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Summability "au plus petit terme"

by

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Abstract. There is a curious phenomenon in the theory of Gevrey asymptotic expansions. In general the asymptotic formal power series is divergent, but there is some partial sum which approaches the value of the function very well. In this note we prove that there exists a truncation of the series which comes near the function in an exponentially flat way.

A *polysector* is a subset of $\mathbb{C}^n$ of the type

$$V = \{(z_1, \ldots, z_n) \in \mathbb{C}^n : 0 < |z_j| < r_j,\ \arg z_j \in (a_j, b_j),\ 1 \le j \le n\},$$

where $|b_j - a_j| < 2\pi$, $j = 1, \ldots, n$.

Let $s \in [0, \infty)$. Let f be a holomorphic function in V. We say that the formal power series $\sum_{\alpha \in \mathbb{N}^n} a_\alpha z^\alpha$ is the *weak asymptotic expansion of Gevrey type s of the function f* if the following condition is satisfied:

There is a constant $K > 0$ such that for any $t \in \mathbb{N}$,

$$\Big| f(z) - \sum_{|\alpha| < t} a_\alpha z^\alpha \Big| < K t!^s |z|^t \quad \text{for } z \in V.$$

THEOREM. *Let $s > 0$. Let $\sum_{\alpha \in \mathbb{N}^n} a_\alpha z^\alpha$ be the weak asymptotic expansion of Gevrey type s of a holomorphic function f in a polysector V. Then there exist constants $A > 0$ and $B > 0$ such that for every z in V,*

$$\Big| f(z) - \sum_{|\alpha|=0}^{p_z} a_\alpha z^\alpha \Big| < A \exp(-B/|z|^{1/s})$$

whith some $p_z \in \mathbb{N}$.

Proof. We take $|z| = \max(|z_1|, \ldots, |z_n|)$ as norm. First assume that

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$\varrho = r_1 = \ldots = r_n < 1$. Then for each p in $\mathbb{N}$,

$$\Big|f(z) - \sum_{|\alpha|=0}^{p-1} a_\alpha z^\alpha\Big| < A(p^p|z|^{p/s})^s = A\left[\exp\left(\frac{p|z|^{1/s}\ln(p|z|^{1/s})}{|z|^{1/s}}\right)\right]^s$$

with some $A > 0$ (by Stirling's formula). Let $\varepsilon > 0$ be such that $[\varepsilon, \varepsilon + \varrho^{1/s}]$ is contained in $(0,1)$, and let $m > 0$ satisfy

$$(*) \qquad t\ln t \le -m \quad \text{in } [\varepsilon, \varepsilon + \varrho^{1/s}].$$

Now, fixing $z \in V$, we have $\alpha_{p-1} \le A\beta_p^s$ with

$$\alpha_p := \Big|f(z) - \sum_{|\alpha|=0}^{p} a_\alpha z^\alpha\Big|, \qquad \beta_p := \exp\left(\frac{p|z|^{1/s}\ln(p|z|^{1/s})}{|z|^{1/s}}\right)$$

for any $p \in \mathbb{N}$. Using the inequality $(*)$ we get the implication

$$p|z|^{1/s} \in [\varepsilon, \varepsilon + \varrho^{1/s}] \Rightarrow \alpha_{p-1} \le A\exp(-ms/|z|^{1/s}).$$

We claim that $\Lambda = \{p \in \mathbb{N} : p|z|^{1/s} \in [\varepsilon, \varepsilon + \varrho^{1/s}]\}$ is not empty. Indeed, let p_0 be the greatest positive integer satisfying $p_0|z|^{1/s} < \varepsilon + \varrho^{1/s}$. Then $p_0 \in \Lambda$, for otherwise one would have $p_0|z|^{1/s} < \varepsilon$, so that $(p_0+1)|z|^{1/s} < \varepsilon + \varrho^{1/s}$, which contradicts the maximality of p_0. Moreover, Λ is a finite set. Therefore,

$$\alpha_{p_z} := \min\{\alpha_p : p \in \Lambda\} < A\exp(-ms/|z|^{1/s}),$$

which implies the statement in this case.

Now put $\varrho = \max\{r_1, \ldots, r_n\}$. Using a homothety we can assume that $\varrho < 1$. We may suppose that $\varrho = r_1$. Then it is sufficient to apply the first case to the function

$$g(z_1, \ldots, z_n) = f\left(z_1, \frac{r_2}{r_1}z_2, \ldots, \frac{r_n}{r_1}z_n\right)$$

in the polysector

$$V' = V_1 \times \frac{r_1}{r_2}V_2 \times \ldots \times \frac{r_1}{r_n}V_n,$$

in order to have our statement.

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