ and the radius a_{nk}.

Let $D_k \subset E^2$ denote the set consisting of all points (x_1, x_2) with $|x_2| \leqslant x_1 \leqslant 2$ in the case where $k = 0$ and the set of all points (x_1, x_2) such that

$$\left(\frac{k}{2k+1}\right)|x_2| \leqslant x_1 \leqslant 2 + \frac{1}{k} \quad \text{if} \quad k = 1, 2, \ldots$$

Setting $W_0 = D_0 \setminus \bigcup_{k=1}^{\infty} B_{k0}$ and $W_n = D_n \setminus \bigcup_{k=1}^{n} B_{kn}$, we get plane continua $W_1 \supset W_2 \supset W_3 \supset \ldots\ W_0 = \bigcap^{\infty} W_n$.

For $n = 0, 1, 2, \ldots$, we denote by X_n the boundary in E^3 of the set consisting of all points $(x_1, x_2, x_3) \in E^3$ such that $(x_1, x_2, 0) \in W_n$ and $|x_3| \leqslant x_1$ if $n = 0$ and $n \cdot |x_3| \leqslant (n+1)x_1$ if $n = 1, 2, \ldots$

By Y_0 we denote the set consisting of all points (x_1, x_2, x_3) with $(x_1, -x_2, x_3) \in X_0$ and by A and B we denote, respectively, the bounded components of $E^3 \setminus X_0$ and $E^3 \setminus Y_0$.

X_0 and Y_0 are homeomorphic to the locally connected nonmovable subcontinuum of E^3 constructed by K. Borsuk in [B$_1$].

Then $X = \bigcup_{n=0}^{\infty} X_n \cup Y_0 \cup A$ and $Y = \bigcup_{n=0}^{\infty} X_n \cup Y_0 \cup B$ are subcontinua of E^3 with homeomorphic complements. It is clear that Y is movable. Since $\operatorname{Sh}(X) \geqslant \operatorname{Sh}(Y_0)$, we infer that X is nonmovable. This completes the proof.

(7.2) Remark. The continua X and Y constructed in the proof of Theorem (7.1) separate S^3. D. R. Mc Millan, Jr., gave in [M] an example of a locally connected nonmovable continuum $M_0 \subset S^3$ such that $S^3 \setminus M_0$ is connected. By an easy modification of this example one can obtain a movable continuum $M_1 \subset S^3$ such that $S^3 \setminus M_0$ is homeomorphic to $S^3 \setminus M_1$.

Added in proof. When the present note was prepared the author did not know the results of E. Lima obtained in 1958 and presented in the paper *The Spanier-Whitehead duality in new homotopy categories*, Summa Bras. Mat. 4 (1959), 91–148. E. Lima introduced independently the notion of *procategory*. His construction leads to the theories of shape and coshape (inverse system approach).

The above mentioned paper contains also Theorem (4.1).

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INSTITUTE OF MATHEMATICS
UNIVERSITY OF WARSAW
00-901 Warsaw
PKiN 9-th floor
Poland

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Révêtements étales

par

Gabriel Picavet (Aubière)

Abstract. An etale covering of a ring A is a finite etale morphism $A \to B$. A morphism of rings $A \to B$ is said to be reduced in the case that each change of base $A \to A'$ gives a reduced ring $B \otimes_A A'$. M. Lazarus has proved the following result: Let A be a Noetherian ring and let there be a flat morphism $A \to A'$; then this morphism is reduced if and only if integral closure is preserved in the change of base $A \to A'$. We change the Noetherian hypothesis to a finiteness hypothesis on the morphism $A \to A'$. With a mild hypothesis, we obtain that the morphism is reduced if and only if it is an etale covering.

1. Introduction. M. Lazarus a montré, dans deux articles, [5] [6], les résultats suivants:

DÉFINITION 1. Soit f: $A \to B$ un morphisme d'anneaux. Le morphisme f est dit *réduit* si pour tout morphisme $A \to A'$, où A' est un anneau réduit, l'anneau $B \otimes_A A'$ est réduit.

On prendra garde que cette définition n'est pas identique à celle des *Eléments de Géométrie Algébrique* de A. Grothendieck et J. Dieudonné.

PROPOSITION 2 [6]. *Soit f: $A \to B$ un morphisme.*

(a) *Si le morphisme f est réduit et l'anneau A réduit, le morphisme f est plat.*

(b) *Si le morphisme f conserve la fermeture intégrale dans le changement de base qu'il définit, alors f est un morphisme réduit.*

PROPOSITION 3 [5]. *Soit f: $A \to A'$ un morphisme d'anneaux, où A est un anneau Noethérien. Les propriétés suivantes sont équivalentes:*

(a) *Le morphisme f est plat, à fibres géométriquement réduites.*
(b) *Le morphisme f est plat et réduit.*
(c) *Le morphisme f conserve la fermeture intégrale dans le changement de base qu'il définit.*