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ON RISK MINIMIZING STRATEGIES FOR DEFAULT-FREE BOND PORTFOLIO IMMUNIZATION

Abstract. This paper presents new strategies for bond portfolio immunization which combine the time-honored duration with the M-Absolute measure defined by Nawalkha and Chambers (1996). The innovation consists in considering an average shock in a fixed time period as a random variable with mean μ or, alternatively, with normal distribution with mean μ and variance σ^2. Additionally, an extension to arbitrage free models of polynomial shocks is provided. Moreover, the Fisher and Weil model, the M-Absolute strategy and a new one are compared empirically with respect to financial liquidity.

Introduction. Management of interest rate risk, the control of changes in the value of a stream of future cash flows as a result of changes in interest rates are important issues for an investor. Therefore many academic researchers have examined the immunization problem for a bond portfolio (see Nawalkha and Chambers, 1999). They have proposed multiple-risk measure models (e.g. Fong and Vasicek, 1984, Balbás and Ibáñez, 1998) or single-risk measure models (e.g. Nawalkha and Chambers, 1996, Kałuszka and Kondratiuk-Janyska, 2004). We propose new strategies for bond portfolio immunization. One is called the duration-dispersion strategy (DD strategy for short) and combines the time-honored duration with the remarkable risk measure M-Absolute defined by Nawalkha and Chambers (1996). The other is named the modified DD strategy and includes, additionally, M-Squared of Fong and Vasicek (1984). We consider a wider set of shocks than examined by Fong and Vasicek (1984) and Nawalkha and Chambers (1996). A new class includes all parallel movements. Considering an average shock in a fixed time period as a random variable with mean μ or with normal distribution with mean μ and variance σ^2 is our innovation. Moreover, we generalize ar-

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bitrage free models of polynomial shocks. In order to empirically investigate a new model, we use the McCulloch and Kwon (1993) term structure data over the observation period 1951 through 1986. Following an empirical test applied by Nawalkha and Chambers (1996), we examine the immunization performance of the Fisher and Weil strategy, the M-Absolute model and the DD strategy. Finally, a numerical comparison of the strategies is provided with respect to financial liquidity.

New strategy for portfolio immunization. In the pioneering work of Fong and Vasicek (1984) an interest risk measure, called *M-Squared*, was introduced which, if minimized with respect to bond portfolio proportions, would produce a duration-matching portfolio that minimizes the deviation of the portfolio return from the promised return. Compared to the classical theory of immunization their approach was completely new because they estimated a lower bound on the change in the end-of-horizon value of a bond portfolio as a result of the effect of an arbitrary interest change on a portfolio immunized against a parallel shift. Notice that the Fong–Vasicek strategy immunizes the value of portfolio against both small and large parallel shifts. But in practice large shocks seldom occur, therefore hedging against them does not seem to be the best strategy. See also the critique of Bierwag *et al.* (1993).

Nawalkha and Chambers (1996) modified the Fong and Vasicek approach to develop a single-risk measure immunization model, *M-Absolute*. They gave an example showing that their strategy is better than Fisher and Weil's. Shocks were bounded functions. However, this implies that parallel movements cannot be too big. On the other hand, one should not exclude this situation in theoretical considerations because it may happen in practice. Moreover, if the level of an average shock converges to an upper border of a band, then its volatility around the average level is decreasing. Knowing these limitations of existing interest rates behaviour models, we propose a stochastic approach. Earlier, Merton (1974), Vasicek (1977), Cox *et al.* (1979) and others discussed different stochastic models of term structures. For example Cox *et al.* (1979) assumed that the instantaneous compounding risk-free interest rate as a state of the market for default-free bonds is Markov with evolution governed by a stochastic differential equation. We simplify it by making only two assumptions about the shock process. First, an average change in the instantaneous forward rate in a fixed time period is assumed to be a random variable with mean μ. The other condition concerns the deviation size of rate changes around the average shock. Following Nawalkha and Chambers (1996), we require its movement to be within a band of width λ. The parameter λ can represent a volatility measure.

Denote by $[0, T]$ the time interval with $t = 0$ the present moment, and let H be the investor planning horizon, $0 < H < T$. Let C_t be the cash flow from a bond portfolio at time $t \le T$ ($t = t_1, \ldots, t_N$). We exclude the short sale, i.e. $C_t \ge 0$ for all t. The percentage change in the expected terminal value of the bond portfolio caused by an instantaneous change in the forward rates can be written as

$$\frac{\Delta I_H}{I_H} = \sum_t C_t^{(H)} \exp\Big(\int_t^H \Delta i(s)\, ds\Big) - 1,$$

where the sum is taken over all values of t ($t = t_1, \ldots, t_N$) and

- $C_t^{(H)} = (C_t/I_H) \exp(\int_t^H i(s)\, ds)$,
- I_H is the reinvested terminal value of the bond portfolio at $t = H$ if forward rates equal future spot rates,
- $i(t)$ is the current instantaneous forward rate,
- $\Delta i(s)$ is a change in the instantaneous forward rate.

We will make the following assumptions:

(i) $T^{-1} \int_0^T \Delta i(s)\, ds$ is a random variable with mean μ.
(ii) $|\Delta i(t) - T^{-1} \int_0^T \Delta i(s)\, ds| \le \lambda$ for all $t \ge 0$, where $\lambda \in [0, \infty)$.

Under conditions (i)–(ii), the class of shocks in the instantaneous forward rate includes all parallel shifts, which is not the case under the Nawalkha and Chambers assumption (1996). On the other hand, what differs Fong and Vasicek's M-Square model (1984) from our proposition is that large values of an average shock are rather unlikely.

PROPOSITION 1. *A lower bound on the expected value of the change in the terminal value of a bond portfolio is given by*

$$\mathbf{E}\Big(\frac{\Delta I_H}{I_H}\Big) \ge \exp(\mu G - \lambda M^A) - 1, \tag{1}$$

where

- $G = H - D$ *is the duration gap*,
- $D = \sum_t t C_t^{(H)}$ *is the Fisher–Weil duration of portfolio*,
- $M^A = \sum_t |H - t| C_t^{(H)}$ *is the M-Absolute of Nawalkha and Chambers.*

Proof. Put $\delta = T^{-1} \int_0^T \Delta i(s) ds$. Recall that

$$\begin{aligned} \frac{\Delta I_H}{I_H} &= \sum_t C_t^{(H)} \exp\Big(\int_t^H \Delta i(s)\, ds\Big) - 1 \\ &= \sum_t C_t^{(H)} \exp\Big(\delta(H - t) + \int_t^H (\Delta i(s) - \delta)\, ds\Big) - 1. \end{aligned} \tag{2}$$

From assumption (ii) it follows that $\mathit{\Delta}i(s)-\delta\geq-\lambda$ for $t\leq H$, thus

$$\int_t^H(\mathit{\Delta}i(s)-\delta)\,ds\geq-\lambda(H-t)\quad\text{for } t\leq H. \tag{3}$$

If $t>H$ then $\mathit{\Delta}i(s)-\delta\leq\lambda$. Consequently,

$$\int_t^H(\mathit{\Delta}i(s)-\delta)\,ds=-\int_H^t(\mathit{\Delta}i(s)-\delta)\,ds\geq-\lambda(t-H)\quad\text{for } t>H. \tag{4}$$

From (3) and (4) we get

$$\int_t^H(\mathit{\Delta}i(s)-\delta)\,ds\geq-\lambda|H-t|\quad\text{for } t\geq 0. \tag{5}$$

Combining (2) and (5) yields

$$\frac{\mathit{\Delta}I_H}{I_H}\geq\sum_t C_t^{(H)}\exp(\delta(H-t)-\lambda|H-t|)-1. \tag{6}$$

By the Jensen inequality and assumption (i) we have

$$\begin{aligned}\mathbf{E}\biggl(\frac{\mathit{\Delta}I_H}{I_H}\biggr)&\geq\sum_t C_t^{(H)}\mathbf{E}\exp(\delta(H-t)-\lambda|H-t|)-1\\&\geq\sum_t C_t^{(H)}\exp((H-t)\mathbf{E}\delta-\lambda|H-t|)-1\\&=\sum_t C_t^{(H)}\exp((H-t)\mu-\lambda|H-t|)-1.\end{aligned} \tag{7}$$

The sequence $C_{t_1}^{(H)},\ldots,C_{t_N}^{(H)}$ defines a probability distribution on $[0,T]$ because $C_t^{(H)}\geq 0$ and $\sum_t C_t^{(H)}=1$. Applying the Jensen inequality again, we obtain

$$\mathbf{E}\biggl(\frac{\mathit{\Delta}I_H}{I_H}\biggr)\geq\exp(\mu G-\lambda M^A)-1,$$

which completes the proof. ∎

As a corollary of Proposition 1 we get the following *DD strategy*:

$$\text{find a portfolio which maximizes } \mu G-\lambda M^A. \tag{8}$$

Remark 1. If μ is an unknown parameter then

$$\inf_\mu\mathbf{E}\biggl(\frac{\mathit{\Delta}I_H}{I_H}\biggr)\geq\exp(-\lambda M^A)-1$$

provided $G=0$. Therefore an investor should apply the strategy which consists in

$$\text{finding a portfolio which minimizes } M^A \text{ subject to } D=H. \tag{9}$$

In many theoretical considerations it is assumed that shock is a Gaussian random process. If empirical studies confirm that an average shock has normal distribution with mean μ and variance σ^2, strategy (8) can be easily modified by replacing assumption (i) with

(i*) $T^{-1}\int_0^T \Delta i(s)\,ds$ is a Gaussian random variable with mean μ and variance $\sigma^2 \geq 0$ (if $\sigma^2 = 0$, the average change is not random). Both the mean and variance may depend on T.

PROPOSITION 2. *Let assumptions* (i*)–(ii) *hold. Then a lower bound on the expected value of the change in the terminal value of a bond portfolio is given by*

$$\mathbf{E}\left(\frac{\Delta I_H}{I_H}\right) \geq \exp\left(\mu G + \frac{1}{2}\,\sigma^2 M^2 - \lambda M^A\right) - 1, \tag{10}$$

where

- $M^2 = \sum_t (H-t)^2\, C_t^{(H)}$ *is the M-Square of Fong and Vasicek*,
- G *and* M^A *are defined in Proposition* 1.

Proof. Analysis similar to that in the proof of Proposition 1 gives (6). Recall that from assumption (i*) it follows that the random variable δ has Gaussian distribution with mean μ and variance σ^2 so $\mathbf{E}\exp(\delta a) = \exp\big(\mu a + \sigma^2 a^2/2\big)$ for every $a \in \mathbb{R}$. In consequence,

$$\mathbf{E}\left(\frac{\Delta I_H}{I_H}\right) \geq \sum_t C_t^{(H)} \exp\left(\mu(H-t) + \frac{1}{2}\,\sigma^2 (H-t)^2 - \lambda|H-t|\right) - 1.$$

The rest of the proof runs as before. Since $C_t^{(H)} \geq 0$ and $\sum_t C_t^{(H)} = 1$, the sequence $C_{t_1}^{(H)}, \ldots, C_{t_N}^{(H)}$ defines a probability distribution on $[0,T]$. From this and the Jensen inequality we get

$$\mathbf{E}\left(\frac{\Delta I_H}{I_H}\right) \geq \exp\left(\mu G + \frac{1}{2}\,\sigma^2 M^2 - \lambda M^A\right) - 1,$$

as desired. ■

As a consequence of Proposition 2 we obtain the *modified DD strategy*:

$$\text{find a portfolio which maximizes } \mu G + \frac{1}{2}\,\sigma^2 M^2 - \lambda M^A. \tag{11}$$

REMARK 2. If μ is an unknown parameter then an investor should

$$\text{find a duration-matching portfolio which maximizes } \frac{1}{2}\,\sigma^2 M^2 - \lambda M^A. \tag{12}$$

Observe that if $\sigma = 0$ then the immunization strategy is to

$$\text{choose a portfolio which minimizes } M^A \text{ subject to } D = H.$$

A solution to the above problem is a bullet portfolio. On the other hand, if $\lambda = 0$ then we obtain the strategy:

choose a portfolio which maximizes M^2 subject to $D = H$.

It is obvious that a barbell portfolio has the maximum M^2 (Zaremba, 1998, Zaremba and Smoleński, 2000). The above strategy differs from the Fong and Vasicek approach (1984) because they proposed to *minimize* M^2 among all duration-matching portfolios. Our result is closer to the traditional approach based on a Taylor series expansion of the end-of-period value around H with respect to a flat perturbation of interest rate.

Arbitrage free generalized polynomial models. In the previous section we have expanded an unknown shock in the instantaneous forward rate into a series but only the first term has been modeled as a random variable. The rest is estimated by λ. The first term of the series measures the average shock. We have decided to focus on it because during the 1980s duration explained 80% to 90% of the return variance for government bonds (see e.g. Ilmanen, 1992). This means that parallel movements have significant role in shock behaviour. Now we generalize this approach, that is, we take into account further terms of the series. It is clear that the corresponding results are more precise but on the other hand the model becomes more complicated.

Let $a_1(t), \ldots, a_d(t)$ be known functions. Define the class of shocks:

$$S = \Big\{ \Delta i : \Delta i(t) = \sum_{k=1}^{d} \delta_k a_k(t),\ 0 \le t \le T,\ \text{for some reals } \delta_1, \ldots, \delta_d \Big\} \tag{13}$$

(see e.g. Rządkowski and Zaremba, 2000). Special cases of (13) are:

(a) the polynomial model

$$\Delta i(t) = \sum_{k=1}^{d} \delta_k t^{k-1} \tag{14}$$

(see Chambers *et al.*, 1988, Prisman and Shores, 1988, Crack and Nawalkha, 2000),

(b) the multiple shocks model

$$\Delta i(t) = \sum_{k=1}^{d} \delta_k I_k(t), \tag{15}$$

where $I_k(t) = 1$ for $t \in [\tau_{k-1}, \tau_k)$, and $I_k(t) = 0$ otherwise, with $0 = \tau_0 < \tau_1 < \cdots < \tau_d = T$ (see Reitano, 1991),

(c) Khang's (1979) model

$$\Delta i(t) = \delta \frac{\ln(1+\alpha t)}{\alpha t} \quad \text{with a positive real } \alpha, \tag{16}$$

(d) the spline model (see e.g. De La Grandville, 2002).

We now introduce the class of shocks:

$$S^* = \Big\{\Delta i : \Delta i(t) = \sum_{k=1}^{d} \delta_k a_k(t) + \varepsilon(t),\, 0 \le t \le T\Big\}. \tag{17}$$

Our assumptions are as follows:

(iii) $(\delta_1, \ldots, \delta_d)$ is a random vector with the vector of expected values $(\mu_1, \ldots, \mu_d)$.
(iv) $|\varepsilon(t)| \le \lambda$ for all $t \ge 0$.

Proposition 3. *Under assumptions* (iii)–(iv), *we have*

$$\mathbf{E}\bigg(\frac{\Delta I_H}{I_H}\bigg) \ge \exp\Big(\sum_{i=1}^{d} \mu_i G_i - \lambda M^A\Big) - 1, \tag{18}$$

where

- $G_i = \sum_t C_t^{(H)} \int_t^H a_i(s)\,ds$ *is the ith duration gap for* $i = 1, \ldots, d$,
- M^A *is defined in Proposition* 1.

Proof. The proof is similar to that of Proposition 1. Clearly

$$\frac{\Delta I_H}{I_H} = \sum_t C_t^{(H)} \exp\Big(\sum_{j=1}^{d} \delta_j \int_t^H a_j(s)\,ds + \int_t^H \Big(\Delta i(s) - \sum_{j=1}^{d} \delta_j a_j(s)\Big)\,ds\Big) - 1.$$

By assumption (iv),

$$\frac{\Delta I_H}{I_H} \ge \sum_t C_t^{(H)} \exp\Big(\sum_{j=1}^{d} \delta_j \int_t^H a_j(s)\,ds - \lambda|H-t|\Big) - 1.$$

Applying the Jensen inequality, we get

$$\begin{aligned}
\mathbf{E}\bigg(\frac{\Delta I_H}{I_H}\bigg) &\ge \sum_t C_t^{(H)} \mathbf{E} \exp\Big(\sum_{j=1}^{d} \delta_j \int_t^H a_j(s)\,ds - \lambda|H-t|\Big) - 1 \\
&\ge \sum_t C_t^{(H)} \exp\Big(\mathbf{E} \sum_{j=1}^{d} \delta_j \int_t^H a_j(s)\,ds - \lambda|H-t|\Big) - 1 \\
&\ge \exp\Big(\sum_{i=1}^{d} \mu_i G_i - \lambda M^A\Big) - 1,
\end{aligned}$$

as desired. ∎

To have a theorem analogous to the one of the previous section, we need the following assumption:

(iii*) $(\delta_1, \ldots, \delta_d)$ is a random vector with multidimensional Gaussian distribution, with expected values $(\mu_1, \ldots, \mu_d)$ and covariance matrix $\Sigma = (\sigma_{ij})$.

PROPOSITION 4. *Under assumptions* (iii*)–(iv), *we have*

$$\mathbf{E}\left(\frac{\Delta I_H}{I_H}\right) \geq \exp\left(\sum_{i=1}^{d} \mu_i G_i + \frac{1}{2}\sum_{i,j=1}^{d} \sigma_{ij} M_{ij}^2 - \lambda M^A\right) - 1, \tag{19}$$

where

- $G_i = \sum_t C_t^{(H)} \int_t^H a_i(s)\,ds$ *is the ith duration gap for* $i = 1, \ldots, d$,
- $M_{ij}^2 = \sum_t C_t^{(H)} \int_t^H a_i(s)\,ds \int_t^H a_j(u)\,du$ *is the* (i,j)*th modified Fong–Vasicek measure*,
- M^A *is the M-Absolute of Nawalkha–Chambers.*

Proof. The proof is similar to that of Proposition 3. We apply the formula $\mathbf{E}\exp(b_1X_1 + \cdots + b_dX_d) = \exp\left(\mu b^T + \frac{1}{2} b\Sigma b^T\right)$ with $b = (b_1, \ldots, b_d)$ provided $(X_1, \ldots, X_d)$ has multidimensional Gaussian distribution with expected value $\mu = (\mu_1, \ldots, \mu_d)$ and covariance matrix $\Sigma = (\sigma_{ij})$. ∎

Proposition 4 yields the following immunization strategy:

$$\text{find a portfolio which maximizes } \sum_{i=1}^{d} \mu_i G_i + \frac{1}{2}\sum_{i,j=1}^{d} \sigma_{ij} M_{ij}^2 - \lambda M^A. \tag{20}$$

The strategy defined by (20) is an extension of (11). It is easy to implement because it leads to a linear programming problem with linear restrictions.

EXAMPLE 1. Consider the polynomial model of Chambers *et al.* (1988). The well known limitation of this model is that there cannot exist portfolios without short position unless $d = 1$. Notice that a solution of problem (20) always exists and one can easily calculate the duration gaps and risk measures. In the polynomial model, $a_i(t) = t^{i-1}$ for $i = 1, \ldots, d$, so the ith duration gap is given by

$$G_i = \frac{1}{i}\,(H^i - D_i) \quad \text{for } i = 1, \ldots, d,$$

where $D_i = \sum_t C_t^{(H)} t^i$ is the ith order Fisher–Weil duration of portfolio. Elementary algebra leads to

$$M_{ij}^2 = \frac{1}{ij}\,(H^{i+j} - H^i D_j - H^j D_i + D_{i+j}) \quad \text{for all } i, j.$$

Taking $\mu_1 = \cdots = \mu_d = 0$ in Proposition 4 we get the strategy:

$$\text{find a portfolio which maximizes } \frac{1}{2}\sum_{i,j=1}^{d} \sigma_{ij} M_{ij}^2 - \lambda M^A.$$

Putting $d = 1$ we obtain $M_{11}^2 = H^2 - 2HD + D_2 = M^2$ and the immunization strategy reduces to that in (11) with $\mu = 0$.

Empirical test. We apply the McCulloch and Kwon (1993) interest rate data. This data set has been used before for empirical studies on bond immunization by Nawalkha and Chambers (1996), Nawalkha *et al.* (2003) and Christiansen (2003) among others. Following a testing methodology proposed by Nawalkha and Chambers (1996), the immunization performance of duration, M-Absolute and a linear combination of duration and M-Absolute is empirically examined. Thus 31 annual coupon bonds are constructed at the end of each year, with 7 different maturities $(1, 2, 3, \ldots, 7)$ and 5 different coupon values (6, 8, 10, 12, 14 percent) for each maturity. Coupon bond prices are simulated using zero-coupon yields. The investor planning period is equal to 4. For December 31, 1951, three bond portfolios are constructed according to the Fisher and Weil duration strategy, the M-Absolute strategy and the strategy given by (8) and rebalanced on December 31 of each of the next three years (1952, 1953, 1954) when annual coupons are received. At the end of the four-year horizon (December 31, 1955), the returns of these portfolios are compared with the return on a hypothetical four-year zero-coupon bond (computed at the beginning of the planning horizon). The difference between the actual values and the target value is defined as deviation in the interest rate risk hedging performance. The immunization procedure is repeated for the next four-year periods: 1951–55, 1952–56, ..., 1982–86.

Alternative models. We select a unique bond portfolio corresponding to the strategy (8) by solving an equivalent problem:

$$\text{(21)} \qquad \text{maximize } \mu\left(\sum_{i=1}^{J} \frac{n_i p_i}{I_0} D_i - H\right) - \lambda \sum_{i=1}^{J} \frac{n_i p_i}{I_0} M_i^A$$

$$\text{subject to } \sum_{i=1}^{J} n_i p_i = I_0,\ n_i \geq 0 \text{ for all } i = 1, \ldots, J,$$

where $J = 31$ is the number of bonds in the portfolio, I_0 is the initial investment amount, p_i is the price of the ith bond, n_i stands for the number of ith bonds held, and D_i, M_i^A are the duration and the M-Absolute of the ith bond, respectively. The portfolio solutions using the M-Absolute strategy are found from the model:

$$\text{(22)} \qquad \text{minimize } \sum_{i=1}^{J} \frac{n_i p_i}{I_0} M_i^A$$

$$\text{subject to } \sum_{i=1}^{J} n_i p_i = I_0,\ n_i \geq 0 \text{ for all } i = 1, \ldots, J.$$

The traditional Fisher and Weil duration strategy is tested in the form:

$$\text{(23)} \qquad \text{minimize } \sum_{i=1}^{J} (p_i n_i)^2 \text{ subject to } \sum_{i=1}^{J} \frac{n_i p_i}{I_0} D_i = H,\ \sum_{i=1}^{J} n_i p_i = I_0.$$

The solutions of (21), (22) and (23) are found using Microsoft Excel 2000 Solver.

The results. To decide which strategy is the best, a wide range of market simulations should be conducted. Our objective is to show that the DD strategy may be implemented just as (22) and (23). We present a likely scenario that is one from thousands. Hence, we observe that the DD strategy is not neutral with respect to the market parameters such as μ, λ. As it is widely known that interest rate volatility differs significantly in the 1950s and 1960s from the 1970s and 1980s, it is reasonable to divide these 32 four-year overlapping periods into two groups: 1951–1970 and 1967–1986. The deviations of actual portfolio values from target value for three hedging strategies (21), (22) and (23) in these two periods are reported in Table 1.

Due to the sum of absolute deviations and sum of negative deviations for each period in this situation, the DD strategy is better than the traditional duration strategy but worse than the M-Absolute model. However, in reality, an investor puts money in the bank account if the return of his portfolio is greater than a liability, otherwise he borrows it to pay it off. Therefore we propose a new criterion to assess hedging strategies. Applying (21), (22) and (23) we analyze balances of three accounts, taking the beginning and end of investment to be 1955 and T, respectively. We give the values at the end of a fixed period of time ($T = 1962, 1970, 1978, 1986$) in Figures 1–4.

It is assumed that the interest rate of savings is equal to i ranging from 0% to 8% on $[1955, T]$, whereas the interest rate of loans is equal to $i + 3\%$. This numerical study illustrates the behaviour of three strategies under fixed conditions. The M-Absolute strategy significantly outperforms the Fisher and Weil model but is occasionally slightly outperformed by the DD strategy. Moreover, we use the McCulloch and Kwon (1993) interest rate data to calculate the balance of account from 1955 to 1986. We consider the account where the initial capital is equal to the deviation of actual portfolio

Table 1. Deviations of actual values from target values for two observation periods 1951–70 and 1967–86 in alternative models

Sample period	Target value	Duration strategy	M-Absolute strategy	DD strategy
1951–55	1.09055	−0.00615	−0.00112	−0.00252
1952–56	1.09825	−0.00642	−0.00147	−0.00188
1953–57	1.08898	0.00089	0.00071	0.00251
1954–58	1.08676	0.00743	0.00243	0.00331
1955–59	1.12183	−0.00752	0.00143	0.00002
1956–60	1.15984	0.01216	−0.00094	0.00160
1957–61	1.11918	0.00445	0.00381	0.00408
1958–62	1.16193	−0.00151	−0.00037	−0.00407
1959–63	1.21327	−0.00896	−0.00620	−0.00665
1960–64	1.14129	−0.00234	−0.00061	−0,00145
1961–65	1.16077	−0.00459	−0.00131	−0.00052
1962–66	1.14747	0.00252	0.00199	0.00260
1963–67	1.17473	0.00354	0.00173	0.00375
1964–68	1.17746	0.00208	0.00419	0.00464
1965–69	1.22336	−0.00678	0.00233	0.00501
1966–70	1.21808	0.02245	0.00678	0.00950
Sum of absolute deviations		0.09979	0.03743	0.05411
Sum of negative deviations		−0.04427	−0.01202	−0.01709
1967–71	1.26319	0.01709	0.00440	0.00525
1968–72	1.29600	−0.00680	0.00095	−0.00678
1969–73	1.37537	−0.02751	−0.01178	−0,01440
1970–74	1.26587	−0.00348	−0.00219	−0.00042
1971–75	1.23560	0.02415	0.00313	0.00450
1972–76	1.27476	0.01369	0.00242	0.00289
1973–77	1.30567	−0.01146	0.00034	−0.00291
1974–78	1.33434	−0.03743	−0.00368	−0.00768
1975–79	1.33771	−0.01697	−0.00390	0.00144
1976–80	1.26592	0.00421	0.01115	0.01828
1977–81	1.34291	0.02042	0.01448	0.02245
1978–82	1.43838	0.02970	0.01670	0.02541
1979–83	1.49900	0.01704	0.01494	0.01471
1980–84	1.62671	0.01764	−0.00117	−0.01401
1981–85	1.73090	−0.01827	−0.02611	−0.02804
1982–86	1.50212	−0.01478	−0.00424	−0.01136
Sum of absolute deviations		0.28064	0.12158	0.18053
Sum of negative deviations		−0.14534	−0.05307	−0.08560

value from target value in 1955. Then we check if this amount is positive or negative in order to multiply it by $1 + i_t$ or $1 + i_t + 3\%$, respectively, where i_t is the annual interest rate taken from the McCulloch and Kwon (1993) term structure at time t. This accumulated value is added to the next deviation which accounts for the balance at the end of 1956. The procedure is repeated by the end of 1986. The result is presented in Figure 5.

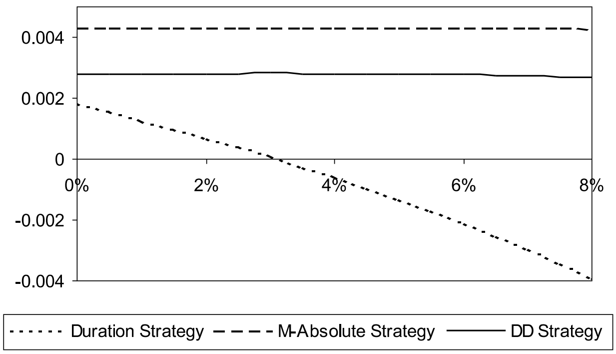

Fig. 1. Period 1955–1962

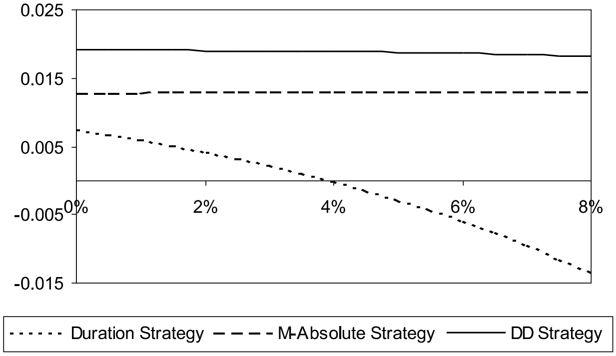

Fig. 2. Period 1955–1970

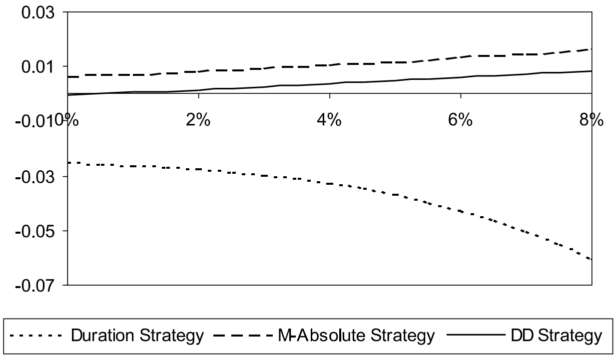

Fig. 3. Period 1955–1978

An interesting discovery is that by testing strategy (11) the results are very close to those for (8). Therefore we omit them.

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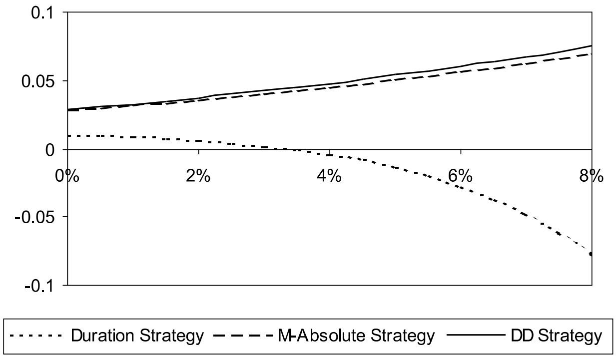

Fig. 4. Period 1955–1986

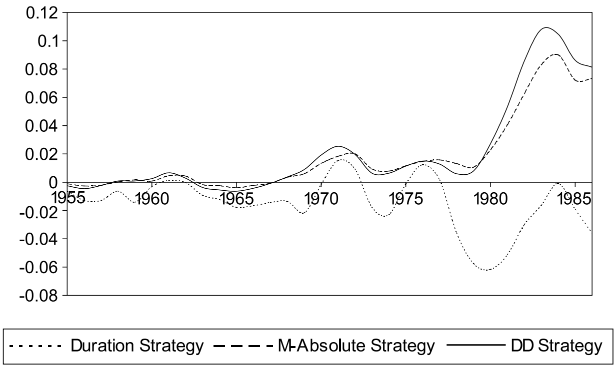

Fig. 5. Period 1955–1986

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