STUDIA MATHEMATICA, T. LXXXIX. (1988)

Zero-one law for subgroups of paths of group-valued stochastic processes

by

H. BYCZKOWSKA and T. BYCZKOWSKI (Wrocław)

Abstract. Let ξ be a symmetric infinitely divisible stochastic process with values in a locally compact separable group G and let η be its Lévy measure. If H is a measurable subgroup of paths of ξ then $\eta(H^c) = \infty$ implies that $P\{\xi \in H\} = 0$ and $\eta(H^c) = 0$ implies that $P\{\xi \in H\} = 0$ or 1.

Let G be a complete separable metric group and let $(\mu_t)_{t>0}$ be a continuous convolution semigroup of probability measures on G. Suppose, further, that H is a Borel subgroup of G. We investigate conditions under which either $\mu_t(H) = 0$ for all $t > 0$ or $\mu_t(H) = 1$ for all $t > 0$.

The results of this paper extend the ones from [2] and [6] in two directions: first of all, we do not assume that H is a normal subgroup, as in these two papers; moreover, our results are valid not only for Gaussian but also for general convolution semigroups.

Since our approach is rather general, the results we obtain here are valid without the local compactness assumption, as in [6]. This enables us to prove 0-1 laws for subgroups of paths of stochastic processes with values in locally compact groups.

1. Preliminaries. Throughout this paper G will stand for a complete separable metric group, unless stated otherwise. By $C_u = C_u(G)$ we denote the space of all left uniformly continuous real bounded functions on G. By a *probability measure* μ on G we mean a σ-additive Borel measure such that $\mu(G) = 1$. For any probability measure μ on G we define the *probability operator* T_μ on C_u by the formula

$$T_\mu f(x) = \int f(xy)\,\mu(dy), \qquad f \in C_u.$$

It is easy to see that $T_\mu f \in C_u$ and that $T_{\mu_n} f \to T_\mu f$ uniformly, for every $f \in C_u$, if and only if $\mu_n \Rightarrow \mu$ weakly. Also, we have $T_{\mu * \nu} = T_\mu T_\nu$.

Now, let $(\mu_t)_{t>0}$ be a convolution semigroup of probability measures on G, i.e.

$$\mu_t * \mu_s = \mu_{t+s} \qquad \text{for all } t, s > 0.$$

$(\mu_t)_{t>0}$ is called *continuous* if $\lim_{t\to 0}\mu_t = \delta_e$. If $(\mu_t)_{t>0}$ is continuous then the corresponding family $(T_{\mu_t})_{t>0}$ of probability operators forms a strongly continuous semigroup of contractions on C_u regarded as a Banach space under the supremum norm. This semigroup is uniquely determined by its infinitesimal generator N defined on its domain $\mathcal{D}(N)$ which is dense in C_u. It is evident that N commutes with left translations: $L_x Nf = NL_x f$ for $f\in\mathcal{D}(N)$. Therefore it is enough to consider the generating functional A,

$$Af = (Nf)(e), \quad f\in\mathcal{D}(N).$$

If $f\in\mathcal{D}(N)$ then $Nf = AL_x f$.

The crucial role in the proof of our theorems is played by

Trotter Approximation Theorem. *Let $T_t^{(n)}$ be a sequence of strongly continuous semigroups of operators on a Banach space X, satisfying the condition*

$$\|T_t^{(n)}\| \leqslant e^{Kt},$$

where K is independent of n and t. Let N_n be the infinitesimal generator of $T_t^{(n)}$. Assume that $\lim N_n x$ exists in the strong sense on a dense linear subspace D. Define

$$Nx = \lim N_n x, \quad x\in D.$$

Suppose additionally that for some $\lambda > K$ the range of $\lambda I - N$ is dense in X. Then the closure of N is the infinitesimal generator of a strongly continuous semigroup T_t such that

$$T_t x = \lim T_t^{(n)} x \quad \textit{for } x\in X.$$

We will also use the following version of the perturbation theorem (cf. [3], see also [6]):

Perturbation Principle. *Let $(\mu_t)_{t>0}$ be a continuous convolution semigroup of probability measures on G with the generating functional A defined on $\mathcal{D}(A)\subseteq C_u$. Suppose that $A = B + c(\nu-\delta_e)$, where B is the generating functional of a convolution semigroup $(\varkappa_t)_{t>0}$, c is a positive constant and ν is a probability measure. Then*

$$e^{ct}\mu_t = \sum_{n=0}^{\infty} \gamma^{(n)}(t), \tag{1}$$

where $\gamma^{(0)}(t) = \varkappa_t$, $\gamma^{(n)}(t) = c\int_0^t \varkappa_{t-u} * \nu * \gamma^{(n-1)}(u)\,du$ for $n\geqslant 1$, and the series is convergent in the total variation norm.

Another important point in our method is the use of the $L^1(\mu)$ space for μ defined by

$$\mu = \int_0^{\infty} e^{-t}\mu_t\,dt. \tag{2}$$

It is easy to see that μ is a probability measure. By L^1 we mean the space of all Borel measurable functions on G that are μ-integrable. The importance of this L^1 space follows from the fact that $(\mu_t)_{t>0}$ acts as a strongly continuous semigroup on L^1 and that

$$\|T_{\mu_s}\|_{L^1,L^1} \leqslant e^s. \tag{3}$$

Moreover, if H is a Borel subgroup of G then

$$\mu(H) > 0 \quad \text{implies} \quad \mu_t(H)\to 1 \text{ as } t\to 0. \tag{4}$$

It is not difficult to show that $\mu(H)>0$ implies $\mu_t(H)>0$ for all $t>0$. On the other hand, if the μ_t are symmetric then $\mu_{t_0}(H)>0$ for a single t_0 implies $\mu_t(H)>0$ for all $t>0$. All these facts concerning the L^1 space are taken from [2].

2. Zero-one laws. Let $(\mu_t)_{t>0}$ be a continuous convolution semigroup of probability measures on G. Suppose, further, that H is a Borel subgroup of G with $\mu(H)>0$, where μ is defined by the formula (2). The following lemma is an important step in proving the main result.

Lemma. *Assume that $(\mu_t)_{t>0}$ and H are as above. Then*

$$\lim_{t\to 0+} (1/t)\big(1-\mu_t(H)\big) = c < \infty. \tag{5}$$

Proof. Let $X = \{X(t);\ t\geqslant 0\}$ be a homogeneous stochastic process with independent increments taking values in G, determined by the condition that the distribution of $X(t)$ is equal to μ_t, i.e. $X(0)=e$ and for all $0\leqslant s_1 < s_2 < \ldots < s_k$ the increments $X(s_1)$, $X(s_2)X(s_1)^{-1}, \ldots, X(s_k)X(s_{k-1})^{-1}$ are independent and have distributions $\mu_{s_1}, \mu_{s_2-s_1}, \ldots, \mu_{s_k-s_{k-1}}$. A routine application of Kolmogorov's Extension Theorem shows the existence of such a process. Next, let $\pi\colon G\to G/H$ be the canonical mapping of G onto the space of left cosets of H, endowed with the σ-algebra induced by π from G. Then $\{\pi(X(t));\ t\geqslant 0\}$ is a homogeneous Markov process on the state space G/H with the following transition probabilities:

$$P\{\pi(X(t+u))\in D \mid \pi(X(t)) = zH\} = P_u(zH, D) = \int \mathbf{1}_D(xzH)\,d\mu_u(x).$$

In particular,

$$P_u(H, H) = \mu_u(H).$$

Since P_u is differentiable (see e.g. [7]) the limit in (5) exists. We only have to show that it is finite. To do this, we apply the following version of Ottaviani's Inequality:

$$\min_{0\leqslant k\leqslant n-1} P\{X(u)X(ku/n)^{-1}\in H\}\cdot P\{X(iu/n)\notin H \text{ for some } i,\ 1\leqslant i\leqslant n\}$$
$$\leqslant P\{X(u)\notin H\},$$

where $\{X(t);\ t \geqslant 0\}$ is the previously determined process. In terms of one-dimensional distributions the above inequality can be rewritten as follows:

$$(6) \qquad \min_{1 \leqslant k \leqslant n} \mu_{ku/n}(H)\left(1-(\mu_{u/n}(H))^n\right) \leqslant 1-\mu_u(H).$$

Since $\mu_s(H) \to 1$ as $s \to 0$, we may choose $u > 0$ such that $\mu_s(H) > 2/3$ for all s such that $0 < s \leqslant u$. Then the inequality (6) yields

$$\left(\mu_{u/n}(H)\right)^n \geqslant 1-(3/2)\left(1-\mu_u(H)\right) \geqslant 1/2,$$

so

$$n\left(1-\mu_{u/n}(H)\right) \leqslant n\left(1-(1/2)^{1/n}\right) \to \ln 2.$$

This shows that the constant c in (5) is finite and completes the proof of the lemma.

Now, we are able to state and prove our main result.

THEOREM. *Let $(\mu_t)_{t>0}$ be a symmetric convolution semigroup of probability measures on G and let H be a Borel subgroup of G such that $\mu_t(H) > 0$ for all $t > 0$. Then the generating functional A of the semigroup $(\mu_t)_{t>0}$ acting on C_u has the following decomposition*:

$$(7) \qquad A = A^H + c(\nu-\delta_e),$$

where A^H is the generating functional of a symmetric convolution semigroup of probability measures which are concentrated on H, $0 \leqslant c < \infty$, and ν is a probability measure such that $c\nu(H) = 0$.

Proof. 1. Let μ_s^H be the conditional probability with respect to H. Observe that if $\mu_s(H) = 1$ for a single $s > 0$ then $\mu_s(H) = 1$ for all $s > 0$ and in that case the conclusion is clearly true with $A = A^H$ and $c = 0$. Therefore we may assume that for every $s > 0$, $\mu_s(H^c) > 0$. Then for all $s > 0$ we have the following identity:

$$(1/s)[\mu_s-\delta_e] = (1/s)\mu_s(H^c)[\mu_s^{H^c}-\delta_e]+(1/s)\mu_s(H)[\mu_s^H-\delta_e].$$

Write the above identity in the form

$$(8) \qquad A_s = A_s^{H^c}+A_s^H,$$

with $A_s = (1/s)[\mu_s-\delta_e]$, $A_s^H = (1/s)\mu_s(H)[\mu_s^H-\delta_e]$ and $A_s^{H^c} = (1/s)\mu_s(H^c)[\mu_s^{H^c}-\delta_e]$. Now, observe that by our lemma $A_s^{H^c}$ is norm bounded. We show that the family $\{\mu_s^{H^c}\}$ is conditionally compact as $s \to 0+$. To do this, we apply the formula (1) with A_s as A, A_s^H as B and $\mu_s^{H^c}$ as ν. We then have

$$(9) \qquad e^{c_s t}\exp\left((t/s)[\mu_s-\delta_e]\right) = \sum_{n=0}^{\infty} \gamma_s^{(n)}(t),$$

where $c_s = (1/s)\mu_s(H^c) \to c < \infty$ as $s \to 0$,

$$\gamma_s^{(0)}(t) = \exp\left((t/s)\mu_s(H)[\mu_s^H-\delta_e]\right),$$

and

$$(10) \qquad \gamma_s^{(k)}(t) = c_s \int_0^t \gamma_s^{(0)}(t-u) * \mu_s^{H^c} * \gamma_s^{(k-1)}(u)\,du \quad \text{for } k \geqslant 1.$$

Next, let $\varepsilon > 0$ and let t be fixed. Since $\exp\left((t/s)[\mu_s-\delta_e]\right) \Rightarrow \mu_t$ as $s \to 0$ ([3]) there exists a compact subset K of G such that for $s \leqslant s_\varepsilon$ we have $\exp\left((t/s)[\mu_s-\delta_e]\right)(K^c) < \varepsilon/2$. Further,

$$(11) \qquad \|\gamma_s^{(k)}(t)\| = (c_s t)^k/k!$$

for $k = 0, 1, \ldots$ Therefore for all $s \leqslant s_\varepsilon$ we obtain

$$\gamma_s^{(0)}(t)(K) \geqslant e^{c_s t}(1-\varepsilon/2)-(e^{c_s t}-1) = 1-(\varepsilon/2)e^{c_s t}.$$

Since $c_s \to c$, this means that $\{\gamma_s^{(0)}(t);\ s \leqslant 1\}$ is conditionally compact. By standard symmetry arguments we get

$$(12) \qquad \gamma_s^{(0)}(u)(K') \geqslant 1-\varepsilon e^{c_s t}$$

for all $0 < u \leqslant t$ and s as before, where $K' = KK^{-1}$. Moreover, by (9) and (11) we get

$$\gamma_s^{(1)}(t)(K) = c_s \int_0^t \gamma_s^{(0)}(t-u) * \mu_s^{H^c} * \gamma_s^{(0)}(u)(K)\,du$$
$$\geqslant e^{c_s t}(1-\varepsilon/2)-1-(e^{c_s t}-1-c_s t) = c_s t-(\varepsilon/2)e^{c_s t}.$$

This and the formula (12) show that for some u, $0 < u \leqslant t$, we have

$$\gamma_s^{(0)}(t-u) * \mu_s^{H^c} * \gamma_s^{(0)}(u)(K) \geqslant 1-(\varepsilon/2c_s)e^{c_s t},$$

with s as before. The above inequality, together with (12), shows that

$$\mu_s^{H^c}(K'KK') > 1-(\varepsilon/c_s-\varepsilon^2 e^{c_s t})e^{c_s t}$$

for all $s \leqslant s_\varepsilon$. This means that the family $\{\mu_s^{H^c};\ 0 < s \leqslant 1\}$ is conditionally compact.

2. Now, observe that A_s converges on $\mathcal{D}(A)$ to A as $s \to 0$. Since $\{\mu_s^{H^c};\ 0 < s \leqslant 1\}$ is conditionally compact, choosing an appropriate subsequence we get the following equality on $\mathcal{D}(A)$:

$$A = c(\nu-\delta_e)+A^H$$

for a probability measure ν and a linear functional A^H. Because A^H is a sum of a generating functional and a norm bounded functional and, by (12), the family $\gamma_s^{(0)}(t) = \exp tA_s^H$, $0 < s \leqslant 1$, is conditionally compact for every fixed t, A^H is the generating functional of a continuous convolution semigroup of probability measures on G. We show that these measures are concentrated on H. This part of the proof is the same as the corresponding part of the proof of the main result of [2]. It is included here for the sake of completeness. We apply here the L^1 method. Namely, observe that $\|\mu_s^{H*k}\|_{L^1,L^1} \leqslant (e^s/\mu_s(H))^k$. Thus we have

$$\|\exp((t/s)\,\mu_s(H)\,[T_{\mu_s^H}-I])\|_{L^1,L^1} \leqslant e^{(-t/s)(\mu_s(H)-e^s)}.$$

Since $\lim_{s\to 0}(1/s)(e^s-\mu_s(H)) = 1+c < \infty$, the family of semigroups

$$T_t^{(s)} = \exp((t/s)\,\mu_s(H)\,[T_{\mu_s^H}-I]), \qquad s\in(0,\,1],$$

has the property

$$\|T_t^{(s)}\|_{L^1,L^1} \leqslant e^{Kt}$$

for a $K > 0$, independent of s.

Now, let N_s^H and N^H be the generators of the convolution semigroups $T_t^{(s)}$ and $\lim_{s\to 0} T_t^{(s)}$ considered as acting on C_u, and let $\mathcal{N}_s^H$ and $\mathcal{N}^H$ be the generators of the same convolution semigroups, regarded as acting on L^1. Denote the convolution semigroup corresponding to N^H by γ_t. Let $\overline{N^H}$ be the closure of N^H in L^1. By standard arguments $\overline{N^H} = \mathcal{N}^H$. Moreover, by the definition of N^H we obtain

$$\lim_{s\to 0}\|\mathcal{N}_s^H f - N^H f\|_{L^1} = 0 \quad \text{for } f\in\mathcal{D}(N^H). \tag{13}$$

Since also $(\lambda - N^H)(\mathcal{D}(N^H))$ is dense in L^1 for $\lambda > K$, (13), by the Trotter Approximation Theorem, gives

$$\lim_{s\to 0}\|T_{\gamma_t} f - T_t^{(s)} f\|_{L^1} = 0 \quad \text{for } f\in L^1. \tag{14}$$

Since the convolution semigroups corresponding to $T_t^{(s)}$ are all concentrated on H, by (14) we get

$$\gamma_t(H)\,\mu(H) = \int_H T_{\gamma_t}\mathbf{1}_H(y)\,\mu(dy) = \int_H \mathbf{1}_H\,d\mu = \mu(H).$$

Hence

$$\gamma_t(H) = 1 \quad \text{for all } t > 0.$$

3. Finally, we show that always $cv(H) = 0$. This, again, is a consequence of the formula (1) applied to the decomposition $A = c(\nu-\delta_e)+A^H$:

$$e^{ct}\mu_t = \sum_{n=0}^{\infty}\gamma^{(n)}(t), \tag{15}$$

where $\gamma^{(0)}(t) = \gamma_t$ and $\gamma^{(n)}(t) = c\int_0^t \gamma_{t-u} * \nu * \gamma^{(n-1)}(u)\,du$, and the above series

converges in the total variation norm. Indeed, observe that as in (11) we have

$$\|\gamma^{(n)}(t)\| = (ct)^n/n!$$

Moreover, since $\gamma_t(H)$ is concentrated on H we get

$$\gamma^{(1)}(t)(H) = c\int_0^t\int \gamma_{t-u}(Hx^{-1})\,d(\nu*\gamma_u)(dx)\,du$$
$$= c\int_0^t\int \nu(y^{-1}H)\,d\gamma_u(y)\,du = c\nu(H)\,t.$$

The above observation, together with (15), shows that

$$e^{ct}\mu_t(H) = 1 + ct\,\nu(H) + O(t^2). \tag{16}$$

Since $(1/t)\,\mu_t(H^c)\to c$ as $t\to 0$, the formula (16) gives

$$c\nu(H) = 0.$$

A convolution semigroup $(\mu_t)_{t>0}$ is called *Gaussian* if

$$\lim_{t\to 0}(1/t)\,\mu_t(V^c) = 0$$

for every neighbourhood of the identity e. Using the same arguments as in [2] we can show that in the decomposition (7) we have $A = A^H$ whenever $(\mu_t)_{t>0}$ is Gaussian. Moreover, in that particular case we do not have to assume that the μ_t are symmetric. Thus we have the following:

COROLLARY 1. *Assume that $(\mu_t)_{t>0}$ is a Gaussian convolution semigroup on G. If H is a Borel subgroup of G such that $\mu_t(H) > 0$ for all $t > 0$ then $\mu_t(H) = 1$ for every $t > 0$.*

Throughout the remainder of the paper G will stand for a locally compact group satisfying the second countability axiom. Suppose that $\xi = \{\xi(t);\, t\in T\}$ is a stochastic process with values in G, where $T\subseteq \boldsymbol{R}$. Assume that ξ is infinitely divisible and symmetric, i.e. for every $s = (s_1, \ldots, s_n)\in T^n$, $s_1 < \ldots < s_n$, there exists a continuous symmetric convolution semigroup $(\mu_t^{(s)})_{t>0}$ on G^n such that $\mu_1^{(s)}$ is the distribution of $(\xi(s_1), \ldots, \xi(s_n))$. Because of the uniqueness of such a semigroup (cf. [1]), applying for every t the Kolmogorov Extension Theorem we obtain a family $(\mu_t)_{t>0}$ of probability measures on G^T endowed with the product σ-field $\mathcal{B}_T$. Moreover, it is clear that $(\mu_t)_{t>0}$ is a continuous convolution semigroup.

Now, let $(\mu_t)_{t>0}$ be a continuous convolution semigroup on G^T. For every $s\in T^n$ the semigroup $(\mu_t^{(s)})_{t>0}$, the corresponding finite-dimensional distribution of $(\mu_t)_{t>0}$, is a continuous convolution semigroup on G^n, which again is a locally compact group. Therefore the generating functional $A^{(s)}$ of $(\mu_t^{(s)})_{t>0}$ is defined on the set $\mathcal{D}(G^n)$ of test functions (see [4], [5]), and can be represented on $\mathscr{D}(G^n)$ in the form

$$Af = \psi_1^{(s)} f + \psi_2^{(s)} f + \int (f - f(e) - \Gamma^n(f))\, d\eta^{(s)},$$

where $\psi_1^{(s)}$ is a primitive form, $\psi_2^{(s)}$ is a quadratic form (which defines a Gaussian semigroup on G^n), Γ^n is a fixed Lévy mapping, and $\eta^{(s)}$ is a Lévy measure on G^n (see [4], [5]). Let now η be the measure on $(G^T, \mathscr{B}_T)$, defined by the family $\eta^{(s)}$. η is called the *Lévy measure* of $(\mu_t)_{t>0}$.

We now reformulate our theorem in terms of subgroups of sample paths of infinitely divisible processes with values in G.

COROLLARY 2. *Let $\xi = \{\xi(t);\ t \in T\}$ be a symmetric infinitely divisible stochastic process with values in G and let η be the Lévy measure of ξ. Suppose that H is a measurable subgroup of G^T. Then:*

(i) *If $\eta(H^c) = \infty$ then $P\{\xi \in H\} = 0$.*

(ii) *If $\eta(H^c) = 0$ then either $P\{\xi \in H\} = 0$ or $P\{\xi \in H\} = 1$.*

Proof. Let us first observe that in view of the structure of $\mathscr{B}_T$ we may restrict our considerations to the case when T is countable. However, G^∞ is then again separable, so our theorem still can be applied. Now, let $(\mu_t)_{t>0}$ be the symmetric continuous convolution semigroup on G^T determined by ξ. Suppose that for every $t > 0$ we have $\mu_t(H) > 0$. By our theorem we obtain

$$A = A^H + c(\nu - \delta_e),$$

where A^H is the generating functional of a continuous convolution semigroup $(\gamma_t)_{t>0}$ which is concentrated on H and ν is a probability measure with the property $c\nu(H) = 0$. Let now η^H, η be the Lévy measures of $(\gamma_t)_{t>0}$ and $(\mu_t)_{t>0}$, respectively. By the uniqueness of η we obtain

$$\eta = \eta^H + c\nu.$$

Now, if $\eta^H(H^c) \neq 0$ then we may decompose A^H as $A_1 + d(m - \delta_e)$, where $0 < d < \infty$ and m is a probability measure concentrated on H^c. Using once more the formula (1) we then obtain $(\exp tA_1)(H) = 1$ for every $t > 0$ and

$$t = \int_0^t (\exp(t-u)A_1) * m * \exp(uA_1)(H)\, du = \int_0^t m(H)\, du = 0.$$

Therefore $\eta^H(H^c) = 0$. Thus $\eta(H^c) = \infty$ implies that $\nu(H^c) = \infty$, which is impossible. This proves the first part of the corollary.

The proof of the second part is even easier: if $\eta(H^c) = 0$ and $\mu_t(H) > 0$ then $c\nu(H^c) = 0$, so $A = A^H$, which means that $\mu_t(H) = \gamma_t(H) = 1$. This concludes the proof.

Remark. Corollary 2 was proved in [6] for infinitely divisible random variables with values in a locally compact group G, instead of stochastic processes, and under the additional assumption that H is a normal subgroup.

The next corollary follows immediately from the fact that the Lévy measure of a Gaussian process vanishes.

COROLLARY 3. *Let $\xi = \{\xi(t);\ t \in T\}$ be a symmetric Gaussian process with values in G and let H be a measurable subgroup of G^T. Then either $P\{\xi \in H\} = 0$ or $P\{\xi \in H\} = 1$.*

Acknowledgement. The authors are grateful to Andrzej Hulanicki for stimulating conversations on the subject of this paper.

References

[1] H. Byczkowska, *The invariance principle for random variables with values in locally compact groups*, Colloq. Math. 45 (1981), 61–70.

[2] T. Byczkowski and A. Hulanicki, *Gaussian measure of normal subgroups*, Ann. Probab. 11 (1983), 685–691.

[3] N. Dunford and J. T. Schwartz, *Linear Operators*, part I, Wiley, New York 1958.

[4] W. Hazod, *Stetige Halbgruppen von Wahrscheinlichkeitsmassen und erzeugende Distributionen*, Lecture Notes in Math. 595, Springer, New York 1977.

[5] H. Heyer, *Probability Measures on Locally Compact Groups*. Springer, Berlin 1977.

[6] A. Janssen, *Zero-one laws for infinitely divisible probability measures on groups*, Z. Wahrsch. Verw. Gebiete 60 (1982), 119–138.

[7] M. Loève, *Probability Theory*, vol. II, Springer, New York 1978.

INSTYTUT MATEMATYKI POLITECHNIKI WROCŁAWSKIEJ
INSTITUTE OF MATHEMATICS, TECHNICAL UNIVERSITY OF WROCŁAW
Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland

Received November 28, 1986
Revised version May 11, 1987

(2248)