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LOCAL ANALYSIS OF A CUBICALLY CONVERGENT METHOD FOR VARIATIONAL INCLUSIONS

Abstract. This paper deals with variational inclusions of the form $0 \in \varphi(x) + F(x)$ where φ is a single-valued function admitting a second order Fréchet derivative and F is a set-valued map from $\mathbb{R}^q$ to the closed subsets of $\mathbb{R}^q$. When a solution $\bar{z}$ of the previous inclusion satisfies some semistability properties, we obtain local superquadratic or cubic convergent sequences.

1. Introduction. The inspiration for this study goes back to the work of Bonnans [3] concerning variational inequalities. Let φ be a twice continuously Fréchet differentiable function from $\mathbb{R}^q$ into $\mathbb{R}^q$. Given a closed convex subset K of $\mathbb{R}^q$, we consider the variational inequality

$$\langle \varphi(z), y - z \rangle \geq 0, \quad \forall y \in K,\ z \in K. \tag{1}$$

We may define the (closed convex) cone of outward normals to K at a point $z \in K$,

$$N(z) := \{x \in \mathbb{R}^q;\ \langle x, y - z \rangle \leq 0,\ \forall y \in K\},$$

and if $z \notin K, N(z) := \emptyset$.

It is easy to observe that (1) is equivalent to the relation

$$0 \in \varphi(z) + N(z).$$

In the rest of the paper, we consider more general inclusions of the form

$$0 \in \varphi(z) + F(z), \tag{2}$$

where F is a multifunction from $\mathbb{R}^q$ to the closed subsets of $\mathbb{R}^q$.

One can notice that when $F = \{0\}$ we recover the equation $\varphi(z) = 0$ and if F is the positive orthant of $\mathbb{R}^q$ we get a system of inequalities.

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Inclusions such as (2), known more specifically as *generalized equations*, were introduced by Robinson in the 1970's and serve as a general tool for describing and solving different problems in a unified manner. Most of the algorithms for solving (2) generate a sequence (x_n) of iterates obtained by subsequently solving implicit subproblems of the form $0 \in A(x_n, x_{n+1}) + F(x_{n+1})$ where A denotes some approximation of the mapping φ.

When the function φ is such that φ' is locally Lipschitz, Dontchev [4, 5] associated to (2) a Newton-type method based on a partial linearization which is locally quadratically convergent. Using a second-degree Taylor polynomial expansion of φ at x_k, Geoffroy et al. [8, 9] considered the iteration

$$0 \in \varphi(x_k) + \varphi'(x_k)(x_{k+1} - x_k) + \tfrac{1}{2}\varphi''(x_k)(x_{k+1} - x_k)^2 + F(x_{k+1}), \tag{3}$$

where $\varphi'(x)$ and $\varphi''(x)$ denote respectively the first and the second Fréchet derivative of φ at x. They proved the cubic convergence of this iterative procedure whenever φ' and φ'' are Lipschitz continuous and the set-valued map $\varphi + F$ is metrically regular. Moreover, they showed that the method (3) enjoys nice stability properties.

The method introduced in the present paper can be described as follows. We start with a point $z_0 \in \mathbb{R}^n$ and if the current point z_k is not a solution of the variational inclusion (2), we obtain z_{k+1} by solving

$$0 \in \varphi(z_k) + \varphi'(z_k)(z_{k+1} - z_k) + M_k(z_{k+1} - z_k)^2 + F(z_{k+1}) \tag{4}$$

where M_k is a $q \times q$ matrix. When $M_k = \frac{1}{2}\varphi''(z_k)$ we recover the method introduced by Geoffroy et al.

Our purpose is to study the local behavior of the iterative method (4) when the solution $\bar{z}$ enjoys some stability properties.

The rest of this paper is organized as follows. In Section 2, we give some notation and collect some definitions regarding semistability of solutions and regularity for set-valued maps. Section 3 is devoted to convergence results for the sequence defined by (4). It is proved that we can obtain cubic convergence when the matrix M_k or the function φ'' has some properties. The final section is concerned with some relations between semistability and regularity. Moreover, it is proved that semistability of a solution implies local superquadratic or cubic convergence of a sequence defined by (4).

2. Notation and preliminaries. Let us recall some notation. We define the graph of a set-valued map $\Gamma : X \rightrightarrows Y$ by

$$\operatorname{gph} \Gamma := \{(x, y) \in X \times Y;\ y \in \Gamma(x)\}$$

and its inverse at a point y by

$$\Gamma^{-1}(y) := \{x \in X;\ y \in \Gamma(x)\}.$$

$\mathbb{B}(x,r)$ is the closed ball with center x and radius r, and $\mathbb{B}$ stands for the closed unit ball. Moreover, we set $\mathbb{B}^\star(x,r) := \mathbb{B}(x,r) \setminus \{x\}$.

Norms in Banach spaces are denoted by $\|\cdot\|$.

We also recall the following definition concerning rates of convergence.

Definition 1. Let (z_n) be a sequence which converges towards $\bar{z}$ in a normed space. If $K_p := \lim \|z_{n+1} - \bar{z}\|/\|z_n - \bar{z}\|^p$ exists and $K_p > 0$, then (z_n) is said to be *convergent of order* p towards $\bar{z}$.

- When $p = 1$, (z_n) is said to be *linearly convergent.*
- When $p = 2$, (z_n) is said to be *quadratically convergent.*
- When $p = 3$, (z_n) is said to be *cubically convergent.*

If $K_1 = 0$, then (z_n) is said to be *superlinearly convergent*, and if $K_2 = 0$, then (z_n) is said to be *superquadratically convergent.*

We will deal with the concept of semistability, introduced by J.-F. Bonnans [3].

Definition 2. A solution $\bar{z}$ of (2) is said to be *semistable* if there exist $c_1, c_2 > 0$ such that, for all $(z,\delta) \in \mathbb{R}^q \times \mathbb{R}^q$, if

$$\delta \in \varphi(z) + F(z)$$

and $\|z - \bar{z}\| \le c_1$, then $\|z - \bar{z}\| \le c_2\|\delta\|$.

Note that a sufficient condition for semistability is the strong regularity of Robinson [15]. Recently, A. F. Izmailov and M. V. Solodov [10] used the concept of semistability to study the convergence of the Inexact Josephy Newton Method for solving generalized equations.

For more details on this subject, the reader is referred to [3].

We will also need some notions related to set-valued maps.

Definition 3. Let (X,d) and (Y,ρ) be metric spaces. Let $F : X \rightrightarrows Y$ be a set-valued map and let $(\bar{x},\bar{y}) \in X \times Y$. Then F is said to be *metrically regular* at $\bar{x}$ for $\bar{y}$ with modulus $k \ge 0$ if $(\bar{x},\bar{y}) \in \operatorname{gph} F$ and there are neighborhoods U of $\bar{x}$ and V of $\bar{y}$ such that

$$d(x, F^{-1}(y)) \le k\rho(y, F(x)), \qquad \forall (x,y) \in U \times V. \tag{5}$$

The infimum of $k \ge 0$ over all (k,U,V) for which (5) is satisfied is called the *exact regularity bound* of F around $(\bar{x},\bar{y})$ and is denoted by $\operatorname{reg} F(\bar{x},\bar{y})$.

Definition 4. Let X and Y be Banach spaces. A map $S : Y \rightrightarrows X$ is said to be *pseudo-Lipschitz* (or to be *Lipschitz-like* or to have the *Aubin property*) with modulus $k \ge 0$ at $\bar{y} \in Y$ for $\bar{x} \in X$ if $(\bar{y},\bar{x}) \in \operatorname{gph} S$ and there are neighborhoods U of $\bar{x}$ and V of $\bar{y}$ such that

$$S(y') \cap U \subset S(y) + k\|y' - y\|\mathbb{B}, \qquad \forall y', y \in V. \tag{6}$$

The infimum of $k \geq 0$ over all (k, U, V) for which (6) holds is called the *exact Lipschitzian bound* of S around $(\bar{y}, \bar{x})$ and is denoted by $\operatorname{lip} S(\bar{y}, \bar{x})$.

DEFINITION 5. Let (X, d) be a metric space and let A and B be two subsets of X. The *excess* from A to B is defined by

$$e(A, B) = \sup_{x \in A} d(x, B),$$

where it is understood that $e(\emptyset, B) = 0$ for $B \neq \emptyset$ and $e(\emptyset, B) = +\infty$ if $B = \emptyset$.

We obtain an equivalent definition of a pseudo-Lipschitz map by replacing (6) in Definition 4 by

$$e(S(y') \cap U, S(y)) \leq k\|y' - y\|, \qquad \forall y', y \in V. \tag{7}$$

The pseudo-Lipschitz property has been introduced by J.-P. Aubin and he was the first to define this concept as a continuity property. Sometimes this property is called "Aubin continuity". Characterizations of the pseudo-Lipschitz property have also been obtained by Rockafellar [17, 16] using the Lipschitz continuity of the distance function $\operatorname{dist}(y, S(x))$ around (x_0, y_0) and by Mordukhovich [11, 12] using the concept of coderivative of a multifunction. Dontchev, Quincampoix and Zlateva [7] gave a derivative criterion of metric regularity of set-valued mappings based on the work of Aubin and co-authors. Relationships between metric regularity and pseudo-Lipschitz property can be found in [13].

PROPOSITION 1. *Let $S : \mathbb{R}^n \rightrightarrows \mathbb{R}^m$ be a set-valued map, and let $(\bar{x}, \bar{y}) \in \operatorname{gph} S$. Then S is metrically regular around $(\bar{x}, \bar{y})$ if and only if its inverse $S^{-1} : \mathbb{R}^m \rightrightarrows \mathbb{R}^n$ is pseudo-Lipschitz around $(\bar{y}, \bar{x})$. Furthermore,*

$$\operatorname{reg} S(\bar{x}, \bar{y}) = \operatorname{lip} S^{-1}(\bar{y}, \bar{x}).$$

For more details and applications of this property, the reader is referred to [2, 1, 6].

The following two definitions will be useful. More details can be found in [7].

DEFINITION 6. A single-valued function f from a normed linear space X into a normed linear space Y is *strictly differentiable* at $x_0 \in X$ if there exists a continuous linear operator from X to Y, denoted $f'(x_0)$, with the property that for every $\varepsilon > 0$ there exists $\delta > 0$ such that

$$\|f(x_1) - f(x_2) - f'(x_0)(x_1 - x_2)\| \leq \varepsilon \|x_1 - x_2\|$$

whenever $\|x_i - x_0\| \leq \delta$, $i = 1, 2$.

A strictly differentiable function is obviously Fréchet differentiable, but the converse is not true. One has however the equivalence of "strict differentiability" and being of "differentiability class C^1". For more information on this topic, the reader is referred to [14].

DEFINITION 7. A single-valued function f from a metric space (X,ρ) into a metric space (Y,d) is *strictly stationary* at $x_0 \in X$ if for every $\varepsilon > 0$ there exists $\delta > 0$ such that

$$d(f(x_1), f(x_2)) \le \varepsilon\rho(x_1, x_2)$$

whenever $\rho(x_i, x_0) \le \delta$, $i = 1, 2$.

3. Convergence properties

THEOREM 2. *Let $\bar z$ be a semistable solution of* (2), *and let* (z_k) *generated by* (4) *converge toward $\bar z$. Then:*

(i) *If* $\left(\frac{1}{2}\varphi''(\bar z) - M_k\right)(z_{k+1} - z_k)^2 = o(\|z_{k+1} - z_k\|^2)$, *then* (z_k) *converges superquadratically.*

(ii) *If* $\left(\frac{1}{2}\varphi''(\bar z) - M_k\right)(z_{k+1} - z_k)^2 = O(\|z_{k+1} - z_k\|^3)$ *and φ'' is locally Lipschitz, then* (z_k) *converges cubically.*

Proof. We write (4) as

$$r_k := \Theta_k + \Phi(z_{k+1}, z_k, \bar z) \in \varphi(z_{k+1}) + F(z_{k+1}), \tag{8}$$

where

$$\Theta_k := \left(\tfrac{1}{2}\varphi''(\bar z) - M_k\right)(z_{k+1} - z_k)^2,$$
$$\Phi(z_{k+1}, z_k, \bar z) := \varphi(z_{k+1}) - \varphi(z_k) - \varphi'(z_k)(z_{k+1} - z_k) - \tfrac{1}{2}\varphi''(\bar z)(z_{k+1} - z_k)^2.$$

We have

$$\varphi(z_{k+1}) = \varphi(z_k) + \varphi'(z_k)(z_{k+1} - z_k) + \tfrac{1}{2}\varphi''(z_k)(z_{k+1} - z_k)^2 + o((z_{k+1} - z_k)^2).$$

Hence

$$\Phi(z_{k+1}, z_k, \bar z) = \tfrac{1}{2}(\varphi''(z_k) - \varphi''(\bar z))(z_{k+1} - z_k)^2 + o((z_{k+1} - z_k)^2).$$

Since φ'' is continuous and $z_k \to \bar z$, we get

$$\Phi(z_{k+1}, z_k, \bar z) = o((z_{k+1} - z_k)^2).$$

Thus

$$\|r_k\| = o(\|z_{k+1} - z_k\|^2).$$

From the semistability of $\bar z$ we get

$$\|z_{k+1} - \bar z\| = O(\|r_k\|).$$

Consequently,

$$\|z_{k+1} - \bar z\| = o(\|z_{k+1} - z_k\|^2) = o(\|z_{k+1} - \bar z\|^2 + \|z_{k+1} - \bar z\|\,\|z_k - \bar z\| + \|z_k - \bar z\|^2),$$

i.e.

$$0 = \lim \frac{\|z_{k+1} - \bar{z}\|}{\|z_{k+1} - \bar{z}\|^2 + \|z_{k+1} - \bar{z}\|\,\|z_k - \bar{z}\| + \|z_k - \bar{z}\|^2} = \lim \frac{1}{\|z_{k+1} - \bar{z}\| + \|z_k - \bar{z}\| + \frac{\|z_k - \bar{z}\|^2}{\|z_{k+1} - \bar{z}\|}}.$$

Since $z_k \to \bar{z}$, the last relation implies that

$$\frac{\|z_k - \bar{z}\|^2}{\|z_{k+1} - \bar{z}\|} \to \infty \quad \text{as } k \to \infty,$$

i.e.

$$\|z_{k+1} - \bar{z}\| = o(\|z_k - \bar{z}\|^2).$$

This proves (i).

We have

$$\begin{aligned}\|\Phi(z_{k+1}, z_k, \bar{z})\| &= \|\Phi(z_{k+1}, z_k, z_k) + \Phi(z_{k+1}, z_k, \bar{z}) - \Phi(z_{k+1}, z_k, z_k)\| \\ &\le \|\Phi(z_{k+1}, z_k, z_k)\| + \|\Phi(z_{k+1}, z_k, \bar{z}) - \Phi(z_{k+1}, z_k, z_k)\| \\ &\le \|\Phi(z_{k+1}, z_k, z_k)\| + \tfrac{1}{2}\|\varphi''(z_k) - \varphi''(\bar{z})\|\,\|z_{k+1} - z_k\|^2.\end{aligned}$$

Since φ'' is locally Lipschitz, we let l be its constant and get

$$\|\Phi(z_{k+1}, z_k, \bar{z})\| \le \frac{l}{6}\|z_{k+1} - z_k\|^3 + \frac{l}{2}\|z_k - \bar{z}\|\,\|z_{k+1} - z_k\|^2.$$

Since (z_k) converges superquadratically, we obtain $\|z_{k+1} - z_k\|/\|z_k - \bar{z}\| \to 1$ and this implies that

$$\|\Phi(z_{k+1}, z_k, \bar{z})\| = O(\|z_{k+1} - z_k\|^3).$$

Hence

$$\|z_{k+1} - \bar{z}\| = O(\|r_k\|) = O(\|z_{k+1} - z_k\|^3) = O(\|z_k - \bar{z}\|^3). \ \blacksquare$$

COROLLARY 3. *Let $\bar{z}$ be a semistable solution of* (2), *and let* (z_k) *generated by* (4) *converge to* $\bar{z}$.

(i) *If $M_k \to \frac{1}{2}\varphi''(\bar{z})$ then (z_k) converges superquadratically.*
(ii) *If φ'' is locally Lipschitz and $M_k = \frac{1}{2}\varphi''(\bar{z}) + O(\|z_k - \bar{z}\|^2)$, then (z_k) converges cubically.*

4. Semistability and regularity

PROPOSITION 4. *If $\bar{z}$ is a semistable solution of* (2) *and* $\mathrm{int}(\varphi(\bar{z}) + F(\bar{z})) \neq \emptyset$ *then the set-valued map $\Gamma : \mathbb{R}^q \rightrightarrows \mathbb{R}^q$ defined by $\Gamma(z) := \varphi(z) + F(z)$ is metrically regular around $(\bar{z}, 0)$.*

Proof. Since $\bar{z}$ is a semistable solution of (2), we have

$$\forall z \in \mathbb{B}^\star(\bar{z}, c_1),\ \forall \delta \in \Gamma(z), \quad d(z, \bar{z}) < c\,d(0, \delta), \quad \text{with } c > c_2,$$

and thus there exists $\varepsilon_1 > 0$ such that $d(z,\bar{z}) + c\varepsilon_1 \le cd(0,\delta)$. Hence $d(z,\bar{z}) \le c(d(0,\delta) - \varepsilon_1)$, which implies $d(z,\bar{z}) \le cd(y,\delta)$ for all $y \in \mathbb{B}(0,\varepsilon_1)$.

Let $\varepsilon_2 > 0$ be such that $\mathbb{B}(0,\varepsilon_2) \subset \Gamma(\bar{z})$ and let $y \in \mathbb{B}(0,\varepsilon)$, where $\varepsilon = \min(\varepsilon_1,\varepsilon_2)$. As $\bar{z} \in \Gamma^{-1}(y)$, we have $d(z,\Gamma^{-1}(y)) \le cd(y,\delta)$. We finally get

$$\forall z \in \mathbb{B}(\bar{z},c_1),\ \forall \delta \in \Gamma(z),\ \forall y \in \mathbb{B}(0,\varepsilon), \qquad d(z,\Gamma^{-1}(y)) \le cd(y,\delta),$$

i.e.

$$\forall z \in \mathbb{B}^\star(\bar{z},c_1),\ \forall y \in \mathbb{B}(0,\varepsilon), \qquad d(z,\Gamma^{-1}(y)) \le cd(y,\Gamma(z)).$$

If $z = \bar{z}$, the inequality still holds since $d(\bar{z},\Gamma^{-1}(y)) = 0$ for all $y \in \mathbb{B}(0,\varepsilon)$. Thus,

$$\forall z \in \mathbb{B}(\bar{z},c_1),\ \forall y \in \mathbb{B}(0,\varepsilon), \qquad d(z,\Gamma^{-1}(y)) \le cd(y,\Gamma(z)). \ ■$$

THEOREM 5 (Cubic convergence). *Let $\bar{z}$ be a semistable solution of* (2) *such that* $\operatorname{int}(\varphi(\bar{z}) + F(\bar{z})) \ne \emptyset$. *If F has a closed graph and φ'' is Lipschitz with constant L on $\mathbb{B}(\bar{z},c_1)$, then there exists $M > 0$ such that for all $C > ML/6$, one can find $\eta > 0$ such that for all initial points $x_0 \in B_\eta(\bar{z})$, there exists a sequence (z_k) generated by*

$$(9) \qquad 0 \in \varphi(z_k) + \varphi'(z_k)(z_{k+1} - z_k) + \tfrac{1}{2}\varphi''(z_k)(z_{k+1} - z_k)^2 + F(z_{k+1})$$

which satisfies

$$(10) \qquad \|z_{k+1} - \bar{z}\| \le C\|z_k - \bar{z}\|^3.$$

Proof. By Proposition 4, the map

$$(\varphi(\cdot) + F(\cdot))^{-1}$$

is pseudo-Lipschitz around $(0,\bar{z})$; denote by M its modulus. Let us consider the map f defined by

$$f(z) = \varphi(z) - \varphi(\bar{z}) - \varphi'(\bar{z})(z - \bar{z}) - \tfrac{1}{2}\varphi''(\bar{z})(z - \bar{z})^2.$$

Let $\varepsilon > 0$. As φ is of differentiability class C^1 and hence strictly differentiable, there exists $\delta > 0$ such that

$$\|\varphi(z_1) - \varphi(z_2) - \varphi'(\bar{z})(z_1 - z_2)\| \le \tfrac{1}{2}\varepsilon\|z_1 - z_2\|$$

whenever $\|z_i - \bar{z}\| \le \delta$, $i = 1,2$. Then, for all z_1, z_2 such that $\|z_i - \bar{z}\| \le \delta$, $i = 1,2$, we have

$$\begin{aligned}
&\|f(z_1) - f(z_2)\| \\
&\quad = \big\|\varphi(z_1) - \varphi(z_2) - \varphi'(\bar{z})(z_1 - z_2) + \tfrac{1}{2}(\varphi''(\bar{z})(z_2 - \bar{z})^2 - \varphi''(\bar{z})(z_1 - \bar{z})^2)\big\| \\
&\quad \le \tfrac{1}{2}\varepsilon\|z_1 - z_2\| + \tfrac{1}{2}\|\varphi''(\bar{z})(z_1 - z_2)(z_1 + z_2 - 2\bar{z})\|.
\end{aligned}$$

The inequality $\|z_1 + z_2 - 2\bar{z}\| \le \|z_1 - \bar{z}\| + \|z_2 - \bar{z}\| \le 2\delta$ implies

$$\|f(z_1) - f(z_2)\| \le (\varepsilon/2 + \delta\|\varphi''(\bar{z})\|)\|z_1 - z_2\|,$$

We can choose δ such that $2\delta\|\varphi''(\bar{z})\| \leq \varepsilon$, which implies that f is strictly stationary at $\bar{z}$.

By [6], the map

$$\left(\varphi(\bar{z}) + \varphi'(\bar{z})(\cdot - \bar{z}) + \tfrac{1}{2}\varphi''(\bar{z})(\cdot - \bar{z})^2 + F(\cdot)\right)^{-1}$$

is M-pseudo-Lipschitz around $(0, \bar{z})$.

The rest of the statement is a consequence of the main theorem given in [8]. ■

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