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Corrigendum to: "Improvements on the star discrepancy of (t,s)-sequences"

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by

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We report on an inaccuracy in the proof of Theorem 3 (page 74, first step, lines 9–17) which arises from the omission of some product terms on the left-hand side of the inequality on line 14, just above equation (14). In order to overcome this problem and take into account all product terms, we proceed as follows.

For any integer i, $1 \le i \le s$, we still define integers $c_j^{(i)} := |a_{2j-1}^{(i)}| + |a_{2j}^{(i)}|$ for any integer $j \ge 1$ and we set $c_0^{(i)} := 1$. Recall that $a_0^{(i)}$ is either 0 or 1 so that we still have $|a_0^{(i)}| \le c_0^{(i)}$.

Then, for simplicity, we first consider the case of two dimensions ($s = 2$). Two consecutive integers $j_i \ge 1$ ($i = 1, 2$), say $2h_i - 1$ and $2h_i$, occurring in S' give the same integer $j'_i = h_i$. Notice that these j'_i are those defined in equation (14). For the four resulting pairs we have

$$|a_{2h_1}^{(1)}|\,|a_{2h_2}^{(2)}| + |a_{2h_1}^{(1)}|\,|a_{2h_2-1}^{(2)}| + |a_{2h_1-1}^{(1)}|\,|a_{2h_2}^{(2)}| + |a_{2h_1-1}^{(1)}|\,|a_{2h_2-1}^{(2)}| = c_{h_1}^{(1)} c_{h_2}^{(2)} = c_{j'_1}^{(1)} c_{j'_2}^{(2)}.$$

If $j_1 = 0$ and $j_2 \ge 1$, there are only two pairs $(0, 2h_2)$ and $(0, 2h_2 - 1)$ giving

$$|a_0^{(1)}|\,|a_{2h_2}^{(2)}| + |a_0^{(1)}|\,|a_{2h-1}^{(2)}| \le c_0^{(1)}(|a_{2h_2}^{(2)}| + |a_{2h_2-1}^{(2)}|) = c_0^{(1)} c_{j'_2}^{(2)} = c_{j'_2}^{(2)}.$$

If $j_1 \ge 1$ and $j_2 = 0$, we have an analogous inequality, and finally, if $j_1 = j_2 = 0$, there is only one pair $(0,0)$ which gives $|a_0^{(1)}|\,|a_0^{(2)}| \le c_0^{(1)} c_0^{(2)} = 1$.

Now, for s dimensions, we first consider the case where all $j_i \ge 1$ and obtain 2^s s-tuples $(j_1, \dots, j_s) \in S'$ of consecutive integers, $j_i = 2h_i - 1$, $j_i + 1 = 2h_i$, for which equation (14) holds. Summing the products for these

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2^s s-tuples, for a given s-tuple $(h_1, \ldots, h_s)$ we get

$$\sum_{\{(j_1,\ldots,j_s)\in S';\, j'_i=h_i\}} \prod_{i=1}^{s} |a_{j_i}^{(i)}| = \prod_{i=1}^{s} c_{j'_i}^{(i)}.$$

In the remaining cases where there exists some $j_i = 0$, there are only 2^k s-tuples where k $(1 \le k \le s-1)$ is the number of $j_i \ge 1$ (for which j'_i is still defined by (14)). Summing the products for such 2^k s-tuples, we obtain (recall that $|a_0^{(i)}| \le c_0^{(i)} = 1$)

$$\sum_{\substack{(j_1,\ldots,j_s)\in S' \\ j'_i=h_i \text{ when } j_i\ge 1}} \prod_{\{i;\, j_i=0\}} |a_0^{(i)}| \prod_{\{i;\, j_i\ge 1\}}^{s} |a_{j_i}^{(i)}| \le \sum_{\substack{(j_1,\ldots,j_s)\in S' \\ j'_i=h_i \text{ when } j_i\ge 1}} \prod_{\{i;\, j_i\ge 1\}}^{s} |a_{j_i}^{(i)}| = \prod_{\{i;\, j_i\ge 1\}} c_{j'_i}^{(i)}$$

(the last product reads $\prod_{i=1}^{s} c_{j'_i}^{(i)}$ if we set $j'_i = 0$ when $j_i = 0$).

This way, we obtain a one-to-one correspondence between the s-tuples $(j_1, \ldots, j_s) \in S'$ and the products occurring in the different cases enumerated above. This ends the first step we took under consideration.

The next two steps remain the same and lead to the desired bound on line 2, page 75 without any other change, so that Theorem 3 is still valid.

Furthermore, we want to point out a misprint in equation (13): In the first expression enclosed in large parentheses, "$+sb$" shoud read "$+s$".

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