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ON UNCOUNTABLE COLLECTIONS OF CONTINUA AND THEIR SPAN

BY

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We prove that if the Euclidean plane $\mathbb{R}^2$ contains an uncountable collection of pairwise disjoint copies of a tree-like continuum X, then the symmetric span of X is zero, $sX = 0$. We also construct a modification of the Oversteegen–Tymchatyn example: for each $\varepsilon > 0$ there exists a tree $X \subset \mathbb{R}^2$ such that $\sigma X < \varepsilon$ but X cannot be covered by any 1-chain. These are partial solutions of some well-known problems in continua theory.

1. Introduction. It is well known that the plane $\mathbb{R}^2$ does not contain uncountably many pairwise disjoint triods [14]. This result has been generalized in various directions [1], [3], [4], [16], [19], [21] and [22]. In the present paper we obtain further strengthenings of some of these results.

Consider the following conditions on a planar tree-like continuum X:

(C) X is chainable;

(U) The plane contains uncountably many disjoint copies of X;

(Σ) $\sigma X = 0$; and

(S) $sX = 0$.

Let $\widetilde{X}^*_\varepsilon = \{(x,y) \in X^2 \mid \operatorname{dist}(x,y) \geq \varepsilon\}$ be the deleted product of X. Consider the involution $t(x,y) = (y,x)$ on $\widetilde{X}^*_\varepsilon$. Then the *span* of X is defined as follows [12]:

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$$\sigma X = \sup\{\varepsilon \geq 0 \mid \text{there is a subcontinuum } Z \subset \widetilde{X}^*_\varepsilon \text{ such that } \operatorname{pr}_1(Z) = \operatorname{pr}_2(Z)\}$$

and the *symmetric span* of X is defined [8] by

$$sX = \sup\{\varepsilon \geq 0 \mid \text{there is a subcontinuum } Z \subset \widetilde{X}^*_\varepsilon \text{ such that } Z = t(Z)\}.$$

The implication (C)$\Rightarrow$(U) was proved in [20] and (C)$\Rightarrow$ (Σ) in [12]. Clearly, (Σ) $\Rightarrow$(S) is obvious. It is an open problem in continua theory whether (U)$\Rightarrow$(C) [10] or (U)$\Rightarrow$(Σ) [7, 430], or (S)$\Rightarrow$(Σ) [7, 434], or (Σ)$\Rightarrow$(C) [7, 435] (see [13], [15]).

$$\begin{array}{ccc} \mathrm{C} & \Rightarrow & \Sigma \\ \Downarrow & & \Downarrow \\ \mathrm{U} & \Rightarrow & \mathrm{S} \end{array}$$

We prove a theorem which provides us with a tool for evaluation of the symmetric span (compare [2, 1.1.2], [16, I, Th. 2.6], [16, II, Th. 4]).

Theorem (1.1). (a) *If $X \subset \mathbb{R}^2$ is a tree-like continuum and $f : X \to \mathbb{R}^2$ is a map ε-close to an inclusion and such that $X \cap f(X) = \emptyset$, then $sX \leq \varepsilon$. Moreover, if there is a vector $\vec{\varepsilon} \in \mathbb{R}^2$ such that $f(x) = x + \vec{\varepsilon}$, then $\sigma X \leq \varepsilon = |\vec{\varepsilon}|$.*

(b) *If $f, g : X \to \mathbb{R}^2$ are ε-close maps with disjoint images from a tree-like continuum, then $sf \leq \varepsilon$.*

Here,

$$sf = \sup\{\varepsilon > 0 \mid \text{there is a subcontinuum } Z \subset X^2 \text{ such that } Z = t(Z) \text{ and } \operatorname{dist}(f(x), f(y)) \geq \varepsilon \text{ for each } (x,y) \in Z\}.$$

Let $\chi : \widetilde{(\mathbb{R}^2)}^*_\varepsilon \to S^1$ be the map defined by $\chi(x,y) = (x-y)/\|x-y\|$. The proof of Theorem (1.1)(a) is based on the fact that under the assumptions of the theorem, $\chi|_{\tilde{X}^*_\varepsilon}$ is an inessential equivariant mapping. Take a covering $\widetilde{\chi} : \widetilde{X}^*_\varepsilon \to \mathbb{R}$ of $\chi|_{\tilde{X}^*_\varepsilon}$ and for $(x,y) \in \widetilde{X}^*_\varepsilon$ define that $x < y$ if $\widetilde{\chi}(x,y) < \widetilde{\chi}(y,x)$. Evidently, "$<$" is a continuous relation (in general it is not transitive). Hence $\widetilde{X}^*_\varepsilon$ cannot contain a subcontinuum Z such that $Z = t(Z)$, so $sX \leq \varepsilon$. If $X \cap (X + \vec{\varepsilon}) = \emptyset$, then each subcontinuum of X has a $<$-minimal point. Hence $\widetilde{X}^*_\varepsilon$ cannot contain a subcontinuum Z such that $\operatorname{pr}_1 Z = \operatorname{pr}_2 Z$, so $\sigma X \leq \varepsilon$.

Conjecture (1.2). The condition "$f(x) = x + \vec{\varepsilon}$" is unnecessary for the existence of a $<$-minimal point in every subcontinuum of X (Conjecture (1.2) implies that (U)$\Rightarrow$(Σ)).

Corollary (1.3). (a) ((U)$\Rightarrow$(S)) *If the plane contains an uncountable collection of disjoint copies of a tree-like continuum X (or even the product* *of X with a convergent sequence), then $sX = 0$. Moreover, if these copies are obtained by parallel transfers from one another, then $\sigma X = 0$.*

(b) *If $f_\alpha : X \to \mathbb{R}^2$ is a collection of maps from a tree-like continuum X with disjoint images, then $sf_\alpha = 0$ for all but countably many α.*

Since from $sX = 0$ it follows that X is atriodic [8], (U)⇒(S) generalizes Moore's and Burgess' [4] theorems.

Ingram has constructed in [10] an uncountable collection of pairwise disjoint, nonhomeomorphic, tree-like continua with the positive symmetric span in the plane. This shows that the implication (U)⇒(S) does not extend to the case of nonhomeomorphic compacta. From (U)⇒(S) it follows that Ingram's continuum K [10], satisfying $sK > 0$, yields an example of an atriodic continuum K such that the plane does not contain an uncountable collection of pairwise disjoint copies of K (this answers a question from [5]).

We also construct an example which is a modification of [16, I, Fig. 1]. The proof that $\sigma K < \varepsilon$ is based on the "moreover" part of Theorem (1.1)(a) and is shorter than in [16].

EXAMPLE (1.4). *For each $\varepsilon > 0$, there is a tree $K \subset \mathbb{R}^2$ such that $\sigma K < \varepsilon$, but K cannot be covered by any chain with link diameters less than* 1.

2. Proofs

Proof of Theorem (1.1)(a). Suppose, to the contrary, that $sX > \varepsilon$. Then there is a subcontinuum $Z \subset \widetilde{X}^*_\varepsilon$ such that $Z = t(Z)$. Let $\chi' : X^2 \to S^1$ be the map defined by $\chi'(x, y) = \chi(x, f(y))$. For each $(x, y) \in Z$, since $\operatorname{dist}(x, y) \geq \varepsilon$ and $\operatorname{dist}(y, f(y)) < \varepsilon$, it follows that $\chi(x, y)$ and $\chi'(x, y)$ are not antipodal points of S^1. Hence $\chi|_Z$ and $\chi'|_Z$ are homotopic. Since X is tree-like, X^2 is acyclic and so χ' is inessential. Therefore $\chi|_Z$ is also inessential.

By the following lemma (which is an improvement of [6, (3.1.2)] for the case $n = 1$), Z is not connected, which is a contradiction (compare [11, proof of Corollary 1]). ■

LEMMA (2.1). *If there exists an inessential equivariant mapping $\chi : Z \to S^1$ (with respect to some involution t on Z and antipodal involution on S^1), then there exists an equivariant mapping $Z \to S^0$ (in particular, Z is not connected).*

Proof. Denote the universal covering of S^1 by $p : \mathbb{R} \to S^1$. Since χ is inessential, it follows that there is a lifting $\widetilde{\chi} : Z \to \mathbb{R}$ of χ:

$$\begin{tikzcd} & \mathbb{R} \arrow[d, "p"] \\ Z \arrow[ur, "\widetilde{\chi}"] \arrow[r, "\chi"'] & S^1 \end{tikzcd}$$

Define $\chi_1 : Z \to S^0$ as

$$\chi_1(z) = \begin{cases} 1, & \widetilde{\chi}(z) > \widetilde{\chi}(t(z)), \\ -1, & \widetilde{\chi}(z) < \widetilde{\chi}(t(z)). \end{cases}$$

Since χ is equivariant, it follows that for each $x \in Z$, $\chi(x) \neq \chi(t(x))$, hence $\widetilde{\chi}(x) \neq \widetilde{\chi}(t(x))$. Therefore χ_1 is well defined. Evidently, χ_1 is equivariant. Since $\{x \in Z \mid \widetilde{\chi}(x) > \widetilde{\chi}(t(x))\}$ and $\{x \in Z \mid \widetilde{\chi}(x) < \widetilde{\chi}(t(x))\}$ are open, χ_1 is continuous. ∎

Now, suppose that $f(x) = x + \vec{\varepsilon}$ and $\sigma X > \varepsilon$. Then there is a subcontinuum $Z \subset \widetilde{X}^*_\varepsilon$ such that $\mathrm{pr}_1 Z = \mathrm{pr}_2 Z$. For each $(x, y) \in \widetilde{X}^*_\varepsilon$ write $x < y$ if $\widetilde{\chi}(x, y) < \widetilde{\chi}(y, x)$ (we use the notation of Lemma (2.1)). By the following lemma there is a $<$-minimal point $u \in \mathrm{pr}_1 Z = \mathrm{pr}_2 Z$. Then there are $v, w \in X$ such that $(u, v), (w, u) \in Z$. Since Z is connected and "$<$" is continuous, either $v < u < w$ or $w < u < v$. This is a contradiction to the $<$-minimality of u.

LEMMA (2.2). *Every subcontinuum of X has a $<$-minimal point* (*i.e. a point u such that $u < x$ whenever $(u, x) \in X^*_\varepsilon$*).

Proof. We may assume that the given subcontinuum is X itself. Let Oxy be a Cartesian coordinate system such that the directions of Ox and $\vec{\varepsilon}$ are the same and the orientation on S^1 induced by this system coincides with the one induced by $p(t)$. Hence we may assume that $\chi((0,0), (\cos 2\pi t, \sin 2\pi t)) = p(t)$. Let us prove that every point $a \in X$ with the minimal y-projection is $<$-minimal.

Since $\mathrm{dist}(\chi, \chi'|_{\widetilde{X}^*_\varepsilon}) < 1/4$, there is a covering $\widetilde{\chi}' : X^2 \to \mathbb{R}$ of χ' which is (1/4)-close to $\widetilde{\chi}$ on $\widetilde{X}^*_\varepsilon$. Since $|\widetilde{\chi}(u, z) - \widetilde{\chi}(z, u)| = 1/2$, the inequality $\widetilde{\chi}(u, z) > \widetilde{\chi}(z, u)$ holds if and only if $\widetilde{\chi}'(u, z) > \widetilde{\chi}'(z, u)$. By the choice of u, $\chi'(u \times X) \subset p[0, 1/2]$. If $\chi'(u, z) = p(1/2)$ for some $z \in X$, then on the line going through u and parallel to $\vec{\varepsilon}$, the points $z, z + \vec{\varepsilon}, u, u + \vec{\varepsilon}$ are situated in this order. But u and z and $z + \vec{\varepsilon}$ and $u + \vec{\varepsilon}$ are joined by the nonintersecting continua X and $\vec{\varepsilon} + X$ lying in the upper half-plane with respect to the line. This is a contradiction, hence $\chi'(u \times X) \subset p[0, 1/2)$. Analogously, $\chi'(X \times u) \subset p(-1/2, 0]$. Because of this and since $\chi'(u, u) = p(0)$, we have $\widetilde{\chi}'(u, z) \geq \widetilde{\chi}'(u, u) \geq \widetilde{\chi}'(z, u)$ for each $z \in X$. Therefore $\widetilde{\chi}(u, z) > \widetilde{\chi}(z, u)$ whenever $(u, z) \in \widetilde{X}^*_\varepsilon$.

Proof of Theorem (1.1)(b). Suppose that $\varepsilon < sf$. Then there is a subcontinuum $Z \subset X^2$ such that $Z = t(Z)$ and $\mathrm{dist}(f(x), f(y)) \geq \varepsilon$ for each $(x, y) \in Z$. Then, as in the proof of (a), the map $\chi \circ (f \times f)|_Z$ is inessential and equivariant. Hence Z is not connected, which is a contradiction. ∎

Proof of Corollary (1.3). (a) By [9], the product of X with the Cantor set embeds in $\mathbb{R}^2$. We obtain the conclusion with a weaker assumption that $X \times C$ embeds in $\mathbb{R}^2$. Here $C = c_0 \cup \bigcup_{m=1}^{\infty} c_m$ is a convergent sequence such that $c_0 = \lim_{m\to\infty} c_m$. If $X \times C \subset \mathbb{R}^2$, then for each $\varepsilon > 0$, there is a map $f : X \times c_0 \cong X \times c_m \hookrightarrow \mathbb{R}^2$ which is ε-close to the inclusion $X \times c_0 \hookrightarrow \mathbb{R}^2$ and such that $X \times c_0 \cap f(X \times c_0) = X \times c_0 \cap X \times c_m = \emptyset$. By Theorem (1.1)(a), $s(X \times c_0) < \varepsilon$ for each $\varepsilon > 0$, therefore $sX = 0$.

The "moreover" part is proved analogously.

(b) Clearly, it suffices to prove that there exists α such that $sf_\alpha = 0$. Similarly to (a), it suffices to prove that if $f : X \times X \to \mathbb{R}^2$ is a map such that $f_m(X) \cap f_n(X) = \emptyset$ for $m \neq n$, then $sf_0 = 0$ (here $f_m(X) = f(X, c_m)$). As in the proof of (a), $sf_0 < \varepsilon$ for each $\varepsilon > 0$, therefore $sf_0 = 0$.

Construction of Example (1.4). Fix an integer n. Let

$$K = \{0\} \times [2,3] \cup \bigcup_{l=1}^{n} ([0,l] \times \{a_{2l-1}\} \cup \{l\} \times [a_{2l-1}, a_{2l}] \\ \cup [0, l+1] \times \{a_{2l}\} \cup \{0\} \times [a_{2l+1}, a_{2l}]),$$

where

$$a_{2l-1} = 2 - \frac{2l-2}{n} \quad \text{and} \quad a_{2l} = 2 - \frac{2l-2}{n} - \frac{2l-1}{n^2}$$

(see Fig. 1 for $n = 4$). Let $\vec{\varepsilon} = (-c, -2/n - b)$, where $c > 0$ and $0 < b < 2/n^2$. Then $K \cap (K + \vec{\varepsilon}) = \emptyset$. By Theorem (1.1)(a),

$$\sigma K \leq \inf_{c,b} \sqrt{c^2 + (2/n + b)^2} = 2/n.$$

Let us prove that for each chain covering K, the diameter of at least one of its links is greater than 1, provided $n \geq 5$. This property was claimed without proof in [16] for their example. Suppose, on the contrary, that $K = C_1 \cup \ldots \cup C_m$, where the C_i are closed subsets of K of diameter less than 1, and $C_i \cap C_j \neq 0$ if and only if $|i - j| \leq 1$. Without loss of generality, we may assume that the intersection of each C_i with any straight line segment contained in K is connected. Let us fix some notation. Let $x_i = (n+1-i, a_{2(n-i)})$, $0 \leq i \leq n-2$, $u_i = (n+1-i, a_{2(n-i-1)})$, $1 \leq i \leq n-2$, $z = (0,3)$, $t = (0,0)$, $v = (1, 1/n^2)$, $y_i = (2, a_{2n+1-i})$, $2 \leq i \leq 2n-2$ (see Fig. 1). For $p, q \in K$, we denote by $\langle pq \rangle$ the closure of the connected component of $K \setminus \{p, q\}$ which contains both p and q.

Evidently, x_0 and z are contained in the first and in the last link of the chain $\{C_i\}$. Without loss of generality, we may assume that $x_0 \in C_1$ and $z \in C_m$. Let k be the greatest integer such that $C_k \cap \langle x_0 y_2 \rangle \neq \emptyset$. Since $\langle x_0 y_2 \rangle$ is connected, for each $s = 1, \ldots, k$, $C_s \cap \langle x_0 y_2 \rangle \neq \emptyset$ (if $C_i \cap \langle x_0 y_2 \rangle = \emptyset$ for some $i = 2, \ldots, k-1$, then $(C_1 \cup \ldots \cup C_{i-1}) \cap \langle x_0 y_2 \rangle$ and $\langle C_{i+1} \cup \ldots \cup C_k \rangle \cap \langle x_0 y_2 \rangle$ are disjoint nonempty subsets of $\langle x_0 y_2 \rangle$ whose union is $\langle x_0 y_2 \rangle$, which is a contradiction).

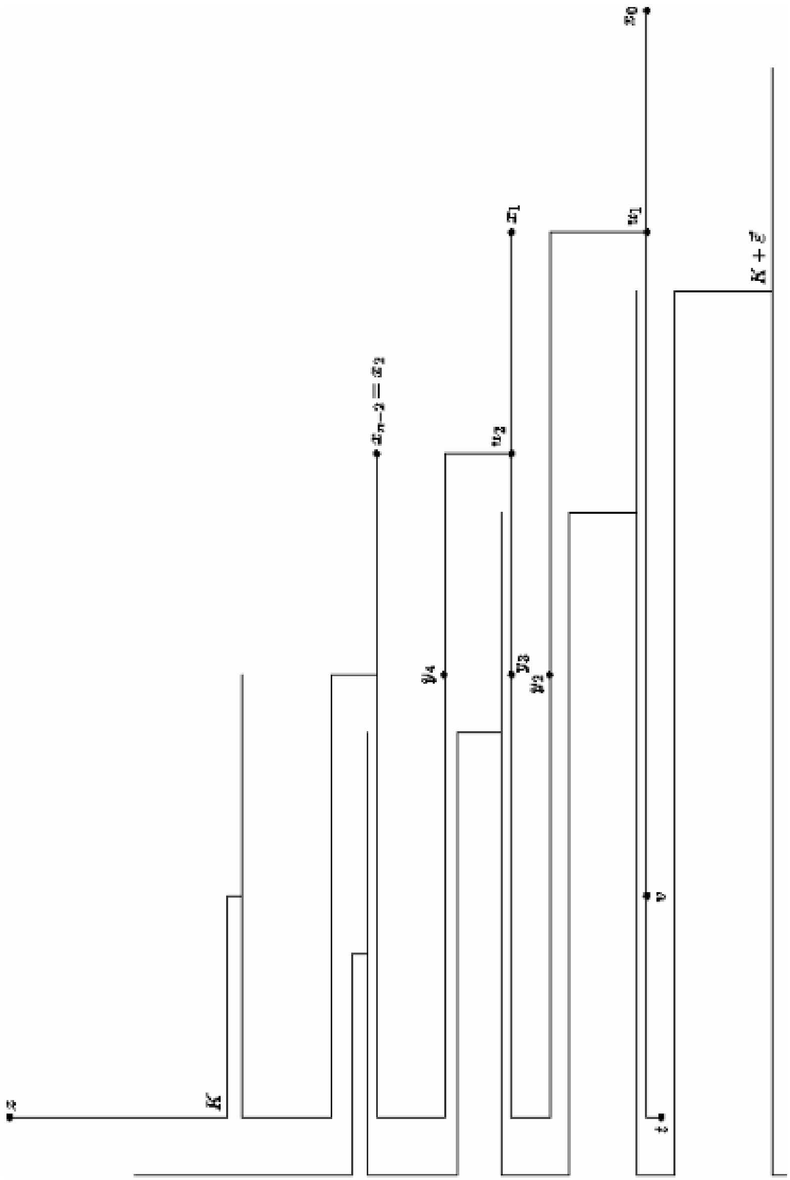

Fig. 1

Also, $C_k \cap \langle tv\rangle \neq \emptyset$. Indeed, in the opposite case there is a point $p \in C_k \cap \langle vy_2\rangle$. As in the previous paragraph, since $\langle pu_1\rangle \cup \langle u_1x_0\rangle$ (when $p \in \langle vu_1\rangle$) or $\langle px_0\rangle$ (when $p \in \langle u_1x_0\rangle$) is connected, it follows that each $C_1, \ldots, C_k$ intersects $\langle pu_1\rangle \cup \langle u_1x_0\rangle$ or $\langle px_0\rangle$, respectively. Hence the link containing t intersects $\langle vy_2\rangle$. Therefore it has diameter greater than $\operatorname{dist}(t, \langle vy_2\rangle) > 1$, which is a contradiction.

Let $C = C_1 \cup \ldots \cup C_k$. Then $x_1 \in C$. Indeed, take an integer i such that $x \in C_i$. Since $\operatorname{dist}(x_1, x_0) > 1$, we have $C_i \cap (\langle zu_2\rangle \cup \langle y_2u_2\rangle) = \emptyset$ and by our assumption, $C_i \cap \langle x_1v\rangle$ is a segment. Therefore $C_i \cap \langle x_0y_2\rangle \neq \emptyset$ and so $x_1 \in C$.

Next, $\langle y_3y_4\rangle \in C$. Indeed, in the opposite case take a point $q \in \langle y_3y_4\rangle$ closest to y_4 such that $\langle qu_2\rangle \cup \langle u_2x_1\rangle \subset C$ (if $q \notin \langle u_2x_1\rangle$) or $\langle qx_1\rangle \subset C$ (if $q \in \langle u_2x_1\rangle$). Then $q \in C_l \cap C_i$, where $l > k \geq i$. Since $C_i \cap C_j = \emptyset$ when $|i-j| > 1$, it follows that $i = k$ and $l = k+1$. Since $C_k \cap \langle vt\rangle \neq \emptyset$ and $q \in C_k \cap \langle y_3y_4\rangle$, it follows that $\operatorname{diam} C_k > \operatorname{dist}(\langle vt\rangle, \langle y_3y_4\rangle) > 1$, which is a contradiction.

Analogously, $x_2 \in C$, then $\langle y_5y_6\rangle \in C$ and so on. Hence $x_{n-2} \in C$. Since each $C_1, \ldots, C_k$ intersects $\langle x_0y_2\rangle$, the diameter of the link $C_i \subset C$ containing x_{n-2} is greater than $\operatorname{dist}(x_{n-2}, \langle x_0y_2\rangle) = 2 - 4/n - 3/n^2 > 1$ when $n \geq 5$, which is a contradiction. ∎

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