and

$$\|\tilde{g}\|_{A(s)} \leqslant \|g\|_{A(s;\Omega)} \leqslant C_{s,A} \|\tilde{g}\|_{A(s)}$$

for all functions g on Ω with $\tilde{g} \in L^1_{\mathrm{loc}}(\boldsymbol{R}^n)$ and for all $s > 0$.

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STUDIA MATHEMATICA, T. XCV (1990)

Weighted inequalities for the Hilbert transform and the adjoint operator in the continuous case

by

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Abstract. We prove two-weight norm inequalities in L^2 for the Hilbert transform in $\boldsymbol{R}$, of the Helson, Szegö and Sarason type.

I. Introduction. Arocena, Cotlar and Sadosky (see [3]) proved that the theory of generalized Toeplitz kernels can be used to obtain the theorems of Helson, Szegö and Sarason type (see [9, 10, 13]), with refinements.

Nevertheless in the case of two measures they do not obtain the Helson, Szegö and Sarason formula and in the case of $\boldsymbol{R}$ they consider, as Adams does (see [1]), functions with vanishing moments. In this paper, we consider two tempered measures, functions with vanishing Fourier transform in an interval, and use the theory of generalized Toeplitz kernels to give a constructive exponential characterization of Helson, Szegö and Sarason type for the Hilbert transform; and we do the same for finite measures, but with the adjoint operator.

The problems considered here arose in a natural way when we studied the following prediction theory problem proposed by Professor Ibragimov (private communication): characterize the continuous parameter weakly stationary completely linearly regular process such that the maximal correlation coefficient ϱ_t is $O(e^{-\lambda t})$ (see also [11]). In the previous papers (cf. [6–8]) this theory and an analogue of Theorem 1 were considered to obtain results about the rate of convergence of the maximal correlation coefficient in the continuous case including a solution to the problem stated by Professor Ibragimov.

An extension, to matrix-valued measures, of the results presented here is given in [5].

II. Basic problems

DEFINITION. A measure μ is *tempered of order* $\leqslant 2$ if $\mu/(x^2+1)$ is a finite measure.

Set

$M(\boldsymbol{R})$ = the positive finite Borel measures in $\boldsymbol{R}$,
$M^2(\boldsymbol{R})$ = the positive Borel tempered measures of order $\leqslant 2$ in $\boldsymbol{R}$.

We use the following notation:

$$e_t(x) = e^{itx},$$
$$E = \operatorname{span}\{e_t\colon t\in \boldsymbol{R}\}, \qquad E' = \operatorname{span}\{e_t\colon t \neq 0\},$$
$$E_1 = \operatorname{span}\{e_t\colon t > 0\}, \qquad E_2 = \operatorname{span}\{e_t\colon t < 0\},$$
$$F_1 = \{\phi(x)/(x+i)\colon \phi\in E_1\}, \qquad F_2 = \{\phi(x)/(x-i)\colon \phi\in E_2\},$$
$$E_{(t)} = e_t E_1 + E_2, \qquad F_{(t)} = e_t F_1 + F_2,$$
$$F = \{f\in C^\infty(\boldsymbol{R})\colon (1+|x|)f(x) \text{ is bounded}\}.$$

We have $F_1 \subset F$, $F_2 \subset F$, $F_1 \subset H^{2+} \subset L^2(\boldsymbol{R}, dx)$, $F_2 \subset H^{2-} \subset L^2(\boldsymbol{R}, dx)$ and $F \subset L^2(\boldsymbol{R}, \mu)$ if $\mu\in M^2(\boldsymbol{R})$.

We shall study the following problems:

1) Characterization of the pairs of measures $(\mu, \nu)\in M^2(\boldsymbol{R})\times M^2(\boldsymbol{R})$ such that

$$\int_{-\infty}^{\infty} |H(f)|^2\, d\mu \leqslant a \int_{-\infty}^{\infty} |f|^2\, d\nu$$

for every function $f\in F_{(t)}$ asociated to a function $\phi\in E_{(t)}$ with vanishing Fourier transform in an interval via

$$f = e_t\frac{\phi_1}{x+i}+\frac{\phi_2}{x-i}, \qquad \phi = e_t\phi_1+\phi_2.$$

Here $a > 1$ and H is the Hilbert transform.

2) Characterization of the pairs of measures $(\mu, \nu)\in M(\boldsymbol{R})\times M(\boldsymbol{R})$ such that

$$\int_{-\infty}^{\infty} |A(f)|^2\, d\mu \leqslant a \int_{-\infty}^{\infty} |f|^2\, d\nu$$

for every function $f\in F_{(t)}$ related to a function $\phi\in E_{(t)}$ with vanishing Fourier transform in an interval via $f(x) = \phi(x)/(x-i)$, with $a > 1$ and A the adjoint operator in $L^\infty(\boldsymbol{R})$, which associates to a function f its adjoint function $A(f) = \tilde{f}$.

For the Hilbert transform H (see [4]) we have

$$H\left(\frac{\phi}{x+i}\right) = -i\frac{\phi}{x+i} \quad \text{if } \phi\in E_1,$$
$$H\left(\frac{\phi}{x-i}\right) = +i\frac{\phi}{x-i} \quad \text{if } \phi\in E_2.$$

Therefore if $f = e_t\frac{\phi_1}{x+i}+\frac{\phi_2}{x-i}$ with $\phi_1\in E_1$ and $\phi_2\in E_2$, then $H(f) = -ie_t\frac{\phi_1}{x+i}+i\frac{\phi_2}{x-i}$.

For $a \geqslant 1$, we set

$$R_{2,t}(a) = \Big\{(\mu, \nu)\in M^2(\boldsymbol{R})\times M^2(\boldsymbol{R})\colon \int_{-\infty}^{\infty} |H(f)|^2\, d\mu \leqslant a \int_{-\infty}^{\infty} |f|^2\, d\nu \text{ for all } f\in F_{(t)}\Big\}.$$

Now we shall explain what we understand by the adjoint of a bounded function. If $\phi\in L^\infty(\boldsymbol{R}, dx)$ and $f(x) = \phi(x)/(x-i)$ then

$$A(\phi)(x) = (x-i)H(f)(x) \quad \text{(see [12], p. 193)}.$$

As $E_{(t)} \subset L^\infty$, if $\phi = e_t\phi_1+\phi_2$, with $\phi_1\in E_1$, $\phi_2\in E_2$, and $f(x) = (e_t\phi_1+\phi_2)(x)/(x-i)$ then $A(\phi)(x) = (x-i)H(f)(x)$.

For $a \geqslant 1$, we set

$$Q_t(a) = \Big\{(\mu, \nu)\in M(\boldsymbol{R})\times M(\boldsymbol{R})\colon \int_{-\infty}^{\infty} |A(\phi)|^2\, d\mu \leqslant a \int_{-\infty}^{\infty} |\phi|^2\, d\nu \text{ for all } \phi\in F_{(t)}\Big\}.$$

Using this notation we specify problems 1) and 2):

1) Characterization of $R_{2,t}(a)$ for $a \geqslant 1$.
2) Characterization of $Q_t(a)$ for $a \geqslant 1$.

For solving problems 1) and 2) we shall make use of the lifting theorem for weakly positive matrices of measures in $\boldsymbol{R}$.

III. Lifting theorem. We generally follow the notation of [2, 3]. Let $M = (\mu_{\alpha\beta})_{\alpha,\beta=1,2}$ be a matrix of measures in $\boldsymbol{R}$.

DEFINITION. M is *positive* if for every Borel set $\Delta \subset \boldsymbol{R}$, $(\mu_{\alpha\beta}(\Delta))_{\alpha,\beta=1,2}$ is a positive-definite numerical matrix.

That is equivalent to the following three conditions:

(i) $\mu_{11} \geqslant 0$, $\mu_{22} \geqslant 0$.
(ii) $\bar{\mu}_{12} = \mu_{21}$.
(iii) $|\mu_{12}(\Delta)|^2 \leqslant \mu_{11}(\Delta)\mu_{22}(\Delta)$ for every Borel set $\Delta \subset \boldsymbol{R}$.

Let M be such that (i) and (ii) hold. It can be seen that M is positive if and only if

(iv) $\sum_{\alpha=1}^{2}\sum_{\beta=1}^{2}\int_{-\infty}^{\infty} \phi_\alpha\bar{\phi}_\beta\, d\mu_{\alpha\beta} \geqslant 0$ for all $(\phi_1, \phi_2)\in E\times E$.

A weaker condition is the following.

DEFINITION. M is *weakly positive* if M satisfies (i), (ii) and

(v) $\sum_{\alpha=1}^{2}\sum_{\beta=1}^{2}\int_{-\infty}^{\infty} \phi_\alpha\bar{\phi}_\beta\, d\mu_{\alpha\beta} \geqslant 0$ for all $(\phi_1, \phi_2)\in E_1\times E_2$.

THEOREM (Lifting property for weakly positive matrices of measures in $\boldsymbol{R}$) [2]. *Let $M = (\mu_{\alpha\beta})_{\alpha,\beta=1,2}$ be a matrix of finite Borel measures in $\boldsymbol{R}$. Then the following conditions are equivalent:*

(a) M *is weakly positive.*

(b) *There exists* $h \in H^1(\boldsymbol{R})$ *such that* $M + \begin{bmatrix} 0 & h\,dx \\ \bar{h}\,dx & 0 \end{bmatrix}$ *is positive.*

PROPOSITION. *Let* $\mu, \nu \in M^2(\boldsymbol{R})$, $a > 1$,

$$\mu_{11} = \mu_{22} = \frac{a\nu - \mu}{x^2+1}, \qquad \mu_{12} = \bar{\mu}_{21} = \frac{(a\nu+\mu)e_t}{(x+i)^2}.$$

M is weakly positive if and only if $(\mu, \nu) \in R_{2,t}(a)$. In this case, μ is an absolutely continuous measure.

Proof. Let $f \in F_{(t)}$, $f = \dfrac{e_t \phi_1}{x+i} + \dfrac{\phi_2}{x-i}$ with $\phi_1 \in E_1$ and $\phi_2 \in E_2$. Then

$$\int_{-\infty}^{\infty} |\phi_1|^2 \frac{d(a\nu-\mu)}{x^2+1} + \int_{-\infty}^{\infty} \phi_1 \bar{\Phi}_2 \frac{e_t d(a\nu+\mu)}{(x+i)^2}$$

$$+ \int_{-\infty}^{\infty} \phi_2 \bar{\Phi}_1 \frac{e_{-t} d(a\nu+\mu)}{(x-i)^2} + \int_{-\infty}^{\infty} |\phi_2|^2 \frac{d(a\nu-\mu)}{x^2+1}$$

$$= a \int_{-\infty}^{\infty} \left(\frac{|\phi_1|^2}{x^2+1} + \frac{e_t \phi_1 \bar{\Phi}_2}{(x+i)^2} + \frac{e_{-t}\phi_2 \bar{\Phi}_1}{(x-i)^2} + \frac{|\phi_2|^2}{x^2+1} \right) d\nu$$

$$- \int_{-\infty}^{\infty} \left(\frac{|\phi_1|^2}{x^2+1} + \frac{e_t \phi_1 \bar{\Phi}_2}{(x+i)^2} + \frac{e_{-t}\phi_2 \bar{\Phi}_1}{(x-i)^2} + \frac{|\phi_2|^2}{x^2+1} \right) d\mu$$

$$= a \int_{-\infty}^{\infty} |f|^2 \, d\nu - \int_{-\infty}^{\infty} |Hf|^2 \, d\mu.$$

From this calculation we obtain the first result.

For the second part notice that

$$\Big|\mu_{12}(\Delta) + \int_{\Delta} h(x)\,dx\Big|^2 \leqslant \mu_{11}(\Delta)\mu_{22}(\Delta) \quad \text{and} \quad |\Delta| = 0$$

implies $|(a\nu+\mu)e_t(\Delta)| \leqslant (a\nu-\mu)(\Delta)$ and then $\mu(\Delta) = 0$.

IV. Characterization of $R_{2,t}(a)$ and $Q_t(a)$ for $a \geqslant 1$. We obtain a characterization of a class of weakly positive matrix measures, which is the crucial tool for solving our problems.

PROPOSITION. *Let $W = (w_{\alpha\beta})_{\alpha,\beta=1,2}$ be a hermitian matrix of densities of tempered measures of order $\leqslant 2$ in $\boldsymbol{R}$, that is, functions $w_{\alpha\beta}$ with $w_{\alpha\beta}/(x^2+1) \in L^1(\boldsymbol{R})$, such that* (i) $w_{11}(x) = w_{22}(x) > 0$ *for almost all* $x \in \boldsymbol{R}$, (ii) *there exists $t \geqslant 0$ such that $w_{12}(x) = |w_{12}(x)|e_t(x)$ for almost all $x \in \boldsymbol{R}$, and*

(iii) $w_{11}(x) < |w_{12}(x)|$ *for almost all* $x \in \boldsymbol{R}$. *Let* $q = w_{12}/w_{11}$. *Then the following conditions are equivalent:*

(a) $Z = \begin{bmatrix} \dfrac{w_{11}}{x^2+1} & \dfrac{w_{12}}{(x+i)^2} \\ \dfrac{w_{21}}{(x-i)^2} & \dfrac{w_{22}}{x^2+1} \end{bmatrix}$ *is weakly positive.*

(b) *There exists $h \in H^1(\boldsymbol{R})$ such that if $v(x) = -\arg\big((x+i)^2 e_{-t}(x)h(x)\big)$ then*

$$w_{11}(x) = \frac{(x^2+1)|h(x)|e^{u(x)}}{\sqrt{|q(x)|^2-1}},$$

$$|u(x)| \leqslant \operatorname{arcosh}\left(\frac{|q(x)|\cos v(x)}{\sqrt{|q(x)|^2-1}}\right),$$

$$|v(x)| \leqslant \arcsin\left(\frac{1}{|q(x)|}\right) < \frac{\pi}{2}.$$

Proof. Z is weakly positive if and only if there exists $h \in H^1(\boldsymbol{R})$ such that

$$\left|\frac{w_{12}(x)}{(x+i)^2} - h(x)\right|^2 \leqslant \frac{w_{11}^2(x)}{(x^2+1)^2} \quad \text{for almost all } x \in \boldsymbol{R}.$$

Let $\phi = (x+i)^2 e_{-t} h$ and $v(x) = -\arg\phi(x)$. We have

$$\left|\frac{w_{12}}{(x+i)^2} - h(x)\right|^2 - \frac{w_{11}^2}{(x^2+1)^2} = \frac{(|q|^2-1)w_{11}^2 - 2|q|(\operatorname{Re}\phi)w_{11} + |\phi|^2}{(x^2+1)^2}.$$

Then $z_1 \leqslant w_{11} \leqslant z_2$, where for $k = 1, 2$ we have set

$$z_k = \frac{|\phi|}{\sqrt{|q|^2-1}} \exp\left((-1)^k \operatorname{arcosh}\left(\frac{|q|\cos v}{\sqrt{|q|^2-1}}\right)\right)$$

because $(-1)^k \operatorname{arcosh}(x) = \log\big(x + (-1)^k\sqrt{x^2-1}\big)$.

Finally, $z_1 \leqslant w_{11} \leqslant z_2$ if and only if

$$\left|\log\left(\frac{w_{11}\sqrt{|q|^2-1}}{|\phi|}\right)\right| \leqslant \operatorname{arcosh}\left(\frac{|q|\cos v(x)}{\sqrt{|q|^2-1}}\right).$$

Let $u(x) = \log\big(w_{11}\sqrt{|q(x)|^2-1}/|\phi|\big)$. Then

$$w_{11}(x) = \frac{(x^2+1)|h(x)|e^{u(x)}}{\sqrt{|q(x)|^2-1}}$$

and we have the first inequality.

Now, $|w_{12}/(x+i)^2 - h| \leqslant w_{11}/(x^2+1)$ implies

$$\left||q|e_t\left(\frac{x-i}{x+i}\right) - \frac{(x^2+1)h}{w_{11}}\right| \leqslant 1, \qquad \left||q| - \frac{(x+i)^2 e_{-t} h}{w_{11}}\right| \leqslant 1.$$

Then $v = -\arg(\phi/w_{11})$ and $\left||q|-\phi/w_{11}\right| \leqslant 1$. Thus $|v| < \pi/2$ and $\sin|v| \leqslant 1/|q|$.

THEOREM 1. *Let μ, $\nu \in M^2(\boldsymbol{R})$, $a > 1$, $t \geqslant 0$. Then the following properties are equivalent*:

(a) $(\mu, \nu) \in R_{2,t}(a)$.

(b) $d\mu = w(x)dx$, $d\nu = y(x)dx + d\nu_s$ (*where ν_s is the singular part of ν with respect to Lebesgue measure*) *and there exists $h \in H^1(\boldsymbol{R})$ such that*

$$\sqrt{y(x)w(x)} = (x^2+1)|h(x)|e^{u(x)}$$

where $v(x) = -\arg\left((x+i)^2 e_{-t}(x)h(x)\right)$ *and* $r_a(x) = \dfrac{ay(x)-w(x)}{ay(x)+w(x)}$ *satisfy*

$$|u(x)| \leqslant \operatorname{arcosh}\left(\frac{\cos v(x)}{\sqrt{1-r_a^2(x)}}\right),$$

$$|v(x)| \leqslant \frac{\pi}{2} - \arccos r_a(x) < \frac{\pi}{2}.$$

Proof. Let $(\mu, \nu) \in R_{2,t}(a)$. Then

$$Z = \begin{bmatrix} \dfrac{a\nu-\mu}{x^2+1} & \dfrac{(a\nu+\mu)e_t}{(x+i)^2} \\ \dfrac{(a\nu+\mu)e_{-t}}{(x-i)^2} & \dfrac{a\nu-\mu}{x^2+1} \end{bmatrix}$$

is weakly positive.

We know that μ is absolutely continuous. Let $d\mu = w(x)dx$ and let y be the density of the nonsingular part of ν. From Lemma 4 of [3] we see that

$$Z = \begin{bmatrix} \dfrac{ay-w}{x^2+1} & \dfrac{(ay+w)e_t}{(x+i)^2} \\ \dfrac{(ay+w)e_{-t}}{(x-i)^2} & \dfrac{ay-w}{x^2+1} \end{bmatrix}$$

is weakly positive.

Using the last proposition we deduce that there exists $h \in H^1(\boldsymbol{R})$ such that

$$ay(x)-w(x) = \frac{(x^2+1)|h(x)|e^{u(x)}}{\sqrt{|q(x)|^2-1}},$$

and if

$$v(x) = -\arg\left((x+i)^2 e_{-t}(x)h(x)\right), \qquad q(x) = \frac{(ay(x)+w(x))e_t(x)}{ay(x)-w(x)},$$

then

$$|u(x)| \leqslant \operatorname{arcosh}\left(\frac{|q(x)|\cos v(x)}{\sqrt{|q(x)|^2-1}}\right),$$

$$|v(x)| \leqslant \arcsin\left(\frac{1}{|q(x)|}\right) < \frac{\pi}{2}.$$

Since

$$\sqrt{|q(x)|^2-1} = \frac{\sqrt{y(x)w(x)}}{ay(x)-w(x)}, \qquad r_a(x) = \frac{1}{|q(x)|},$$

it follows that

$$\frac{|q(x)|}{\sqrt{|q(x)|^2-1}} = \frac{1}{\sqrt{1-r_a^2(x)}}, \qquad \sqrt{y(x)w(x)} = (x^2+1)|h(x)|e^{u(x)}.$$

For the converse, we just notice that at all steps of the proof we have equivalences.

This theorem is a generalization to the continuous case and to tempered measures of order $\leqslant 2$ of the theorem of Helson and Sarason. In the case $t = 0$, it is a generalization of the theorem of Helson and Szegö (see [9, 10, 13]). The theorems study the boundedness of the Hilbert transform in spaces with weights in the discrete case.

THEOREM 2. *Let μ, $\nu \in M(\boldsymbol{R})$, $a > 1$, $t \geqslant 0$. Then the following properties are equivalent*:

(a) $(\mu, \nu) \in Q_t(a)$.

(b) $d\mu = w(x)dx$ *and* $d\nu = y(x)dx + d\nu_s$ *and there exists* $h \in H^1(\boldsymbol{R})$ *such that* $\sqrt{y(x)w(x)} = |h(x)|e^{u(x)}$ *where* $v(x) = -\arg\left((x+i)^2 e_{-t}(x)h(x)\right)$ *and* $r_a(x) = \dfrac{ay(x)-w(x)}{ay(x)+w(x)}$ *satisfy*

$$|u(x)| \leqslant \operatorname{arcosh}\left(\frac{\cos v(x)}{\sqrt{1-r_a^2(x)}}\right),$$

$$|v(x)| \leqslant \frac{\pi}{2} - \arccos r_a(x) < \frac{\pi}{2}.$$

Proof. Consider $d\mu' = (x^2+1)d\mu$ and $d\nu' = (x^2+1)d\nu$, and apply Theorem 1 to the measures μ' and ν'.

Let us finally observe that this theorem also generalizes the theorems of Helson–Sarason and Helson–Szegö to finite measures in the continuous case.

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Absolutely p-summing operators and Banach spaces containing all l_p^n uniformly complemented

by

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Abstract. It is proved that for $p = 1$, 2 and ∞ a Banach space G contains uniformly complemented all l_p^n's if (and only if) each operator $T: E \to F$ such that $\mathrm{id}_G \otimes T: G \otimes_\varepsilon E \to G \otimes_\pi F$ is continuous splits into a product $T = RS$ of an absolutely p-summing operator S and an operator R with an absolutely p'-summing dual.

0. Introduction. In [4] Jarchow conjectured that for a fixed real number $1 \leqslant p \leqslant \infty$ a Banach space G contains all l_p^n uniformly complemented if (and only if) it satisfies the following condition $(*)$: Every operator $T \in \mathscr{L}(E, F)$ such that

$$\mathrm{id}_G \otimes T\colon\ G \otimes_\varepsilon E \to G \otimes_\pi F$$

is continuous can be written as a product RS of two appropriate operators R and S where R' is absolutely p'-summing and S is absolutely p-summing. We give an affirmative answer for $p = 1$, 2 and ∞. For arbitrary $1 \leqslant p \leqslant \infty$ it is proved as a by-product that G satisfies $(*)$ if and only if there is a constant $\lambda \geqslant 1$ such that for every natural number n there are finitely many operators $I_1, \ldots, I_m \in \mathscr{L}(l_p^n, G)$ and $P_1, \ldots, P_m \in \mathscr{L}(G, l_p^n)$ (where m depends on n) with

$$\mathrm{id}_{l_p^n} = \sum_{k=1}^{m} P_k I_k, \qquad \sum_{k=1}^{m} \|P_k\| \, \|I_k\| \leqslant \lambda.$$

Standard notions and notations from Banach space theory are used, as presented in [5]. For the general theory of Banach operator ideals we refer the reader to [8].

1. S_p-spaces and T_p-spaces. As usual l_p^n stands for the space $\boldsymbol{R}^n$ equipped with the l_p-norm. A *real* Banach space G is said to be an S_p-*space* if it contains all l_p^n uniformly complemented, i.e., there is a sequence (G_n) of n-dimensional subspaces of G and projections $P_n \in \mathscr{L}(G, G)$ onto G_n such that

$$\sup_n d(G_n, l_p^n) < \infty, \qquad \sup_n \|P_n\| < \infty$$

(here as usual $d(\cdot, \cdot)$ denotes the Banach–Mazur distance). Clearly, G is an S_p-space if and only if there is a $\lambda \geqslant 1$ such that for every n there are operators