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Corrections to "Irreducibility of the iterates of a quadratic polynomial over a field"

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by

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We use the notations and terminology of Sections 1, 2 and 3 of the paper.

The converse part of Theorem 6 is already false when $A = \mathbb{Z}$ while the other half of the theorem is true in a more general setting.

Example. Let $f(x) = x^2 + 2x - 2$ in $\mathbb{Z}[x]$. Then $d = 12$ and hence $f(x)$ is stable over $\mathbb{Q}$ by Theorem 3. However, $f(-d/4) = 1$, a square.

We prove

Theorem A. *Let K be a field whose characteristic is different from 2. Let $f(x) = x^2 - lx + m$ be irreducible in $K[x]$. Then $f(x)$ is stable over K if $f_n(-d/4)$ is not a square in K for every $n \geq 1$.*

Proof. It is easy to see by induction on n that $f_n(x) = g_{n-1}(x-l_0)^2 - d_0$. Indeed, the case $n = 1$ is trivial and assuming the equality holds for n, we conclude that

$$\begin{aligned} f_{n+1}(x) &= f_n(f(x)) = g_{n-1}(f(x) - l_0)^2 - d_0 \\ &= g_{n-1}(g(x - l_0))^2 - d_0 \\ &= g_n(x - l_0)^2 - d_0. \end{aligned}$$

Hence, $f_n(-d/4) = g_{n-1}(\delta_0)^2 - l_0 = g_{n-1}^2 - d_0$ and the result follows from Theorem 1.

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