

## Jackson's $q$ -Clausen formula and two almost-poised ${}_4\phi_3$ -series

WENCHANG CHU (Zhoukou and Lecce)

**Abstract.** Two almost-poised  $q$ -hypergeometric  ${}_4\phi_3$ -summation formulae rediscovered by Andrews are discussed. One is equivalent to Jackson's  $q$ -Clausen formula and the other is hidden among the almost-poised  ${}_5\phi_4$ -series identities. Three further examples are presented.

**1. Introduction and motivation.** In searching for  $q$ -analogues of Shapiro's convolution formulae on Catalan numbers, Andrews [1, Theorem 3] claims to have found the following  $q$ -hypergeometric summation formula

$$(1.1) \quad {}_4\phi_3 \left[ \begin{matrix} q^{-2m}, b, d, q^{1-2m}/bd \\ q^{2-2m}/b, q^{2-2m}/d, qbd \end{matrix} \middle| q^2; q^2 \right] \\ = q^{-m} \left[ \begin{matrix} b, d \\ q, bd \end{matrix} \middle| q \right]_m \left[ \begin{matrix} q^2, bd \\ b, d \end{matrix} \middle| q^2 \right]_m.$$

In the same paper, he also proposed as an open problem to confirm another formula [1, (6.1)]:

$$(1.2) \quad {}_4\phi_3 \left[ \begin{matrix} q^{-2m}, b, d, q^{3-2m}/bd \\ q^{2-2m}/b, q^{4-2m}/d, qbd \end{matrix} \middle| q^2; q^2 \right] = q^{-m} \left[ \begin{matrix} q^2, bd/q^2 \\ b, d/q^2 \end{matrix} \middle| q^2 \right]_m \\ \times \frac{1 - bd/q}{1 - bd/q^2} \left\{ 1 - \frac{q^{m-2}bd(1-q)(1-q^{m-1}d)}{(1-d/q)(1-q^{2m-1}bd)} \right\} \left[ \begin{matrix} b, d/q \\ q, bd/q \end{matrix} \middle| q \right]_m.$$

Unfortunately, neither of these series are really new, just as commented by Andrews, "it is seldom that a new  $q$ -hypergeometric summation turns up". The first one (1.1) is well-known for its connection to the  $q$ -Clausen

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formula on the product of two  ${}_2\phi_1$ -series established 70 years ago by Jackson [18, 19]. Because the almost well-poised series like (1.1) and (1.2) have been treated separately according to the parity of the summation order  $m$ , for example, by Bailey [3], Bressoud [4] and Carlitz [5], the second one (1.2) hides behind a couple of more general  ${}_5\phi_4$ -series identities obtained by the author [6, Examples 2.5 & 2.29].

Both series (1.1) and (1.2) have spurred several followers to prove “Andrews’ conjecture”, for example, Guo [16, 15], Mu [20] and Wei–Wang [25]. In order to avoid further misleading, it seems necessary to clear these facts. That is the aim of this short commentary paper. In the next section, we shall briefly review Jackson’s  $q$ -Clausen formula and its ties with (1.1). Then in the third section, we shall explain how to recover (1.2) and further identities from an encyclopedic work on almost-poised series by the author [6]. The paper will end with an open question about evaluation of terminating almost-poised  ${}_4\phi_3$ -series.

Throughout the paper, we shall adopt the notation of Bailey [2] for the basic hypergeometric series

$${}_{1+p}\phi_p \left[ \begin{matrix} a_0, & a_1, & \dots, & a_p \\ & b_1, & \dots, & b_p \end{matrix} \middle| q; z \right] = \sum_{n=0}^{\infty} \left[ \begin{matrix} a_0, & a_1, & \dots, & a_p \\ q, & b_1, & \dots, & b_p \end{matrix} \middle| q \right]_n z^n$$

where the shifted factorial is defined by  $(x; q)_0 = 1$  and

$$(x; q)_n = (1 - x)(1 - xq) \cdots (1 - xq^{n-1}) \quad \text{for } n = 1, 2, \dots$$

with its quotient form being abbreviated compactly to

$$\left[ \begin{matrix} a, & b, & \dots, & c \\ \alpha, & \beta, & \dots, & \gamma \end{matrix} \middle| q \right]_n = \frac{(a; q)_n (b; q)_n \cdots (c; q)_n}{(\alpha; q)_n (\beta; q)_n \cdots (\gamma; q)_n}.$$

The  ${}_{1+p}\phi_p$ -series is said to be *balanced* when the product of its denominator parameters exceeds by  $q$  the product of its numerator parameters, and *well-poised* when its parameters can be paired off in columns so that these pairs have almost the same product.

**2. Jackson’s  $q$ -Clausen formula.** In 1828, Clausen [11] discovered the following well-known product formula (cf. Bailey [2, §10.1]):

$$\begin{aligned} {}_2F_1 \left[ \begin{matrix} b, & d \\ b + d + 1/2 \end{matrix} \middle| z \right] &\times {}_2F_1 \left[ \begin{matrix} b, & d \\ b + d + 1/2 \end{matrix} \middle| z \right] \\ &= {}_3F_2 \left[ \begin{matrix} 2b, & 2d, & b + d \\ 2b + 2d, & b + d + 1/2 \end{matrix} \middle| z \right]. \end{aligned}$$

The first  $q$ -analogue of it was found by Jackson [18, 19]:

$$(2.1) \quad {}_2\phi_1 \left[ \begin{matrix} b, d \\ qbd \end{matrix} \middle| q^2; z \right] \times {}_2\phi_1 \left[ \begin{matrix} b, d \\ qbd \end{matrix} \middle| q^2; qz \right] = {}_4\phi_3 \left[ \begin{matrix} b, d, \sqrt{bd}, -\sqrt{bd} \\ bd, \sqrt{qbd}, -\sqrt{qbd} \end{matrix} \middle| q; z \right].$$

For proofs of the  $q$ -Clausen formula (2.1), refer to Singh [22], Chu–Jia [9] and Srivastava–Jain [23]. Different  $q$ -analogues can be found in Gasper [12], Nassrallah [21] and Gasper–Rahman [13].

As done by Jackson [19], extracting the coefficient of  $z^m$  across (2.1), one gets the  $q$ -series identity

$${}_4\phi_3 \left[ \begin{matrix} q^{-2m}, b, d, q^{1-2m}/bd \\ q^{2-2m}/b, q^{2-2m}/d, qbd \end{matrix} \middle| q^2; q^2 \right] = q^{-m} \left[ \begin{matrix} b, d \\ q, bd \end{matrix} \middle| q \right]_m \left[ \begin{matrix} q^2, bd \\ b, d \end{matrix} \middle| q^2 \right]_m,$$

which is exactly (1.1) rediscovered by Andrews [1, Theorem 3].

For the last series, combining it with its reversal (cf. Carlitz [5, (1.3)] for  $m = 2n$ )

$${}_4\phi_3 \left[ \begin{matrix} q^{-2m}, b, d, q^{1-2m}/bd \\ q^{2-2m}/b, q^{2-2m}/d, qbd \end{matrix} \middle| q^2; q^4 \right] = \left[ \begin{matrix} b, d \\ q, bd \end{matrix} \middle| q \right]_m \left[ \begin{matrix} q^2, bd \\ b, d \end{matrix} \middle| q^2 \right]_m$$

we can unify them with an extra free parameter  $w$  as follows:

$$(2.2) \quad {}_5\phi_4 \left[ \begin{matrix} q^2w, q^{-2m}, b, d, q^{1-2m}/bd \\ w, q^{2-2m}/b, q^{2-2m}/d, qbd \end{matrix} \middle| q^2; q^2 \right] \\ = \frac{w - q^{-m}}{w - 1} \left[ \begin{matrix} b, d \\ q, bd \end{matrix} \middle| q \right]_m \left[ \begin{matrix} q^2, bd \\ b, d \end{matrix} \middle| q^2 \right]_m,$$

which has appeared explicitly in a paper on well-poised series by Chu [7].

**3. Almost-poised  ${}_4\phi_3$ -series.** The following  $q$ -analogue of Dougall's very well-poised series identity was also found by Jackson [17] (cf. [2, §8.3] and [14, II-22]):

$${}_8\phi_7 \left[ \begin{matrix} a, q\sqrt{a}, -q\sqrt{a}, b, c, d, e, q^{-n} \\ \sqrt{a}, -\sqrt{a}, qa/b, qa/c, qa/d, qa/e, q^{1+n}a \end{matrix} \middle| q; q \right] \\ = \left[ \begin{matrix} qa, qa/bc, qa/bd, qa/cd \\ qa/b, qa/c, qa/d, qa/bcd \end{matrix} \middle| q \right]_n \quad \text{where} \quad q^{1+n}a^2 = bcde.$$

When the series is terminated by the parameter  $a$ , the usual limiting process does not work. However, there are several unusual formulae in this case. The following formula with the summation order being even is essentially

due to Jackson [19] (cf. Bailey [3], Bressoud [4] and Verma–Joshi [24]):

$$(3.1) \quad {}_5\phi_4 \left[ \begin{matrix} q^{-2n}, & b, & c, & d, & q^{1-3n}/bcd \\ & q^{1-2n}/b, & q^{1-2n}/c, & q^{1-2n}/d, & q^n bcd \end{matrix} \middle| q; q^\varepsilon \right] \\ \stackrel{\varepsilon=1,2}{=} q^{n(\varepsilon-1)} \left[ \begin{matrix} q^{-2n}, & q^{1-2n}/bc, & q^{1-2n}/bd, & q^{1-2n}/cd \\ & q^{1-2n}/b, & q^{1-2n}/c, & q^{1-2n}/d, & q^{1-2n}/bcd \end{matrix} \middle| q \right]_n.$$

The counterpart of the last formula with the summation order being odd was found by Bailey [3] (cf. Carlitz [5]):

$$(3.2) \quad {}_5\phi_4 \left[ \begin{matrix} q^{-1-2n}, & b, & c, & d, & q^{-1-3n}/bcd \\ & q^{-2n}/b, & q^{-2n}/c, & q^{-2n}/d, & q^{1+n}bcd \end{matrix} \middle| q; q^\varepsilon \right] \\ \stackrel{\varepsilon=1,2,3}{=} (1 - q^{(\varepsilon-2)(1+2n)}) \left[ \begin{matrix} q^{-2n}, & q^{-2n}/bc, & q^{-2n}/bd, & q^{-2n}/cd \\ & q^{-2n}/b, & q^{-2n}/c, & q^{-2n}/d, & q^{-2n}/bcd \end{matrix} \middle| q \right]_n.$$

For different proofs of the last two summation formulae, refer to Chu–Jia [9], Chu–Wang [10], Guo [15] and Verma–Joshi [24, (3.12)]. The initial intention of mine was to derive (2.2) by combining the last two formulae under some parameter restriction. Unfortunately, they are not compatible in this sense. In place of (3.2), there does exist an identity due to the author [6, Example 2.11]

$$(3.3) \quad {}_5\phi_4 \left[ \begin{matrix} q^{-1-2n}, & b, & c, & d, & q^{-3n}/bcd \\ & q^{-2n}/b, & q^{1-2n}/c, & q^{-2n}/d, & q^n bcd \end{matrix} \middle| q; q^\varepsilon \right] \\ \stackrel{\varepsilon=1,2}{=} (q^{2n}c)^{\varepsilon-2} \left[ \begin{matrix} q^{-2n}, & q^{1-2n}/bc, & q^{-2n}/bd, & q^{1-2n}/cd \\ & q^{-2n}/b, & q^{1-2n}/c, & q^{-2n}/d, & q^{1-2n}/bcd \end{matrix} \middle| q \right]_n \frac{1 - q^{2n+1}}{1 - q/c}.$$

Letting  $c = q^{1/2-n}$  in both (3.1) and (3.3), we can unify the resulting identities as the following single one (cf. Chu [6, (5.2a/b)])

$$(3.4) \quad {}_4\phi_3 \left[ \begin{matrix} q^{-m}, & b, & d, & q^{1/2-m}/bd \\ & q^{1-m}/b, & q^{1-m}/d, & q^{1/2}bd \end{matrix} \middle| q; q^\varepsilon \right] \\ \stackrel{\varepsilon=1,2}{=} q^{m(\varepsilon-2)/2} \left[ \begin{matrix} q, & bd \\ & b, & d \end{matrix} \middle| q \right]_m \left[ \begin{matrix} b, & d \\ & q^{1/2}, & bd \end{matrix} \middle| q^{1/2} \right]_m.$$

The reader can verify that this is equivalent to (2.2) under the replacement  $q \rightarrow q^2$ .

This method works as long as we can find a couple of summation formulae of even and odd orders so that they can be unified. We can illustrate it again by examining another couple, of Example 2.5 from [6]:

$$(3.5) \quad {}_5\phi_4 \left[ \begin{matrix} q^{-2n}, & b, & c, & d, & q^{2-3n}/bcd \\ & q^{2-2n}/b, & q^{1-2n}/c, & q^{2-2n}/d, & q^{n-1}bcd \end{matrix} \middle| q; q^\varepsilon \right] \\ \stackrel{\varepsilon=1,2}{=} q^{n(\varepsilon-2)} \left\{ 1 - (q^{2n-2}bcd)^{\varepsilon-1} \frac{(1-q^{-n})(1-q^n c)}{(1-q^{n-1}bc)(1-q^{n-1}cd)} \right\} \\ \times \left[ \begin{matrix} b, d, q^{-2n}, q^{2-2n}/bc, q^{2-2n}/bd, q^{2-2n}/cd \\ b/q, d/q, q^{2-2n}/b, q^{1-2n}/c, q^{2-2n}/d, q^{2-2n}/bcd \end{matrix} \middle| q \right]_n,$$

and Example 2.29 from [6] with a typo corrected,

$$(3.6) \quad {}_5\phi_4 \left[ \begin{matrix} q^{-2n-1}, & b, & c, & d, & q^{1-3n}/bcd \\ & q^{1-2n}/b, & q^{1-2n}/c, & q^{1-2n}/d, & q^{n-1}bcd \end{matrix} \middle| q; q^\varepsilon \right] \\ \stackrel{\varepsilon=1,2}{=} q^{n\varepsilon} (q^n c)^{\varepsilon-2} \frac{(1-q^{-n-1})(1-q^{n-1}bc)(1-q^{n-1}cd)}{(1-b/q)(1-c/q)(1-d/q)} \\ \times \left[ \begin{matrix} q^{-1-2n}, q^{1-2n}/bc, q^{1-2n}/bd, q^{1-2n}/cd \\ q^{1-2n}/b, q^{1-2n}/c, q^{1-2n}/d, q^{2-2n}/bcd \end{matrix} \middle| q \right]_n \\ \times \left\{ 1 - (q^{n-1}bd)^{\varepsilon-1} \frac{(1-q^n c)(1-c/q)}{(1-q^{n-1}bc)(1-q^{n-1}cd)} \right\}.$$

By letting  $c = q^{1/2-n}$  in both (3.5) and (3.6), we can unify the resulting identities as the following single one:

$$(3.7) \quad {}_4\phi_3 \left[ \begin{matrix} q^{-m}, & b, & d, & q^{3/2-m}/bd \\ & q^{2-m}/b, & q^{2-m}/d, & bd/q^{1/2} \end{matrix} \middle| q; q^\varepsilon \right] \stackrel{\varepsilon=1,2}{=} q^{m/2(\varepsilon-3)} \left[ \begin{matrix} q, & bd/q \\ b/q, & d/q \end{matrix} \middle| q \right]_m \\ \times \left[ \begin{matrix} b/q^{1/2}, & d/q^{1/2} \\ q^{1/2}, & bd/q \end{matrix} \middle| q^{1/2} \right]_m \left\{ 1 - (q^{(m-3)/2}bd)^{\varepsilon-1} \frac{(1-q^{1/2})(1-q^{-m/2})}{(1-b/q^{1/2})(1-d/q^{1/2})} \right\}.$$

It is trivial to check that the last formula for  $\varepsilon = 1$  confirms the  ${}_4\phi_3$ -series identity conjectured by Andrews under the replacements  $b \rightarrow q^{3/2-m}/(bd)$  and  $q \rightarrow q^2$ .

There are numerous almost-poised  ${}_5\phi_4$ -series identities in [6]. By carrying on the above procedure, more almost-poised  ${}_4\phi_3$ -series identities can be derived. Three further examples are given below.

- Combination of Examples 2.20 and 2.22 from [6, (5.11c/d)]:

$$(3.8) \quad {}_4\phi_3 \left[ \begin{matrix} q^{-m}, & b, & d, & q^{-1/2-m}/bd \\ & q^{-m}/b, & q^{-m}/d, & q^{1/2}bd \end{matrix} \middle| q; q \right] \\ = \left[ \begin{matrix} q, & qbd \\ qb, & qd \end{matrix} \middle| q \right]_m \times \left[ \begin{matrix} q^{1/2}b, & q^{1/2}d \\ q^{1/2}, & q^{1/2}bd \end{matrix} \middle| q^{1/2} \right]_m.$$

- Combination of Examples 2.4 and 2.14 from [6, (5.5c/d)]:

$$(3.9) \quad {}_4\phi_3 \left[ \begin{matrix} q^{-m}, & b, & d, & q^{1/2-m}/bd \\ & q^{1-m}/b, & q^{1-m}/d, & q^{-1/2}bd \end{matrix} \middle| q; q \right] \\ = q^{-m/2} \left[ \begin{matrix} q, & bd \\ b, & d \end{matrix} \middle| q \right]_m \left[ \begin{matrix} b, & d \\ q^{1/2}, & q^{-1/2}bd \end{matrix} \middle| q^{1/2} \right]_m.$$

- Combination of Examples 2.7 and 2.30 from [6] (where the latter contains a typo: the terminating parameter should be  $q^{-2n-1}$ , instead of  $q^{-2n}$ ):

$$(3.10) \quad {}_4\phi_3 \left[ \begin{matrix} q^{-m}, & b, & d, & q^{5/2-m}/bd \\ & q^{2-m}/b, & q^{3-m}/d, & q^{-3/2}bd \end{matrix} \middle| q; q \right] = \left[ \begin{matrix} b, & d/q^{1/2} \\ q^{1/2}, & bd/q^2 \end{matrix} \middle| q^{1/2} \right]_m \\ \times q^{-3m/2} \left\{ 1 - \frac{q^{1/2}(1 - q^{m/2})(1 - q^{m-5/2}bd)}{(1 - q^{(m-1)/2}b)(1 - q^{(m-2)/2}d)} \right\} \left[ \begin{matrix} q, & bd/q^2 \\ b/q, & d/q^2 \end{matrix} \middle| q \right]_m.$$

Observe that the identities displayed in (3.4) and (3.7) are 2-balanced series, while those in (3.8)–(3.10) are balanced ones. The experiments with *Mathematica* suggest that the following series with four extra integer parameters  $\alpha, \beta, \gamma, \lambda$  subject to  $\alpha + \beta + \gamma \geq \lambda$  should be always expressible in analytical formulae with the number of terms being polynomial in  $\alpha, \beta, \gamma, \lambda$ , but independent of the summation order  $m$ :

$$(3.11) \quad {}_4\phi_3 \left[ \begin{matrix} q^{-m}, & b, & d, & q^{\lambda-1/2-m}/bd \\ & q^{\alpha-m}/b, & q^{\beta-m}/d, & q^{\gamma+1/2}bd \end{matrix} \middle| q; q \right].$$

The remaining problem is to find such a formula explicitly, as done by the author [8] for the  ${}_3F_2$ -series of Watson, Whipple and Dixon.

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Wenchang Chu

School of Mathematics and Statistics

Zhoukou Normal University

Zhoukou 466001, P.R. China

*Current address:*

Dipartimento di Matematica e Fisica “Ennio De Giorgi”

Università del Salento, Lecce–Arnesano

P.O. Box 193

73100 Lecce, Italy

E-mail: chu.wenchang@unisalento.it

