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On the exponential stability and dichotomy of C_0-semigroups

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Abstract. A characterization of exponentially dichotomic and exponentially stable C_0-semigroups in terms of solutions of an operator equation of Lyapunov type is presented. As a corollary a new and shorter proof of van Neerven's recent characterization of exponential stability in terms of boundedness of convolutions of a semigroup with almost periodic functions is given.

1. Let $T(t)$, $t \geq 0$, be a strongly continuous semigroup (C_0-semigroup) of bounded linear operators on a Banach space E, with generator A. A projection operator P on E is called a *dichotomic projection* for the semigroup $T(t)$ if the following conditions hold:

(i) $PT(t) = T(t)P$ for all $t \geq 0$;

(ii) There are positive constants M, ω such that $\|T(t)x\| \leq Me^{-\omega t}\|x\|$ for all $x \in P(E)$, $t \geq 0$;

(iii) The restriction $T(t)|\ker(P)$ extends to a C_0-group (we use the same notation without ambiguity) and $\|T(-t)x\| \leq Me^{-\omega t}\|x\|$ for all $x \in \ker(P)$, $t \geq 0$.

If the semigroup $T(t)$ has a dichotomic projection, then it is called *exponentially dichotomic*. If $\|T(t)x\| \leq Me^{-\omega t}\|x\|$ for all $x \in E$ and some positive constants M, ω, then $T(t)$ is called *exponentially stable*. Thus, exponential stability is a particular case of exponential dichotomy when the corresponding dichotomic projection is the identity operator, or, equivalently, when the semigroup is (uniformly) bounded.

There are various conditions characterizing exponentially stable or dichotomic semigroups on Banach or Hilbert spaces. Among known results let us mention a theorem of Datko–Pazy [3, 12] which states that a semigroup $T(t)$ is exponentially stable if and only if, for some $1 \leq p < \infty$,

$$\int_0^\infty \|T(t)x\|^p \, dt < \infty \qquad \text{for all } x \in E.$$

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If $T(t)$ is a semigroup on a Hilbert space, then $T(t)$ is exponentially stable if and only if there exists a bounded solution to the Lyapunov equation $AX + XA^* = -I$ (see [3]).

The main result of this paper is a characterization of exponential stability and dichotomy for arbitrary C_0-semigroups on Banach spaces in terms of solutions of an operator equation of Lyapunov type (Theorem 3). This theorem implies a spectral mapping theorem for the evolutionary semigroups associated with $T(t)$. Combined with some standard arguments of operator theory, this also leads to a remarkable fact that exponential dichotomy is equivalent to the admissibility of the space $\mathrm{BUC}(\mathbb{R}, E)$ (of bounded uniformly continuous E-valued functions on $\mathbb{R}$) or $\mathrm{AP}(\mathbb{R}, E)$ (of almost periodic functions), i.e. $T(t)$ is exponentially dichotomic if and only if for every $f \in \mathrm{BUC}(\mathbb{R}, E)$ (resp. $f \in \mathrm{AP}(\mathbb{R}, E)$) there exists a (mild) solution $u \in \mathrm{BUC}(\mathbb{R}, E)$ (resp. $u \in \mathrm{AP}(\mathbb{R}, E)$) of the differential equation $u'(t) = Au(t) + f(t)$. The uniqueness of the solution follows, of course, as a corollary.

Another consequence of our results is a new and more transparent proof of the following result recently obtained by J. van Neerven [11]: *a C_0-semigroup $T(t)$ is exponentially stable if and only if for every almost periodic function $f : \mathbb{R}_+ \to E$, the function $t \mapsto \int_0^t T(t-s)f(s)\,ds$ is bounded.*

2. Assume that F is another Banach space, $S(t)$ is a C_0-semigroup on F with generator $-B$, and $C : F \to E$ is a bounded (linear) operator. A bounded linear operator $X : F \to E$ is called a *solution* of the operator equation

$$AX - XB = C \tag{1}$$

if $X(D(B)) \subset D(A)$ (where $D(A)$ and $D(B)$ are the domains of A and B, respectively) and

$$AXf - XBf = Cf \qquad \text{for all } f \in D(B).$$

The operator equation (1) on Banach spaces was first considered by Krein (see [2]) and Rosenblum [15] for bounded operators A and B with disjoint spectra (see also [9, 14])). Equation (1) for unbounded A and B was also studied by many authors (see e.g. [1, 5, 6, 8, 14]).

We will need the following lemma (see e.g. [5] or [16, Corollary 8]).

LEMMA 1. *If $T(t)$ is exponentially stable and $S(t)$ is an isometric semigroup, then for every bounded operator $C : F \to E$ equation* (1) *admits a unique bounded solution. Moreover, the solution has the form*

$$X = -\int_0^\infty T(t)CS(t)\,dt. \tag{2}$$

From Lemma 1 we obtain the following slightly more general fact which will be used in the sequel.

LEMMA 2. *If $T(t)$ is exponentially dichotomic and $S(t)$ is an isometric group, then for every bounded $C : F \to E$ equation* (1) *admits a unique bounded solution. Moreover, the solution has the form*

$$X = -\int_{-\infty}^\infty G_A(t)CS(t)\,dt, \tag{3}$$

where

$$G_A(t) = \begin{cases} T(t)P, & t \ge 0, \\ -T(t)(I-P), & t < 0 \end{cases}$$

(P is the corresponding dichotomic projection).

Proof. Put $C_1 = PC, C_2 = (I-P)C$, $A_1 = A|P(E), A_2 = A|\ker(P)$. By Lemma 1, and by (2), there exists an operator $X_1 : F \to P(E)$ such that

$$A_1X_1 - X_1B = C_1 \tag{4}$$

and

$$X_1 = -\int_0^\infty T(t)PCS(t)\,dt. \tag{5}$$

Since $-A_2 = -A|\ker(P)$ generates an exponentially stable semigroup and $-B$ also generates an isometric group, again by Lemma 1 there exists a bounded linear operator $X_2 : F \to \ker(P)$ such that

$$-A_2X_2 + X_2B = -C_2, \quad \text{or} \quad A_2X_2 - X_2B = C_2, \tag{6}$$

and

$$X_2 = \int_0^\infty T(-t)(I-P)CS(-t)\,dt = \int_{-\infty}^0 T(s)(I-P)CS(s)\,ds. \tag{7}$$

Let $X : F \to E$ be defined by $Xf = X_1Pf + X_2(I-P)f$. Then from (4) and (6) it follows that X is a bounded solution of (1) and that X has the form (3).

To see the uniqueness observe that if X is a solution of (1), then $X_1 = PX$, $X_2 = (I-P)X$ are solutions of (4) and (6), respectively, so they are unique. Hence X is unique. ∎

Consider the differential equation

$$u'(t) = Au(t) + f(t), \qquad t \in \mathbb{R}, \tag{8}$$

where $f : \mathbb{R} \to E$ is a continuous function. A continuous function $u : \mathbb{R} \to E$

is called a *mild solution* of (8) if

$$u(t) = T(t-s)u(s) + \int_s^t T(t-\tau)f(\tau)\,d\tau \qquad \text{for all } t \ge s.$$

Let $\mathrm{BUC}(\mathbb{R}, E)$ be the Banach space of bounded uniformly continuous functions on $\mathbb{R}$ with values in E, with the sup-norm, and $\mathrm{AP}(\mathbb{R}, E)$ its subspace of almost periodic functions. Let $\mathcal{S}(t)$ be the translation group on F, where F is either $\mathrm{BUC}(\mathbb{R}, E)$ or $\mathrm{AP}(\mathbb{R}, E)$, with generator $\mathcal{D}$, i.e. $(\mathcal{S}(t)f)(s) = f_t(s) = f(s+t), \mathcal{D}f = f'$, and let $\delta_0 : F \to E$ be defined by $\delta_0 f = f(0)$, $f \in F$. If A is a bounded operator, then it is well known that A generates an exponentially dichotomic semigroup if and only if $i\mathbb{R} \cap \sigma(A) = \emptyset$ (see e.g. [2]). This fact is not true in general for unbounded generators (see e.g. [10]). For general C_0-semigroups we show that the following theorem is valid.

THEOREM 3. *Let $T(t)$ be a C_0-semigroup on a Banach space E, with generator A. The following are equivalent*:

(i) *$T(t)$ is exponentially dichotomic.*

(ii) *For every B which is the generator of an isometric C_0-group $S(t)$ on a Banach space F, and for every bounded operator $C : F \to E$, the operator equation $AX - XB = C$ has a unique bounded solution.*

(iii) *The operator equation $AX - X\mathcal{D} = -\delta_0$ has a unique bounded solution.*

(iv) *For every $f \in \mathrm{BUC}(\mathbb{R}, E)$ there is a unique mild solution $u \in \mathrm{BUC}(\mathbb{R}, E)$ of equation* (8).

(v) *For every $f \in \mathrm{AP}(\mathbb{R}, E)$ there is a unique mild solution $u \in \mathrm{AP}(\mathbb{R}, E)$ of equation* (8).

(vi) $\sigma(T(1)) \cap \{\lambda \in \mathbb{C} : |\lambda| = 1\} = \emptyset$.

Proof. (i)⇒(ii) follows from Lemma 2.

(ii)⇒(iii) is trivial.

(iii)⇔(iv). Assume that there exists precisely one bounded solution to the operator equation $AX - X\mathcal{D} = -\delta_0$. We first show that for every bounded operator $C : \mathrm{BUC}(\mathbb{R}, E) \to E$, the operator equation $AY - Y\mathcal{D} = C$ has a unique solution. In fact, the operator $Y : \mathrm{BUC}(\mathbb{R}, E) \to E$ defined by $Yf = X\widetilde{f}$, where $\widetilde{f}(t) \equiv -C(\mathcal{S}(t)f)$, $t \in \mathbb{R}$, is a bounded solution, and the uniqueness follows from the fact that the homogeneous equation $AX - X\mathcal{D} = 0$ has only the zero solution $X = 0$. This implies that $i\mathbb{R} \cap \sigma(A) = \emptyset$ (see [1, Theorem 2.1]).

Now let $f \in D(\mathcal{D})$, and the function $u(t)$ be defined by $u(t) = X\mathcal{S}(t)f$, $t \in \mathbb{R}$. It follows that

$$u'(t) = X\mathcal{D}\mathcal{S}(t)f = (AX + \delta_0)\mathcal{S}(t)f = Au(t) + f(t),$$

i.e. $u(t)$ is a classical solution of (8). From this it follows that for every $f \in \mathrm{BUC}(\mathbb{R}, E)$ the function $u(t) \equiv X\mathcal{S}(t)f$ is a mild solution of (8). It remains to show that the mild solution is unique. Assume there exist two distinct solutions $u_1(t)$ and $u_2(t)$ of (8). Then $v(t) \equiv u_1(t) - u_2(t)$ is a bounded non-trivial mild solution of the homogeneous equation $v'(t) = Av(t)$, $t \in \mathbb{R}$, i.e. $v(t)$ is a non-trivial complete trajectory of the semigroup $T(t)$. It is the well-known Tauberian Theorem that the spectrum, $\mathrm{Sp}(v)$, of the function v is a non-empty subset of $\mathbb{R}$, and $i\mathrm{Sp}(v) \subset \sigma(A)$ (see [17]). Hence $\sigma(A) \cap i\mathbb{R} \ne \emptyset$, which is a contradiction.

Conversely, if (iv) holds then one can define a bounded linear operator G on $\mathrm{BUC}(\mathbb{R}, E)$ by $Gf = u$, where u is the unique mild solution in $\mathrm{BUC}(\mathbb{R}, E)$ of equation (8). Define $Xf = (Gf)(0)$. Since G commutes with $\mathcal{S}(t)$ and $\mathcal{D}$, for continuously differentiable f we have

$$\frac{d}{dt}(Gf)(t) = A(Gf)(t) + f(t),$$

so that $(Gf)'(0) = A(Gf)'(0) + f(0)$, or $AXf - X\mathcal{D}f = -\delta_0 f$, for all $f \in D(\mathcal{D})$. That is, X is a bounded solution of $AX - X\mathcal{D} = -\delta_0$. On the other hand, as shown above, if X is a solution to $AX - X\mathcal{D} = -\delta_0$, then for every $f \in \mathrm{BUC}(\mathbb{R}, E)$, the function $u(t) = X\mathcal{S}(t)f$ is a mild solution in $\mathrm{BUC}(\mathbb{R}, E)$ of (8). The uniqueness of u implies that the solution X to $AX - X\mathcal{D} = -\delta_0$ is unique.

(iv)⇒(v) follows from the formula $u(t) = X\mathcal{S}(t)f$ for the solution $u(t)$ of (8).

(v)⇒(vi). Observe again that from the formula $u(t) = X\mathcal{S}(t)f$ it follows that if (v) holds then for every continuous ω-periodic function $f(t)$ equation (8) has precisely one ω-periodic mild solution. We show that $I - T(\omega)$ is invertible for every ω, which implies (vi). Since the ω-periodic solution of (8) is unique (for ω-periodic f), it is easily seen that $I - T(\omega)$ is injective. To show that $I - T(\omega)$ is surjective, take an arbitrary $x \in E$, and let $g(t)$ be a real continuous function on $[0, \omega]$ such that $g(0) = g(\omega)$ and

$$\int_0^\omega g(t)\,dt = 1.$$

Let $f(t) = T(t)[g(t)x]$, $0 \le t \le \omega$. Since $f(0) = f(\omega) = 0$, the function f can be continued to be an ω-periodic function on $\mathbb{R}$. By the above remark, there exists an ω-periodic solution $u(t)$ of (8). Then

$$u(\omega) = u(0) = T(\omega)u(0) + \int_0^\omega T(\omega - s)[T(s)g(s)x]\,ds = T(\omega)u(0) + T(\omega)x,$$

hence $(I - T(\omega))(u(0) + x) = x$, so that $I - T(\omega)$ is surjective.

(vi)⇒(i) is well known. ■

From Theorem 3 we obtain the following corollary.

COROLLARY 4. *Let $T(t)$ be a bounded C_0-semigroup on E. Then $T(t)$ is exponentially stable if and only if one of the conditions* (ii)–(vi) *holds.*

Let us give an interpretation of Theorem 3 in terms of a spectral mapping theorem. Consider the evolutionary semigroup e^{tL} on $\mathrm{BUC}(\mathbb{R},E)$ defined by $(e^{tL}f)(s)=T(t)f(s-t)$, $t\ge 0$, $s\in\mathbb{R}$. The generator of this semigroup is the closure L of the operator

$$(L_0u)(s)=-\frac{du}{ds}+Au(s),\qquad s\in\mathbb{R},$$

with $D(L_0)=\{u\in \mathrm{BUC}(\mathbb{R},E): u(t)$ is continuously differentiable, $u'\in \mathrm{BUC}(\mathbb{R},E)$ and $u(t)\in D(A)$ for all $t\in\mathbb{R}\}$. The equivalence (i)$\Leftrightarrow$(iv)$\Leftrightarrow$(v) in Theorem 3 means, in other words, that $T(t)$ is exponentially dichotomic if and only if L is invertible (in one of the spaces $\mathrm{BUC}(\mathbb{R},E)$ or $\mathrm{AP}(\mathbb{R},E)$). It is easy to see that if $T(t)$ is exponentially dichotomic, then so is e^{tL} (with the dichotomic projection $\mathcal{P}$ defined by $(\mathcal{P}u)(s)=Pu(s)$). Therefore, if $0\in\varrho(L)$ then $1\in\varrho(e^{tL})$, $t>0$. It is also easy to see that if L is invertible (in $\mathrm{BUC}(\mathbb{R},E)$ or $\mathrm{AP}(\mathbb{R},E)$), then so is $L-i\lambda$ for every $\lambda\in\mathbb{R}$, which implies that the spectrum of L is invariant w.r.t. translations parallel to $i\mathbb{R}$. From this and the implication $0\in\varrho(L)\Rightarrow 1\in\varrho(e^{tL})$ it immediately follows that

$$\sigma(e^{tL})\setminus\{0\}=\exp\{t\sigma(L)\},$$

i.e. the Spectral Mapping Theorem holds for the semigroup e^{tL} (cf. [7]) ([1]).

The equivalence of (i) and (iv) in Theorem 3 is well known (see [13]), while the other statements are new. However, we show below that a stronger result holds. First, we recall that the space $\mathrm{BUC}(\mathbb{R},E)$ is called *admissible* if for every $f\in \mathrm{BUC}(\mathbb{R},E)$ equation (8) has a solution $u(t)$ in $\mathrm{BUC}(\mathbb{R},E)$. (Note that the uniqueness of the solution is not required in this definition.) The admissibility of the space $\mathrm{AP}(\mathbb{R},E)$ is defined analogously.

THEOREM 5. *The semigroup $T(t)$ is exponentially dichotomic if and only if one of the spaces* $\mathrm{BUC}(\mathbb{R},E)$ *and* $\mathrm{AP}(\mathbb{R},E)$ *is admissible.*

Proof. Since the "only if" part is obvious, we need only prove the "if" part. Assume for definiteness that $\mathrm{BUC}(\mathbb{R},E)$ is admissible. This means that the range of L is the whole $\mathrm{BUC}(\mathbb{R},E)$. In view of the remark preceding Theorem 4, it is enough to show that L is invertible. Assuming that, on the contrary, L is not invertible, we can conclude that, firstly, $i\mathbb{R}\subset\sigma(L)$ (by the previous remark) and, secondly, $N=\ker L=\{u: Lu=0\}\ne\{0\}$, by the Open Mapping Theorem. Since L is closed, N is a closed subspace of $\mathrm{BUC}(\mathbb{R},E)$. Let $\mathcal{F}=\mathrm{BUC}(\mathbb{R},E)$ and $\widehat{L}$ be the operator induced by L in $\mathcal{F}$ (i.e. $\widehat{L}\widehat{u}=\widehat{Lu}$, where $\widehat{u}$ is the class containing the function u). It is easy to see that $\widehat{L}$ is also closed, surjective and therefore, again by the Open Mapping Theorem, $\widehat{L}$ is invertible. Moreover, it is well known that $\sigma(L)\subset\sigma(L|N)\cup\sigma(\widehat{L})=\{0\}\cup\sigma(\widehat{L})$ (see [4]), hence $i\mathbb{R}\subset\{0\}\cup\sigma(\widehat{L})$, which is a contradiction since 0 is in the resolvent set of $\widehat{L}$. ∎

([1]) The same argument applies to the non-autonomous equation $u'(t)=A(t)u(t)+f(t)$ under the condition that the equation is well posed, so that there exists a strongly continuous evolution family.

The argument in Theorem 5 applies without changes to the non-autonomous equation $u'(t)=A(t)u(t)+f(t)$ under the well-posedness condition, so that we can associate with it an evolution family (and hence an evolutionary semigroup as, say, in [7]).

As another corollary of Theorem 3 we give a new and shorter proof of the following result obtained earlier by J. van Neerven [11]. Below let $\mathrm{AP}(\mathbb{R}_+,E)$ be the space of almost periodic functions on $\mathbb{R}_+(\equiv[0,\infty))$ with values in E (i.e. functions on $\mathbb{R}_+$ which are restrictions of almost periodic functions).

COROLLARY 6. *Assume that*

(9) *for all $f\in \mathrm{AP}(\mathbb{R}_+,E)$ the function $\varphi(t)\equiv\int_0^t T(s)f(s)\,ds$ is bounded on $\mathbb{R}_+$.*

Then $T(t)$ is exponentially stable.

Proof. Let $\mathcal{L}(F,E)$ be the space of all bounded linear operators from F to E, and $U(s):\mathcal{L}(F,E)\to\mathcal{L}(F,E)$ be defined by $U(s)X=T(s)XS(s)$, $s\ge 0$. It is shown in [1, Proposition 3.7] that if

$$M\equiv\sup_{t\ge0}\Big\|\int_0^t U(s)\,ds\Big\|<\infty,$$

then the equation $AX-XB=C$ has a unique bounded solution for every bounded C. Below we apply this fact to the equation $AX-X\mathcal{D}=-\delta_0$.

If condition (9) holds, then, by a standard argument involving the Closed Graph Theorem and the Uniform Boundedness Principle, it follows that there exists a constant M such that

$$\sup_{t\ge0}\Big\|\int_0^t T(s)f(s)\,ds\Big\|\le M\sup_{t\ge0}\|f(t)\|\qquad\text{for all } f\in \mathrm{AP}(\mathbb{R}_+,E).$$

Therefore, the equation $AX-X\mathcal{D}=-\delta_0$ has a unique bounded solution. By Theorem 3(v), the semigroup is exponentially dichotomic. From

$$T(t)x=\int_0^t T(s)Ax\,ds+x,\qquad x\in D(A),$$

it follows that $T(t)x$ is bounded for every $x\in D(A)$. Hence the dichotomic projection P is I, i.e. the semigroup $T(t)$ is exponentially stable. ∎

It is easy to see that condition (9) is equivalent to the following:

$$t \mapsto \int_0^t T(t-s)f(s)\,ds \in \mathrm{BUC}(\mathbb{R}_+, E) \quad \text{for all } f \in \mathrm{AP}(\mathbb{R}_+, E). \tag{10}$$

Consider (8) on the half-line $[0,\infty)$. It is well known that every mild solution of the abstract Cauchy problem

$$\begin{cases} u'(t) = Au(t) + f(t), & t \ge 0, \\ u(0) = x, \end{cases} \tag{11}$$

is given by the formula

$$u(t) = T(t)x + \int_0^t T(t-s)f(s)\,ds, \qquad t \ge 0.$$

Thus, condition (10) means the solution of (11) with initial condition $u(0) = 0$ is bounded. If $T(t)$ is a bounded semigroup, then condition (10) holds if and only if every solution of (11) ($f \in \mathrm{AP}(\mathbb{R}_+, E)$) is bounded. Therefore, Corollary 6 implies the following result.

COROLLARY 7. *Assume that $T(t)$ is a bounded C_0-semigroup with generator A. If for every $f \in \mathrm{AP}(\mathbb{R}_+, E)$ there exists a bounded solution u of* (11), *then $T(t)$ is exponentially stable.*

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