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Solving parametrized systems of Pell equations

by

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To my long time friend Pierre Samuel,
octogenarian in love with Diophantus

1. Introduction. Let $F > 1$ be a square-free integer. In his papers [3]–[6] Ljunggren studied the quartic equation

$$x^4 - Fy^2 = 1$$

(and similar equations) and proved that the equation has at most two solutions (x, y) in positive integers. He also gave an algorithm to find the solutions.

When $F = p$ is a prime number, Ljunggren showed that

$$x^4 - py^2 = 1$$

has no solution in positive integers when $p \neq 5$, 29. Moreover if $p = 5$ the only solution is $(3, 4)$ and if $p = 29$ the only solution is $(99, 1820)$. In [12] Samuel gave another proof for $p = 5$.

The proof of this result leads to the study of the systems

$$\begin{cases} x^2 - 2y^2 = \pm 1, \\ x^2 - 2py^2 = \mp 1. \end{cases}$$

In our paper we shall need binary recurring sequences. Let $P > 0$, $Q \neq 0$ be integers such that $D = P^2 - 4Q > 0$. We shall consider the Lucas sequences $(U_n)_n$, $(V_n)_n$ with parameters (P, Q):

$$\begin{aligned} &U_0 = 0, \quad U_1 = 1, && V_0 = 2, \quad V_1 = P, \\ &U_n = PU_{n-1} - QU_{n-2} \text{ for } n \geq 2, && V_n = PV_{n-1} - QV_{n-2} \text{ for } n \geq 2, \\ &U_n = (-1/Q^n)U_{-n} \text{ for } n < 0; && V_n = (1/Q^n)V_{-n} \text{ for } n < 0. \end{aligned}$$

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As is easily seen, the above recurrences still hold for any integer n. When needed, we shall use the notation $U_n = U_n(P,Q)$, $V_n = V_n(P,Q)$.

Given the square-free integer $F > 1$, let $\varepsilon = c + d\sqrt{F}$ be the fundamental unit of the ring $\mathbb{Z}[\sqrt{F}]$, so $c > 0$, $d > 0$. As is well known, c and d are effectively bounded in terms of F. Let $P = 2c$, $Q = c^2 - Dd^2 = \pm 1$, let $U_n = U_n(2c,Q)$, $V_n = V_n(2c,Q)$. Then

$$\varepsilon^n = \frac{V_n}{2} + dU_n\sqrt{F}.$$

As is easily seen, V_n is even for every $n \in \mathbb{Z}_-$ and $V_{2n} \equiv 2 \pmod 4$ for every $n \in \mathbb{Z}$. If $s \geq 1$ we define

$$k_s = \frac{1}{4}(2 + Q^s V_{2s}), \quad h_s = \frac{1}{4}(2 - Q^s V_{2s}),$$

so $h_s, k_s \neq 0, 1$ and $k_s + h_s = 1$.

(1.1) Theorem. *Let $F > 1$, $G \geq 1$ be square-free integers, let $s \geq 1$, $f \neq 0$ and $g = fk_s$ or $g = fh_s$. Then there exists an integer $N > 0$, effectively computable in terms of F, G, f and s, such that if $p = 1$ or p is a prime number, if $x \geq 0$, $y \geq 0$, $z > 0$ are integers such that*

$$\begin{cases} x^2 - Fy^2 = f, \\ x^2 - pGz^2 = g, \end{cases}$$

then $x, y, z, p < N$.

2. Preliminaries

A. Binary recurring sequences. Let $P > 0$, $Q \neq 0$ with $D = P^2 - 4Q > 0$. We gather some properties of $U_n = U_n(P,Q)$ and $V_n = V_n(P,Q)$ which will be needed in this paper.

Let α, β be the roots of $X^2 - PX + Q$, so

$$\alpha = \frac{P + \sqrt{D}}{2}, \quad \beta = \frac{P - \sqrt{D}}{2},$$
$$\alpha + \beta = P, \quad \alpha\beta = Q, \quad \alpha - \beta = \sqrt{D}.$$

For each $n \in \mathbb{Z}$:

$$U_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, \quad V_n = \alpha^n + \beta^n.$$

If $m, n \in \mathbb{Z}$:

$$2U_{m+n} = U_m V_n + U_n V_m,$$
$$U_{m+n} = U_m V_n - Q^n U_{m-n}, \quad V_{m+n} = V_m V_n - Q^n V_{m-n},$$
$$U_{2m} = U_m V_m, \quad V_{2m} = V_m^2 - 2Q^m,$$
$$V_m^2 - DU_m^2 = 4Q^m.$$

The following lemma will be required:

(2.1) Lemma. *For every $s \geq 1$ and $n \in \mathbb{Z}$ we have*
(a) $V_n^2 - V_{n-s}V_{n+s} = Q^{n-s}(2Q^s - V_{2s})$,
(b) $V_n^2 - DU_{n-s}U_{n+s} = Q^{n-s}(2Q^s + V_{2s})$,
(c) $D(U_n^2 - U_{n-s}U_{n+s}) = Q^{n-s}(-2Q^s + V_{2s})$,
(d) $DU_n^2 - V_{n-s}V_{n+s} = -Q^{n-s}(2Q^s + V_{2s})$.

Proof. We just prove (a):

$$\begin{aligned} V_n^2 - V_{n-s}V_{n+s} &= (\alpha^n + \beta^n)^2 - (\alpha^{n-s} + \beta^{n-s})(\alpha^{n+s} + \beta^{n+s}) \\ &= \alpha^{2n} + \beta^{2n} + 2(\alpha\beta)^n \\ &\quad - [\alpha^{2n} + \beta^{2n} - (\alpha\beta)^{n-s}(\alpha^{2s} + \beta^{2s})] \\ &= Q^{n-s}(2Q^s - V_{2s}). \ \blacksquare \end{aligned}$$

Now we assume $\gcd(P,Q) = 1$. If $m, n \geq 1$ and $d = \gcd(m,n)$ then

$$\gcd(U_m, U_n) = U_d,$$

$$\gcd(V_m, V_n) = \begin{cases} V_d & \text{if } m/d \text{ and } n/d \text{ are odd,} \\ 1 \text{ or } 2 & \text{otherwise,} \end{cases}$$

$$\gcd(U_m, V_n) = \begin{cases} V_d & \text{if } m/d \text{ is even,} \\ 1 \text{ or } 2 & \text{otherwise,} \end{cases}$$

$$\gcd(U_n, Q) = 1, \qquad \gcd(V_n, Q) = 1.$$

If $a, b \in \mathbb{Z}$, not both equal to 0, let

$$W_n = aU_n + bV_n \quad \text{for all } n \in \mathbb{Z}.$$

Then

$$W_n = PW_{n-1} - QW_{n-2} \quad \text{for all } n \in \mathbb{Z}.$$

We also have

$$W_{m+n} = W_m V_n - Q^n W_{m-n} \quad \text{for all } m, n \in \mathbb{Z}.$$

If $\gcd(P,Q) = 1$ then

$$\gcd(W_n, Q) = \gcd(W_1, Q) \quad \text{for all } n \geq 1.$$

The following lemma will also be required:

(2.2) Lemma. *Assume that $W_n \neq 0$ for all $n \geq 1$ and $\gcd(W_1, Q) = 1$. If $t \geq 1$ then $\gcd(W_n, W_{n+t})$ divides $W_1 W_2 \cdots W_t$ for all $n \geq 1$.*

Proof. The lemma is trivial for $n = 1, \ldots, t$. Let $t \leq n$ and assume the lemma true for $1, 2, \ldots, n$. Let $d = \gcd(W_{n+1}, W_{n+1+t})$. We have $W_{n+1+t} = W_{n+1}V_t - Q^t W_{n+1-t}$. Since $1 \leq n+1-t \leq n$ and $\gcd(W_{n+1}, Q) = \gcd(W_1, Q) = 1$, it follows that d divides $\gcd(W_{n+1-t}, W_{n+1})$, which, by induction, divides $W_1 W_2 \cdots W_t$, thus concluding the proof. $\blacksquare$

We shall need the following theorem (see Shorey & Tijdeman [14], Shorey & Stewart [13], Pethő [9]), which we quote in the special case needed in this paper.

With the preceding notations:

(2.3) THEOREM. *Assume that* $\gcd(P,Q)=1$ *and* $D\neq 0$. *Let* $a,b\in\mathbb{Z}$, *not both equal to* 0, *let* $W_n = aU_n + bV_n$ *for all* $n\in\mathbb{Z}$. *Let* $A>0$ *be a square-free integer. Then there exists* $N>0$, *effectively computable in terms of* P,Q,a,b,A *such that if* $n\geq 0$ *and* $W_n = A\square$ (*where* $\square$ *denotes a non-zero integer which is a square*) *then* $n<N$.

The proof of this theorem involves inequalities of Baker for linear forms in logarithms and the constant N provided by the proof is usually very large.

For special sequences, the explicit determination of squares and double-squares has been achieved. We quote a few results for sequences with parameters P even and $Q=\pm 1$.

For $P=2$, $Q=1$, U_n and V_n are the Pell numbers and we have:

$$\{n \mid U_n = \square\} = \{1,7,-1,-7\}, \qquad \{n \mid U_n = 2\square\} = \{2\},$$
$$\{n \mid V_n = \square\} = \emptyset, \qquad \{n \mid V_n = 2\square\} = \{0,1\}.$$

The above results are due to Ljunggren [5]; the determination of the square Pell numbers required deep arguments.

Ljunggren [5] and Cohn [1] studied the sequences of numbers $U_n(4,-1)$ and $V_n(4,-1)$:

$$\{n \mid U_n(4,-1) = \square\} = \{1,2,-1\}, \qquad \{n \mid U_n(4,-1) = 2\square\} = \{4\},$$
$$\{n \mid V_n(4,-1) = \square\} = \{1\}, \qquad \{n \mid V_n(4,-1) = 2\square\} = \{0,2,-2\}.$$

In [1] Cohn obtained more results about squares and double squares in the sequences $U_n(2c,\pm 1)$ and $V_n(2c,\pm 1)$, for special values of $2c$.

The reader may obtain more information about recurring sequences in Ribenboim [11] (see Chapter 1 entitled "The Fibonacci Numbers and the Arctic Ocean"). More specifically about Pell numbers, see Ribenboim [10].

B. Pell equations. We keep the same notations: $F>1$, $f\neq 0$, $\varepsilon = c+d\sqrt{F}$ is the fundamental unit of $\mathbb{Z}[\sqrt{F}]$, so $c\geq 1$, $d\geq 1$; $P=2c$, $Q=c^2-Fd^2=\pm 1$, $U_n = U_n(2c,Q)$, $V_n=V_n(2c,Q)$. We consider solutions of $x^2-Fy^2=f$.

Two solutions (x,y) and (x',y') of the Pell equation are said to be *equivalent* if there exists $n\in\mathbb{Z}$ such that

$$Q^n = 1 \quad \text{and} \quad \frac{x+y\sqrt{F}}{x'+y'\sqrt{F}} = \varepsilon^n.$$

If $Q=1$ let $c'+d'\sqrt{F} = c+d\sqrt{F} = \varepsilon$. If $Q=-1$ let

$$c'+d'\sqrt{F} = (c^2+d^2F)+2cd\sqrt{F} = \varepsilon^2.$$

We note that if $c=1$ then $Q=-1$ so $c'>1$.

A solution (a,b) with $a\geq 0$ and $b\geq 0$ is called a *fundamental solution* if the following inequalities are satisfied:

$$0\leq a\leq\sqrt{\frac{(c'+\delta)|f|}{2}},\qquad 0\leq b\leq d'\sqrt{\frac{|f|}{2(c'+\delta)}}$$

where $\delta=|f|/f$.

Nagell proved (see [7] and [8]):

(2.4) THEOREM. *Every solution (x,y) with $x\geq 0$, $y\geq 0$ of $x^2-Fy^2=f$ is equivalent to a fundamental solution.*

3. Proof of Theorem (1.1). We divide the proof into three parts.

1°) Let S be the set of all (x,y,z,p) such that $x\geq 0$, $y\geq 0$, $z>0$, $p=1$ or p is a prime number and

$$\begin{cases} x^2-Fy^2=f,\\ x^2-pGz^2=fk_s.\end{cases}$$

[The proof when $g=fh_s$ is similar and will not be given.]

Let T be the set of all (x,y) such that $x\geq 0$, $y\geq 0$, $x^2-Fy^2=f$ and there exists (z,p) such that $(x,y,z,p)\in S$. Clearly, it suffices to show that the set T is effectively computable.

By the theorem of Nagell (2.4) if the equation $x^2-Fy^2=f$ has solutions, then it has a non-empty effectively computable set of fundamental solutions and every solution (x,y), with $x\geq 0$, $y\geq 0$ is given by a relation $x+y\sqrt{F}=(a+b\sqrt{F})\varepsilon^n$, where $a\geq 0$, $b\geq 0$, (a,b) is a fundamental solution, $\varepsilon=c+d\sqrt{F}$ is the fundamental unit of $\mathbb{Z}[\sqrt{F}]$, $Q=c^2-d^2F$, $Q^n=1$.

We fix an arbitrary fundamental solution (a,b) and write

$$x_n+y_n\sqrt{F}=(a+b\sqrt{F})\varepsilon^n=(a+b\sqrt{F})\left(\frac{V_n}{2}+dU_n\sqrt{F}\right),$$

where $U_n=U_n(2c,Q)$ and $V_n=V_n(2c,Q)$. So

$$x_n=a\,\frac{V_n}{2}+bdFU_n,\qquad y_n=adU_n+b\,\frac{V_n}{2}.$$

It suffices to show that the set $R=\{n>s\mid(x_n,y_n)\in T\}$ is effectively bounded.

2°) We show that if $Q^n=1$ and $s\geq 1$ then

$$x_n^2-Fy_{n-s}y_{n+s}=fk_s.$$

Indeed:

$$x_n^2=\left(a\,\frac{V_n}{2}+bdFU_n\right)^2=a^2\,\frac{V_n^2}{4}+\frac{b^2F}{4}\,DU_n^2+abdFU_{2n}.$$

Next

$$\begin{aligned} Fy_{n-s}y_{n+s} &= F\left(b\,\frac{V_{n-s}}{2} + adU_{n-s}\right)\left(b\,\frac{V_{n+s}}{2} + adU_{n+s}\right) \\ &= \frac{b^2F}{4}\,V_{n-s}V_{n+s} + abdFU_{2n} + a^2d^2FU_{n-s}U_{n+s} \\ &= \frac{b^2F}{4}\,V_{n-s}V_{n+s} + abdFU_{2n} + \frac{a^2}{4}\,DU_{n-s}U_{n+s}. \end{aligned}$$

In the above calculation we used identities indicated in Section 2. It follows that

$$x_n^2 - Fy_{n-s}y_{n+s} = \frac{a^2}{4}\,(V_n^2 - DU_{n-s}U_{n+s}) + \frac{b^2F}{4}\,(DU_n^2 - V_{n-s}V_{n+s}).$$

By Lemma (2.1) we have

$$\begin{aligned} x_n^2 - Fy_{n-s}y_{n+s} &= \frac{a^2}{4}\,Q^{ns}(2Q^s + V_{2s}) - \frac{b^2F}{4}\,Q^{ns}(2Q^s + V_{2s}) \\ &= \frac{f}{4}\,(2 + Q^sV_{2s}) = fk_s, \end{aligned}$$

because $Q^n = 1$. [For the proof of the theorem when $g = fh_s$ we need the relation

$$x_n^2 - x_{n-s}x_{n+s} = fh_s,$$

which is established in a similar way.]

3°) By Lemma (2.2) for every $n > s$, $\gcd(y_{n-s}, y_{n+s})$ divides $y_1y_2\cdots y_{2s}$; we note that the integer $y_1y_2\cdots y_{2s}$ is effectively computable in terms of F, s and the chosen fundamental solution (a,b). For every positive integer e dividing $y_1y_2\cdots y_{2s}$, let

$$R_e = \{n \in R \mid \gcd(y_{n-s}, y_{n+s}) = e\}.$$

Let $n \in R_e$, so from $Fy_{n-s}y_{n+s} = pGz^2$ it follows that

$$e^2F^2\,\frac{y_{n-s}}{e}\cdot\frac{y_{n+s}}{e} = pFG\square = p^\delta H\square,$$

where $\delta = 0$ or 1, $p\nmid H$, H is square-free and $H \mid FG$. Hence

$$\frac{y_{n-s}}{e}\cdot\frac{y_{n+s}}{e} = p^\delta H\square.$$

Let $\mathcal{H} = \{(H', H'') \mid H'H''$ is square-free, $H'H'' \mid FG$ and $\gcd(H', H'') = 1\}$, so $\mathcal{H}$ is effectively computable. For each (H', H'') let $R'_{e,(H',H'')}$ be the set of all $n \in R_e$ such that

$$\frac{y_{n-s}}{e} = p^\delta H'\square, \qquad \frac{y_{n+s}}{e} = H''\square.$$

By Theorem (2.3) the set $\{n+s \mid n \in R'_{e,(H',H'')}\}$, hence also $R'_{e,(H',H'')}$ is effectively bounded. Similarly, let $R''_{e,(H',H'')}$ be the set of $n \in R_e$ such that

$$\frac{y_{n-s}}{e} = H'\square, \qquad \frac{y_{n+s}}{e} = p^\delta H''\square.$$

Then again $R''_{e,(H',H'')}$ is effectively bounded. Since this holds for each $(H',H'') \in \mathcal{H}$ the set R_e is effectively bounded for each $e \mid y_1 y_2 \cdots y_{2s}$. Hence R is effectively bounded and this concludes the proof of the theorem. ∎

4. A numerical example. We give an example where our method is applied with success to determine explicitly all solutions. To begin we prove a lemma.

(4.1) Lemma. *Let U_n, V_n be the Pell numbers for all $n \in \mathbb{Z}$. Then*

(a) $\{n \neq 0 \mid U_n = 3\square\} = \{4\}$, $\{n \mid V_n = 3\square\} = \emptyset$,
(b) $\{n \neq 0 \mid U_n = 6\square\} = \emptyset$.

Proof. (a) Let $U_n = 3\square$. By considering the sequence U_n modulo 3 we see that 4 divides n. Let $n = 4h$, so $U_n = U_{2h}V_{2h}$, with $\gcd(U_{2h}, V_{2h}) = 2$. Then either $V_{2h} = 2\square$ or $U_{2h} = 2\square$. So $h = 1$ and $n = 4$. If $V_n = 3\square$, since $2 \mid V_n$ but $4 \nmid V_n$ this is impossible.

(b) If $U_n = 6\square$ then $n = 4h$ and we have the following cases:

$$\begin{array}{l|c|c|l} U_{2h} = 3\square & 6\square & \square & 2\square, \\ V_{2h} = 2\square & \square & 6\square & 3\square. \end{array}$$

From (a) and the knowledge of m such that $U_m = \square, 2\square$, $V_m = \square, 2\square$ we conclude that $n = 0$. ∎

(4.2) Example. *If p is a prime, or $p = 1$, if x, y, z are positive integers and*

$$\begin{cases} x^2 - 2y^2 = 1, \\ x^2 - pz^2 = 9, \end{cases}$$

then $(x, y, z, p) = (99, 70, 24, 17)$.

Proof. $\varepsilon = 1 + \sqrt{2}$ is the fundamental unit of $\mathbb{Z}[\sqrt{2}]$, let $P = 2$, $Q = -1$, U_n, V_n are the Pell numbers, $k_2 = 9$, so the method is applicable. $\varepsilon^2 = 3 + 2\sqrt{2}$ is the fundamental solution of the first equation, $x_n + y_n\sqrt{2} = \varepsilon^{n+2} = V_{n+2}/2 + U_{n+2}\sqrt{2}$ and we work with n even since $Q = -1$. We have:

$$2y_{n-2}y_{n+2} = pz^2 \neq 0,$$

that is, $U_n U_{n+4} = 2p\square \neq 0$.

(a) $p = 2$, so $U_n U_{n+4} = \square$. If $n \equiv 2 \pmod 4$ then $\gcd(U_n, U_{n+4}) = U_2 = 2$, so $U_n = 2\square$ and $U_{n+4} = 2\square$, which is impossible. If $n \equiv 0 \pmod 4$ then $\gcd(U_n, U_{n+4}) = U_4 = 12$, hence $U_n = 3\square$, $U_{n+4} = 3\square$, which is impossible by (4.1).

(b) $p \neq 2$, so $U_n U_{n+4} = 2p\Box$. If $n \equiv 2 \pmod 4$ then $U_n \equiv U_{n+4} \equiv 2 \pmod 4$ so the 2-adic value of $U_n U_{n+4}$ is even, hence $U_n U_{n+4} = 2p\Box$ is impossible.

Now let $n \equiv 0 \pmod 4$, so the following cases are possible:

$$\begin{array}{r|c|c|c} U_n = \ 3\Box & 6\Box & 3p\Box & 6p\Box, \\ U_{n+4} = 6p\Box & 3p\Box & 6\Box & 3\Box. \\ (1) & (2) & (3) & (4) \end{array}$$

(1) It was seen that $n = 4$, hence $U_8 = 408 = 6 \times 17\Box$ so $p = 17$, $x_4 = V_6/2 = 99$, $y_4 = U_6 = 70$ and this gives the solution $(x, y, z, p) = (99, 70, 24, 17)$.

(4) $n + 4 = 4$, $n = 0$ which is impossible, since then $z = 0$.

(2) and (3) are impossible as it was shown in Lemma (4.1). ∎

As an exercise the reader may wish to show that if x, y, z are positive, if p is a prime number and if

$$\begin{cases} x^2 - 3y^2 = -3, \\ x^2 - 3pz^2 = -12, \end{cases}$$

then $(x, y, z, p) = (3, 2, 1, 7)$.

The reader may employ the same method to prove the original result of Ljunggren mentioned in the Introduction: if x, y are positive integers, if p is a prime number and $x^4 - py^2 = 1$ then $(x, y, p) = (3, 4, 5)$ or $(99, 1820, 29)$.

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