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AN APPLICATION OF THE SPECIAL REISSNER–NORDSTRÖM SPACE-TIME TO DESCRIPTION OF THE UNIVERSE IN THE NEIGHBOURHOOD OF THE PLANCK ERA

Abstract. The description of the special Reissner–Nordström space-time in the Planck era is presented and its comparison with the Robertson–Walker space-time is given.

1. Introduction. The aim of this paper is to present the conjectural description of Universe by means of a special 4-dimensional Reissner–Nordström (briefly R-N) space-time and to compare it with the description given by the 4-dimensional Robertson–Walker (R-W) space-time. Let us recall that according to one of the cosmological conjectures the history of Universe began with Big Bang [6]. We will start our description at time $t_1 = 10^{-44}$ sec, i.e. at the end of the Planck era, and conclude it at time $t_2 = 10^{-34}$ sec [6].

Definition 1. The *generalized* 4*-dimensional R-N space-time* is defined by the metric tensor

$$\operatorname{diag}(-E^{a+1}, E^{a-1}, r^2, r^2 \sin^2\theta), \tag{1}$$

where

$$E = 1 - \frac{r_0}{r} + \frac{Kr_0^2}{r^2}, \qquad a \in [0,1],$$
$$K \in \left(\frac{1}{4}, \frac{9}{32}\right), \qquad r_0 = \text{const} > 0,$$
$$(x^1, x^2, x^3, x^4) = (t, r, \theta, \varphi).$$

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For $a = 1$ the metric tensor (1) has the form

$$\operatorname{diag}(-E^2, 1, r^2, r^2 \sin^2\theta). \tag{2}$$

DEFINITION 2. The *special R-N space-time* is one with metric tensor (2) and parameter $K = (1+\varepsilon)/4$, where $\varepsilon > 0$ is close to zero.

Diagram (9) of [4] shows the graph of scalar curvatures $T = T(r)$ in the model of 4-dimensional generalized R-N space-time in dependance on the radius r of the Universe. We will determine the radii r' and r'' for the scalar curvatures T_1, T_2 corresponding to t_1 and t_2. In [4] the scalar curvature T_ε of the special R-N space-time ($a = 1$) was determined in the form

$$T_\varepsilon = \frac{4(1+\varepsilon)r_0^2}{r^2\left[4\left(r - \frac{r_0}{2}\right)^2 + \varepsilon r_0^2\right]} \tag{3}$$

or equivalently

$$r^2 - \frac{r_0}{2}r - \frac{r_0}{\sqrt{T_\varepsilon}} = 0. \tag{4}$$

The solutions of (4) are

$$\widehat{r}_1 = \frac{r_0}{4} - \frac{1}{2}\sqrt{\frac{r_0^2}{4} + \frac{4r_0}{\sqrt{T_\varepsilon}}}, \qquad \widehat{r}_2 = \frac{r_0}{4} + \frac{1}{2}\sqrt{\frac{r_0^2}{4} + \frac{4r_0}{\sqrt{T_\varepsilon}}}. \tag{5}$$

It follows from (3) that

$$\lim_{r\to 0^+} T = +\infty, \qquad \lim_{\substack{r\to r_0/2\\ \varepsilon\to 0^+}} T = +\infty,$$

so we can assume, for ε close to zero, that $4r_0/\sqrt{T_\varepsilon} = 0$.

The radius r is close to zero so the approximation $\sqrt{x} \approx \frac{1}{2}x$ allows one to obtain the following values:

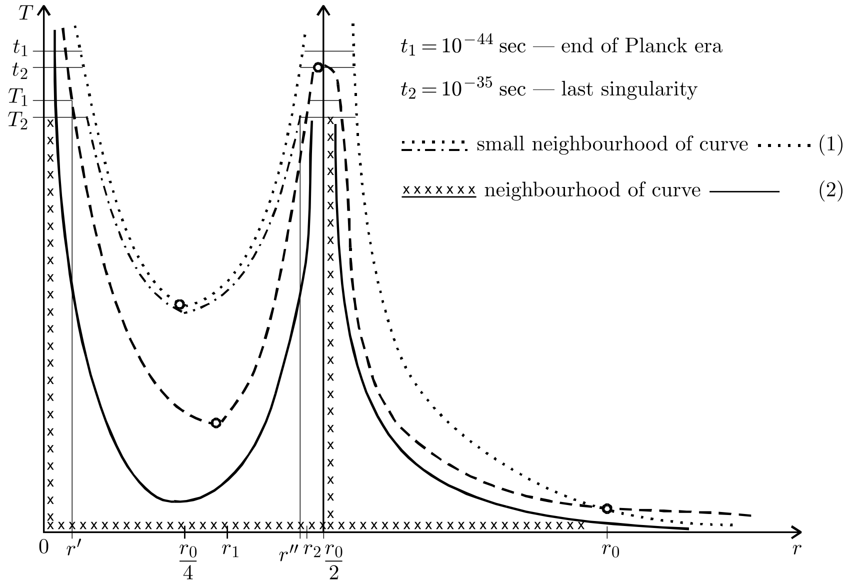
$$r' = \frac{r_0}{4} - \frac{r_0^2}{16}, \qquad r'' = \frac{r_0}{4} + \frac{r_0^2}{16}. \tag{6}$$

The above data are illustrated in diagram (7) below.

We can summarize this in the following theorem.

THEOREM 1. *In the special* 4*-dimensional R-N space-time with metric tensor* (2) *there exist two Big Bangs for* $r = 0$ *and* $r = r_0/2$ *given in diagram* (7).

(7)



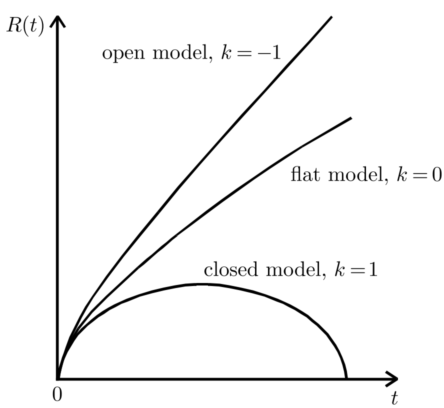
DEFINITION 3. The 4-*dimensional Robertson–Walker* (R-W) *space-time* is defined by the following metric tensor ([2, p. 190]):

(8)
$$\operatorname{diag}\left(-1, \frac{R^2}{1-kr^2}, R^2r^2, R^2r^2\sin^2\theta\right),$$

where $R = R(t)$ denotes the dimension free scale factor and $k \in [-1, 1]$ is a parameter related with space curvature.

There are three Friedmann models of this space-time [2] described as follows:

(9)



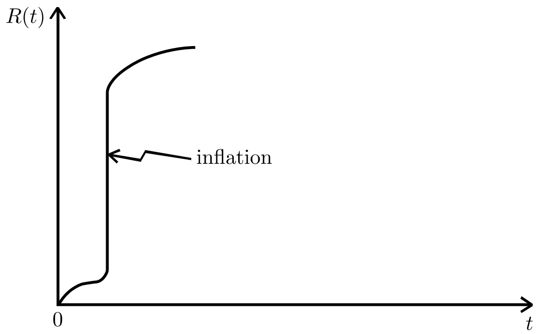
We are interested in the flat Friedmann model ($k = 0$). In this model the scale factor $R(t)$ has the form [2]

(10) $$R(t) = \left(\frac{3A}{2}\right)^{2/3} t^{2/3}, \quad A > 0, \quad A = 8\pi G\varrho_0 R_0^3/3$$

and inflation occurs.

The inflation model can be described by the diagram [1]

(11)



In the inflation model, immediately after Big Bang there occurs a rapid and impetuous growth of the Universe. The inflation process ends at time $t_2 = 10^{-34}$ sec [6].

THEOREM 2. *In the flat model of the R-W space-time with metric tensor* (8) *there occurs a Big Bang for* $r = 0$ *and inflation for* $r = r_0/2$.

COROLLARY 1. *In the flat model of the R-W space-time inflation occurs exactly at the moment when the second Big Bang occurs in the special R-N space-time.*

COROLLARY 2. *The graph of the scalar curvature of the special R-N space-time is the curve* (1) *in diagram* (7).

Observe that in the generalized R-N space-time the graph of the scalar curvature tends for $a \to 0^+$ to the limit curvature $T = 0$ for $r \neq r_0/2$ and to $T = +\infty$ for $r = r_0/2$ (see (2) in diagram (7)).

In [3, p. 223, (2.14)] we have presented a formula for the Weyl curvature. Now we can give the following complement.

COROLLARY 3. *The neighbourhood of a Big Bang or the neighbourhood of a Black Hole has for* $0 < a < 1$ *the structure of a generalized R-N space-time* (see (2) in diagram (7)).

REMARK. It follows from the above corollaries that in the model of the Universe described by the flat R-W space-time, inflation occurs at time t_2 while in the model described by the special R-N space-time the second Big Bang occurs.

It is a problem of experimental cosmology to investigate which of these conjectures is valid [5].

References

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