ACTA ARITHMETICA
165.1 (2014)

Explicit estimates on the summatory functions of the Möbius function with coprimality restrictions

by

Olivier Ramaré (Lille)

1. Introduction. In explicit analytic number theory, one very often needs to evaluate the average of a multiplicative function, say f. The usual strategy is to compare this function to a more usual model f_0, as in [12, Lemma 3.1]. This process is also well detailed in [3]. When the model is $f_0 = 1$, the situation is readily cleared out; it is also well studied when this model is the divisor function [2, Corollary 2.2]. We signal here that the case of the characteristic function of the squarefree numbers is specifically handled in [5].

The next problem is to use the Möbius function as a model. In this case the necessary material can be found in [13], though of course the saving is much smaller and may be insufficient: when the model is 1 or the divisor function, or the characteristic function of the squarefree integers, the saving is a power of the size of the variable, while now it is only a logarithm (or the square of one according to whether one says that the trivial estimate for $\sum_{d\le D}\mu(d)/d$ is 1 or $\log D$). One of the consequences is that one has to be more careful, and thrifty, when it comes to small variations. The variations we consider here is the addition of a coprimality condition $(d,q)=1$, for some fixed q, on the variable d. Our first aim is thus to show how to get explicit estimates for the family of functions

$$m_q(x) = \sum_{\substack{n\le x\\(n,q)=1}} \mu(n)/n, \qquad m(x) = m_1(x) \tag{1.1}$$

from explicit estimates concerning solely $m(x)$. The definition of the Liouville function $\lambda(n)$, appearing in the result below, is recalled in (1.3), while the auxiliary function ℓ_q is defined in (1.4).

2010 *Mathematics Subject Classification*: Primary 11N37, 11Y35; Secondary 11A25.
Key words and phrases: explicit estimates, Möbius function.

DOI: 10.4064/aa165-1-1

Theorem 1.1. *When $1 \le q < x$, where q is an integer and x a real number, we have*

$$\Big|\sum_{\substack{n\le x\\(n,q)=1}}\frac{\mu(n)}{n}\Big|\le\frac{q}{\varphi(q)}\,\frac{2.4}{\log(x/q)},\qquad\Big|\sum_{\substack{n\le x\\(n,q)=1}}\frac{\lambda(n)}{n}\Big|\le\frac{q}{\varphi(q)}\,\frac{0.79}{\log(x/q)}.$$

Moreover $\log(x/q)|\ell_q(x)|\le 0.155q/\varphi(q)$ *and* $\log(x/q)|m_q(x)|\le\frac{3}{2}q/\varphi(q)$ *when* $x/q\ge 3310$. *We also have* $\log(x/q)|m_q(x)|\le\frac{7}{8}q/\varphi(q)$ *when* $x/q\ge 9960$.

The sole previous estimate on $m_q(x)$ seems to be [7, Lemma 10.2], which bounds $|m_q(x)|$ uniformly by 1. The estimate for $m(x)$ that will provide the initial step comes from [13]:

$$|m(x)|\le 0.03/\log x\qquad(x\ge X_0=11\,815).\tag{1.2}$$

Let us first note that the simplest treatment of this condition via the Möbius function, i.e. writing

$$\mathbb{1}_{(d,q)=1}=\sum_{\substack{\delta|q\\\delta|d}}\mu(\delta),$$

does not work here. Indeed, we get

$$\sum_{\substack{d\le D\\(d,q)=1}}\frac{\mu(d)}{d}=\sum_{\delta|q}\mu(\delta)\sum_{\delta|d\le D}\frac{\mu(d)}{d}=\sum_{\delta|q}\frac{\mu(\delta)^2}{\delta}\sum_{\substack{d\le D/\delta\\(d,\delta)=1}}\frac{\mu(d)}{d}$$

and we are back to the initial problem with different parameters. The classical workaround (used for instance in [10, near (7)] but already known by Landau) runs as follows: we determine a function g_q such that $\mathbb{1}_{(n,q)=1}\mu(n)=g_q\star\mu(n)$, where $\star$ denotes the arithmetic convolution product. The drawback of this method is that the support of g is not bounded (determining g_q by comparing the Dirichlet series is a simple exercise). So if we write

$$\sum_{\substack{d\le D\\(d,q)=1}}\mu(d)/d=\sum_{\delta\le D}\frac{g_q(\delta)}{\delta}\sum_{d\le D/\delta}\frac{\mu(d)}{d},$$

we are forced to:

1. use estimates for $\sum_{d\le D/\delta}\mu(d)/d$ when D/δ can be small,
2. complete the sum over δ to get a decent result.

Both steps introduce quite a loss when q is not specified. We propose here a different approach by introducing the Liouville function as an intermediary. This function $\lambda(n)$ is the completely multiplicative function that is 1 on the integers that have an even number of prime factors—counted with multiplicity—and -1 otherwise. It satisfies

$$\sum_{n\geq 1}\frac{\lambda(n)}{n^s}=\frac{\zeta(2s)}{\zeta(s)}. \tag{1.3}$$

We introduce the family of auxiliary functions

$$\ell_q(x)=\sum_{\substack{n\leq x\\(n,q)=1}}\lambda(n)/n,\qquad \ell(x)=\ell_1(x). \tag{1.4}$$

Our process runs as follows: we derive bounds for $\ell(x)$ from bounds on $m(x)$ and some computations, derive bounds on $\ell_q(x)$ from bounds on $\ell(x)$, and finally derive bounds on $\mu_q(x)$ from bounds on $\ell_q(x)$. The theoretical steps are contained in Lemmas 2.3, 2.5 and 3.2.

We complete this introduction by signalling that [14] contains explicit estimates with a large range of uniformity for sums of the shape

$$\sum_{\substack{d\leq x\\(d,r)=1}}\frac{\mu(d)}{d^{1+\varepsilon}}$$

and for a similar sum but with the summand $\mu(d)\log(x/d)/d^{1+\varepsilon}$. The path we followed there is essentially elementary and the saving is smaller.

2. From the Möbius function to the Liouville function

Lemma 2.1. *For $2\leq x\leq 906\,000\,000$, we have $|\ell(x)|\leq 1.347/\sqrt{x}$.*
For $2\leq x\leq 1.1\cdot 10^{10}$, we have $|\ell(x)|\leq 1.41/\sqrt{x}$.
For $1\leq x\leq 1.1\cdot 10^{10}$, we have $|\ell(x)|\leq\sqrt{2/x}$.

The computations have been run with PARI/GP (see [11]), speeded up by using gp2c as described for instance in [2]. We mention here that [6] proposes an algorithm to compute isolated values of $M(x)$. This can most probably be adapted to compute isolated values of $\ell(x)$, but does not seem to offer any improvement for bounding $|\ell(x)|$ on a large range. In [4], the authors show that

$$\ell(x)\geq 0\qquad (x<72\,185\,376\,951\,205)$$

and that

$$\ell(x)\geq -2.0757642\cdot 10^{-9}\qquad (x\leq 75\,000\,000\,000\,000).$$

This takes care of the lower bound for $\ell(x)$. The computations we ran are much less demanding in time and algorithm, but rely on a large enough sieve-kind table to compute the values of $\lambda(n)$ on some very large range. Harald Helfgott (indirectly) pointed out to me that the RAM-memory can be very large nowadays, allowing one to precompute large quantities to which one has almost immediate access. Here is a simplified version of the main loop:

```
{getbounds(zmin:small, valini:real, zmax:small) =
   my(maxi:real, valuesliouville:vecsmall, gotit:vecsmall,
      valuel:real, bound:small, pa:small);

   /* Precomputing lambda(n): */
   valuesliouville = vectorsmall(zmax-zmin+1, m, 1);
   gotit = vectorsmall(zmax-zmin+1, m, 1);
   forprime (p:small = 2, floor(sqrt(zmax+0.0)),
            bound = floor(log(zmax+0.0)/log(p+0.0));
            pa = 1;
            for(a:small = 1, bound,
               pa *= p;
               for(k:small = 1, floor((zmax+0.0)/pa),
                  if(k*pa >= zmin,
                     valuesliouville[k*pa-zmin+1] *= -1;
                     gotit[k*pa-zmin+1] *= p,))));

   /* Correction in case of a large prime factor: */
   for(n:small = zmin, zmax,
      if(gotit[n-zmin+1] < n,
         valuesliouville[n-zmin+1] *= -1,));

   valuel = (valini + 0.0) + valuesliouville[1]/zmin;
   maxi = max( valini*sqrt(zmin+0.0), abs(valuesl*sqrt(zmin+1.0)));

   /* Main loop: */
   for(n:small = zmin+1, zmax,
      valuel += valuesliouville[n-zmin+1]/n;
      maxi = max(maxi, abs(valuel)*sqrt(n+1.0)));

   return([maxi, valuel]);
}
```

We used this loop to compute our maximum on intervals of length $2\cdot 10^7$. The main function aggregates these results by making the interval vary. The computations took about half a day on a 64-bit fast desktop with 8G of RAM. In the actual script, we also checked that the computed value of $\ell(x)$ is non-negative in this range. Going farther would improve on the final constants, but only when x/q is large. We compared $|\ell(x)|$ with $1/\sqrt{x}$, and this seems correct for small values, but [9] and [8] suggest that the maximal order is larger.

Lemma 2.2. *The function*

$$T(y): y \mapsto \frac{\log y}{y}\int_{\sqrt{X_0}}^{y}\frac{dv}{\log v}$$

satisfies $T(y) \leq 1.119$ *for* $y \geq 10^5$.

Proof. Repeated integration by parts shows that

$$T(y) = \frac{\log y}{y}\biggl(\frac{y}{\log y} - \frac{\sqrt{X_0}}{\log\sqrt{X_0}} + \frac{y}{(\log y)^2} - \frac{\sqrt{X_0}}{(\log\sqrt{X_0})^2} + 2\int_{\sqrt{X_0}}^{y}\frac{dv}{(\log v)^3}\biggr)$$
$$\leq \frac{\log y}{y}\biggl(\frac{y}{\log y} - \frac{\sqrt{X_0}}{\log\sqrt{X_0}} + \frac{y}{(\log y)^2} - \frac{\sqrt{X_0}}{(\log\sqrt{X_0})^2}\biggr) + \frac{2T(y)}{(\log\sqrt{X_0})^2},$$

from which we deduce that

$$T(y) \leq 1.1001\cdot\Bigl(1+\frac{1}{\log y}\Bigr).$$

This shows that $T(y) \leq 1.113$ when $y \geq 10^{40}$. We then check *numerically* that the function T is increasing and then decreasing, with a maximum around 12478.8 with value $1.118\,598 + \mathcal{O}^*(10^{-6})$. But this is only an *observation*, since the computer gives only a sample of values. Since the derivative of T can easily be bounded, we obtain the claimed upper bound. The reader may also consult [1] where a similar process is fully detailed. ∎

The following lemma is a simple exercise:

Lemma 2.3. *We have*

$$\ell_q(x) = \sum_{\substack{u^2\leq x\\(u,q)=1}} m_q(x/u^2)/u^2. \tag{2.1}$$

We shall use it only when $q = 1$, but it is equally easy to state it in general.

Lemma 2.4. *For $x > 1$, we have $|\ell(x)| \leq 0.79/\log x$.*
For $x \geq 3310$, we have $|\ell(x)| \leq 0.155/\log x$.
For $x \geq 8918$, we have $|\ell(x)| \leq 0.099/\log x$.

Proof. We appeal to Lemma 2.3 (with $q = 1$) and separate the sum according to $u \leq U$ or $u > U$ where $x/U^2 \geq X_0$. When $u \leq U$ we apply (1.2), in the other case we use the fact that $|m(x)| \leq 1$ to obtain

$$|\ell(x)| \leq 0.03\sum_{u\leq U}\frac{1}{u^2\log(x/u^2)} + \frac{1+U^{-1}}{U}$$

With $f(t) = 1/(t^2\log(x/t^2))$, we check that

$$f'(t) = -\frac{2}{t^3\log(x/t^2)} + \frac{2}{t^3\log^2(x/t^2)}.$$

This quantity is negative for $1 \leq t \leq U$, since then $x/t^2 \geq x/U^2 \geq X_0 > e$.

We thus have

$$\sum_{u\le U}\frac{1}{u^2\log(x/u^2)}\le f(1)+\int_1^U f(t)\,dt=\frac{1}{\log x}+\int_1^U\frac{dt}{t^2\log(x/t^2)}.$$

Changing variables we get

$$\sum_{u\le U}\frac{1}{u^2\log(x/u^2)}\le\frac{1}{\log x}+\frac{1}{\sqrt{x}}\int_{\sqrt{x/U^2}}^{\sqrt{x}}\frac{dv}{2\log v}.$$

It follows that

$$|\ell(x)|\le\frac{0.03}{\log x}+\frac{0.03}{\sqrt{x}}\int_{\sqrt{X_0}}^{\sqrt{x}}\frac{dv}{2\log v}+\frac{1+\sqrt{X_0/x}}{\sqrt{x/X_0}}.$$

We apply Lemma 2.2 at this level. Hence, when $x\ge 10^{10}$,

$$\begin{aligned}|\ell(x)|&\le\frac{0.03}{\log x}+\frac{0.03\cdot 1.119}{\log x}+\frac{1+\sqrt{X_0/x}}{\sqrt{x/X_0}}\\&\le\frac{0.06357}{\log x}+\frac{(1+\sqrt{X_0/x})\log x}{\sqrt{x/X_0}}\frac{1}{\log x}\\&\le\frac{0.089}{\log x}\le\frac{0.099}{\log x}.\end{aligned}$$

We extend it to $x\ge 17\,715$ via Lemma 2.1, part one and two, and to $x\ge 8918$ by direct inspection. This inequality extends to $x\ge 1$ by weakening the constant 0.099 to 0.79. Straightforward computations yield the bound 0.155 when $x\ge 3310$. ∎

Adding coprimality conditions. Our tool is provided by a simple elementary lemma.

Lemma 2.5. *We have*

$$\ell_q(x)=\sum_{d|q}\frac{\mu^2(d)}{d}\ell(x/d).$$

The second part of Theorem 1.1 follows immediately by combining Lemma 2.5 with Lemma 2.4. Actually, what comes out is the bound

$$|\ell_q(x)|\le\frac{0.79}{\log(x/q)}\sum_{d|q}\frac{\mu^2(d)}{d}=\frac{0.79}{\log(x/q)}\prod_{p|q}\frac{p+1}{p}.$$

As the function $q/\varphi(q)$ is easier to remember and $\prod_{p|q}\frac{p+1}{p}\le q/\varphi(q)$, we simplify the above to

$$|\ell_q(x)|\le\frac{0.79}{\log(x/q)}\frac{q}{\varphi(q)}.$$

When $x/q \geq 3310$, one can replace 0.79 by 0.155, and when $x/q \geq 8918$, by 1/10.

3. Back to the Möbius function with coprimality coditions. Let us start with a wide ranging estimate:

LEMMA 3.1. *For every integer $q \geq 1$ and every real number $x \geq 1$, we have $|\ell_q(x)| \leq \pi^2/6$.*

Proof. Apply Lemma 2.3 and [7, Lemma 10.2] ([1]). ∎

The following lemma is again a simple exercise.

LEMMA 3.2. *We have*

$$m_q(x) = \sum_{\substack{u^2 \leq x \\ (u,q)=1}} \frac{\mu(u)}{u^2} \ell_q\big(x/u^2\big).$$

Proof of Theorem 1.1. We have to prove several estimates of type

$$\varphi(q) \log(x/q) |m_q(x)| \leq c, \qquad x/q \geq N.$$

We put $x^* = x/q$ and $y = \log x^* = \log(x/q)$ and divide the proof into two parts. First we consider the case $1 \leq y \leq 8$, and later the case $y > 8$.

CASE (A): $1 \leq y \leq 8$. We appeal to Lemma 3.2. For a real parameter U such that $U^2 \leq x^*$ we have

$$\begin{aligned} (3.1) \qquad |m_q(x)| &\leq \sum_{u^2 \leq x} \frac{\mu^2(u)}{u^2} |\ell_q(x/u^2)| \\ &\leq \sum_{u \leq U} \frac{q}{\varphi(q)} \frac{0.79\mu^2(u)}{u^2 \log(x/(u^2 q))} + \frac{\pi^2}{6} \sum_{u > U} \frac{\mu^2(u)}{u^2} \\ &\leq \frac{q/\varphi(q)}{\log(x/q)} \Bigg(\sum_{u \leq U} \frac{0.79\mu^2(u)}{u^2 \big(1 - \frac{2\log u}{\log(x/q)}\big)} + \frac{\pi^2}{6} \sum_{u > U} \frac{\mu^2(u)}{u^2} \log(x/q) \Bigg). \end{aligned}$$

This is our starting inequality. We define

$$(3.2) \qquad \rho(U, y) = 0.79 \sum_{u \leq U} \frac{\mu^2(u)}{u^2 \big(1 - \frac{2\log u}{y}\big)} + \frac{\pi^2}{6} \sum_{u > U} \frac{\mu^2(u)}{u^2} y.$$

Note that $\rho(U, y) = \rho([U], y)$ where $[U]$ is the integer part of U. For each y we need to select one U such that $\rho(U, y) \leq 2.4$. We choose $U = 1$ for $y \in [1, a_1]$; $U = 2$ for $y \in [a_1, a_2]$; $U = 3$ for $y \in [a_2, a_3]$; and $U = 7$ for $y \in [a_3, 8]$. Here $a_1 = 1.8665\ldots$ is a solution of $\rho(1, y) = \rho(2, y)$; $a_2 = 2.6774\ldots$ is a solution of $\rho(2, y) = \rho(3, y)$; $a_3 = 4.1237\ldots$ is a solution of $\rho(3, y) = \rho(7, y)$.

([1]) If we were to adapt the proof presented in [7] to the case of λ instead of μ, we would reach the bound 2 and not $\pi^2/6$.

Each of these three functions is a sum of a linear term ay and terms of type $Ay/(y-2\log n)$ with $A>0$. These are convex for $y>2\log n$. In this way it is very easy to show that $\rho(1,y)$ is convex in $[1,a_1]$, $\rho(2,y)$ is convex in $[a_1,a_2]$, $\rho(3,y)$ is convex in $[a_2,a_3]$, and finally $\rho(7,y)$ is convex in $[a_3,8]$. So, for example, to show the inequality $\rho(3,y)\le 2.4$ in the interval $[a_2,a_3]$ we only have to show that $\rho(3,a_2),\rho(3,a_3)\le 2.4$. This presents no difficulty. The maximum value obtained is $\rho(2,a_2)=2.38790\ldots$ with

$$a_2=\frac{237+100\pi^2\log 3}{50\pi^2},$$
$$\rho(2,a_2)=\frac{237}{20\pi^2}+\pi^2\left(\frac{79\log 2}{948+400\pi^2\log(3/2)}-\frac{5\log 3}{12}\right)+\log 243.$$

Case (B): $y>8$. We start from Lemma 3.2, from which we deduce a simpler bound:

$$|m_q(x)|\le\sum_{u^2\le x}|\ell_q(x/u^2)|/u^2,$$

which we then exploit in the same way as in the proof of Lemma 2.4, replacing the bound $|m(x)|\le 1$ by Lemma 3.1. With $x=eU^2q$ and $x^*=x/q$, we thus get

$$\begin{aligned}|m_q(x)|&\le\frac{q}{\varphi(q)}\frac{0.79}{\log x^*}+\frac{0.79q}{\varphi(q)}\int_1^{\sqrt{x^*/e}}\frac{du}{u^2\log(x^*/u^2)}+\frac{\pi^2\sqrt{e}}{6}\frac{1+\sqrt{e/x^*}}{\sqrt{x^*}}\\&\le\frac{q}{\varphi(q)}\frac{0.79}{\log x^*}+\frac{0.79q}{\varphi(q)\sqrt{x^*}}\int_{\sqrt{e}}^{\sqrt{x^*}}\frac{dv}{2\log v}+\frac{\pi^2\sqrt{e}}{6}\frac{1+\sqrt{e/x^*}}{\sqrt{x^*}}\\&\le c(x^*)\frac{q}{\varphi(q)\log x^*}\end{aligned}$$

with

$$c(x^*)=0.79+0.79\frac{\log x^*}{\sqrt{x^*}}\int_{\sqrt{e}}^{\sqrt{x^*}}\frac{dv}{2\log v}+\frac{\pi^2\sqrt{e}}{6}\frac{1+\sqrt{e/x^*}}{\sqrt{x^*}}\log x^*.$$

Some numerical work shows that $c(x^*)\le 2.4$ when $x^*\ge 1862$, so our inequality is proved for $y>\log 1862=7.52941\ldots$. This together with part (A) proves that $\varphi(q)\log(x/q)|m_q(x)|\le 2.4$ for $1\le q<x$.

When $x^*\ge 3310$, we can single out the term $u=1$ in (3.1) and modify the coefficient of the bound on this term from 0.79 into 0.155; then we treat the rest of the sum in the same way as before. We get a similar bound with

$c(x^*)$ replaced by

$$c_1(x^*) = 0.155 + 0.79\frac{\log x^*}{4\log(x^*/4)} + 0.79\frac{\log x^*}{\sqrt{x^*}}\int_{\sqrt{e}}^{\sqrt{x^*/4}}\frac{dv}{2\log v} + \frac{\pi^2\sqrt{e}}{6}\,\frac{1+\sqrt{e/x^*}}{\sqrt{x^*}}\log x^*.$$

This yields a maximum of not more than $1.466 < 3/2$. When $x^* \geq 3\cdot 3310$, we single out the terms of index 1, 2, and 3 similarly. This means replacing $c_1(x^*)$ by

$$c_2(x^*) = 0.155 + 0.155\frac{\log x^*}{4\log(x^*/4)} + 0.155\frac{\log x^*}{9\log(x^*/9)} + 0.79\frac{\log x^*}{25\log(x^*/25)} + 0.79\frac{\log x^*}{\sqrt{x^*}}\int_{\sqrt{e}}^{\sqrt{x^*/25}}\frac{dv}{2\log v} + \frac{\pi^2\sqrt{e}}{6}\,\frac{1+\sqrt{e}x^{*-1/2}}{\sqrt{x^*}}\log x^*.$$

This yields a maximum of not more than $0.871 < 7/8$. The proof of Theorem 1.1 is complete. ∎

Acknowledgments. I thank Harald Helfgott for interesting discussions that pushed me into pulling this note out of its drawer. Special thanks are also due to the referee for his/her very careful reading: several numerical errors have been corrected in the process, and the arguments are also now better presented.

References

[1] J. Arias de Reyna and J. van de Lune, *A proof of a trigonometric inequality. A glimpse inside the mathematical kitchen*, J. Math. Inequal. 5 (2011), 341–353.

[2] D. Berkane, O. Bordellès, and O. Ramaré, *Explicit upper bounds for the remainder term in the divisor problem*, Math. Comp. 81 (2012), 1025–1051.

[3] P. Berment et O. Ramaré, *Ordre moyen d'une fonction arithmétique par la méthode de convolution*, Rev. Math. Spéciale 212 (2012), 1–15.

[4] P. Borwein, R. Ferguson, and M. J. Mossinghoff, *Sign changes in sums of the Liouville function*, Math. Comp. 77 (2008), 1681–1694.

[5] H. Cohen, F. Dress, and M. El Marraki, *Explicit estimates for summatory functions linked to the Möbius μ-function*, Funct. Approx. Comment. Math. 37 (2007), 51–63.

[6] M. Deléglise and J. Rivat, *Computing the summation of the Möbius function*, Experiment. Math. 5 (1996), 291–295.

[7] A. Granville and O. Ramaré, *Explicit bounds on exponential sums and the scarcity of squarefree binomial coefficients*, Mathematika 43 (1996), 73–107.

[8] T. Kotnik and J. van de Lune, *On the order of the Mertens function*, Experiment. Math. 13 (2004), 473–481.

[9] H. L. Montgomery, *Zeros of approximations to the zeta function*, in: Studies in Pure Mathematics, Birkhäuser, Basel, 1983, 497–506.

[10] Y. Motohashi, *Primes in arithmetic progressions*, Invent. Math. 44 (1978), 163–178.

[11] The PARI Group, Bordeaux, *PARI/GP, version 2.5.2*, 2011, http://pari.math.u-bordeaux.fr/.

[12] O. Ramaré, *On Šnirel'man's constant*, Ann. Scuola Norm. Sup. Pisa 21 (1995), 645–706; http://math.univ-lille1.fr/~ramare/Maths/Article.pdf.

[13] O. Ramaré, *From explicit estimates for the primes to explicit estimates for the Möbius function*, Acta Arith. 157 (2013), 365–379.

[14] O. Ramaré, *Some elementary explicit bounds for two mollifications of the Möbius function*, Funct. Approx. Comment. Math. 49 (2013), 229–240.

Olivier Ramaré
CNRS, Laboratoire Paul Painlevé
Université Lille 1
59 655 Villeneuve d'Ascq, France
E-mail: ramare@math.univ-lille1.fr

Received on 8.3.2012
and in revised form on 19.3.2014 (7000)