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GLOBAL EXISTENCE OF SOLUTIONS FOR INCOMPRESSIBLE MAGNETOHYDRODYNAMIC EQUATIONS

Abstract. Global-in-time existence of solutions for incompressible magnetohydrodynamic fluid equations in a bounded domain $\Omega \subset \mathbb{R}^3$ with the boundary slip conditions is proved. The proof is based on the potential method. The existence is proved in a class of functions such that the velocity and the magnetic field belong to $W_p^{2,1}(\Omega \times (0,T))$ and the pressure q satisfies $\nabla q \in L_p(\Omega \times (0,T))$ for $p \geq 7/3$.

1. Introduction. In a bounded domain $\Omega \subset \mathbb{R}^3$ with boundary S we consider the initial-boundary value problem for the equations of incompressible magnetohydrodynamics (see [4, 7])

$$
\begin{aligned}
&\partial_t v + v \cdot \nabla v + \nabla(q + H^2/2) \\
&\quad - H \cdot \nabla H - \nu \Delta v = f && \text{in } \Omega^T = \Omega \times (0,T), \\
&\operatorname{div} v = 0 && \text{in } \Omega^T, \\
&\partial_t H + v \cdot \nabla H - H \cdot \nabla v - \nu_\sigma \Delta H = 0 && \text{in } \Omega^T, \\
&\operatorname{div} H = 0 && \text{in } \Omega^T, \\
&\overline{n} \cdot D(v) \cdot \overline{\tau}_\alpha + \gamma v \cdot \overline{\tau}_\alpha = 0 && \text{in } \Omega^T, \\
&v \cdot \overline{n} = 0 && \text{on } S^T = S \times (0,T), \\
&H = 0 && \text{on } S^T, \\
&v|_{t=0} = v(0) && \text{in } \Omega, \\
&H|_{t=0} = H(0) && \text{in } \Omega,
\end{aligned}
\tag{1.1}
$$

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where $v = v(x,t)$ is the velocity of the fluid, $H = H(x,t)$ the magnetic field, $f = f(x,t)$ the external force, $\overline{n}$ the unit outward vector normal to S, $\overline{\tau}_\alpha$, $\alpha = 1,2$, tangent vectors to S, $q = q(x,t)$ the pressure, $\gamma > 0$ the constant slip coefficient. Moreover, $D(v) = \{v_{i,x_j} + v_{j,x_i}\}_{i,j=1,2,3}$ is the dilatation tensor.

The aim of this paper is to prove the global-in-time existence of solutions to (1.1) with small data in the L_p-approach.

Now we recall some results concerning mathematical questions of equations of magnetohydrodynamics (mhd). The first results on global existence of weak solutions to various initial-boundary value problems for mhd equations were given in [5, 6]. In these papers global existence of strong solutions in 2d and in the axially symmetric case was also proved. Moreover, global existence of regular solutions for small data was obtained.

In [8] existence, regularity and global properties of solutions of mhd equations such as global estimates, invariant sets, attracting sets have been obtained.

In [9, 10] by applying the semigroup technique global existence of regular solutions of mhd equations was proved under either smallness assumptions or some geometrical restrictions (2d, axially symmetric case).

Finally in [11] Stupialis has proved the existence of local solutions to the mhd equations such that the displacement term is taken into account.

In this paper we present a very simple and short proof of existence of global regular solutions to problem (1.1).

The main result can be stated as follows

Theorem. *Let* $f \in L_p(\Omega^T)$, $f(0) \in L_2(\Omega)$, $p \ge 7/3$, *and let* $(v(0), H(0))$ *belong to* $W_p^{2-2/p}(\Omega)$. *Assume that* $\|f(t)\|_{L_2(\Omega)} \le \|f(0)\|_{L_2(\Omega)} e^{-\lambda t}$ *for some* $\lambda > 0$ *and* $f(t)$ *describes dependence on time only. Let*

$$A = \|f\|_{L_p(\Omega^T)} + \|v(0)\|_{W_p^{2-2/p}(\Omega)} + \|H(0)\|_{W_p^{2-2/p}(\Omega)}.$$

Assume that A *is so small that* $cT^{1/p} \le 1$. *Assume also that* $S \in C^2$. *Then there exists a solution for problem* (1.1) *such that* $(v,H) \in W_p^{2,1}(\Omega^T)$, $\nabla q \in L_p(\Omega^T)$ *and the following estimate holds*:

$$\begin{aligned}\|v\|_{W_p^{2,1}(\Omega_k)} &+ \|H\|_{W_p^{2,1}(\Omega_k)} + \|\nabla q\|_{L_p(\Omega_k)} \\ &\le c(\|f\|_{L_p(\Omega\times(k-1)T_0,(k+1)T_0)} + \|v(k)\|_{W_p^{2-2/p}(\Omega)} + \|H(k)\|_{W_p^{2-2/p}(\Omega)}),\end{aligned}$$

where $\Omega_k = \Omega \times (kT_0, (k+1)T_0)$ *for* $k \in \mathbb{N}$, $T_0 > 0$ *and* c *is independent of time.*

2. Notation and auxiliary results. In our considerations we will need anisotropic Sobolev spaces $W_p^{m,n}(\Omega^T)$, where $m,n \in \mathbb{R}_+ \cup \{0\}$, $p \ge 1$, and

$\Omega^T = \Omega \times (0,T)$ with the norm

$$\|v\|^p_{W^{m,n}_p(\Omega^T)} = \|v\|^p_{W^{m,0}_p(\Omega^T)} + \|v\|^p_{W^{0,n}_p(\Omega^T)}$$

where

$$\|v\|^p_{W^{m,0}_p(\Omega^T)} = \int_0^T \|v\|^p_{W^m_p(\Omega)}\,dt, \qquad \|v\|^p_{W^{0,n}_p(\Omega^T)} = \int_\Omega \|v\|^p_{W^n_p(0,T)}\,dx$$

for

$$\|v\|^p_{W^m_p(\Omega)} = \sum_{|\alpha|\le[m]} \|D^\alpha_x v\|^p_{L_p(\Omega)} + \sum_{|\alpha|=[m]} \int_\Omega\int_\Omega \frac{|D^\alpha_x v(x,t) - D^\alpha_y v(y,t)|^p}{|x-y|^{s+p(m-[m])}}\,dx\,dy,$$

$$\|v\|^p_{W^n_p(0,T)} = \sum_{|\beta|\le[n]} \|D^\beta_t v\|^p_{L_p(0,T)} + \sum_{|\beta|=[n]} \int_0^T\int_0^T \frac{|D^\beta_t v(x,t) - D^\beta_{t'} v(x,t')|^p}{|t-t'|^{1+p(n-[n])}}\,dt\,dt',$$

where $s \equiv \dim\Omega$; $[m]$ is the integral part of m; D^α is the derivative in the distributional sense; $D^\alpha_x \equiv \partial^{\alpha_1}_{x_1}\ldots\partial^{\alpha_s}_{x_s}$; $\alpha = (\alpha_1,\ldots,\alpha_s)$ is a multiindex.

We will use the following results.

LEMMA 2.1 ([2]). *Let $f \in L_p(\Omega^T)$, $G \in W^{1,0}_p(\Omega^T)$ and $p \ge 2$. Assume that there exist functions $A, B \in L_p(\Omega^T)$ such that $\partial_t G - \operatorname{div} f = \operatorname{div} B + A$ and $\operatorname{diam}\operatorname{supp} A < 2\lambda_1$ for sufficiently small $\lambda_1 > 0$. Let $v(0) \in W^{2-2/p}_p(\Omega)$, $b \in W^{1-1/p,1/2-1/2p}_p(S^T)$, $b_3 \in W^{2-1/p,1-1/2p}_p(S^T)$, where $b \equiv (b_1, b_2, 0)^T$. Then there exists a solution of the problem*

$$\begin{aligned}
&\partial_t v - \nu\Delta v + \nabla q = f,\\
&\operatorname{div} v = G,\\
&\overline{n}\cdot D(v)\cdot\overline{\tau}_\alpha + \gamma v\cdot\overline{\tau}_\alpha|_{S^T} = b_\alpha \qquad (\alpha = 1,2),\\
&v\cdot\overline{n}|_{S^T} = b_3,\\
&v|_{t=0} = v(0),
\end{aligned}$$

such that $v \in W^{2,1}_p(\Omega^T)$, $\nabla q \in L_p(\Omega^T)$, and the following estimate holds:

$$\begin{aligned}
\|v\|_{W^{2,1}_p(\Omega^T)} + \|\nabla q\|_{L_p(\Omega^T)} \le c(T)(&\|f\|_{L_p(\Omega^T)} + \|B\|_{L_p(\Omega^T)} + \lambda_1\|A\|_{L_p(\Omega^T)}\\
&+ \|G\|_{W^{1,0}_p(\Omega^T)} + \|b\|_{W^{1-1/p,1/2-1/2p}_p(S^T)}\\
&+ \|b_3\|_{W^{2-1/p,1-1/2p}_p(S^T)} + \|v(0)\|_{W^{2-2/p}_p(\Omega)}),
\end{aligned}$$

where $c(T)$ is an increasing positive function of T.

LEMMA 2.2. *Let $u, v \in W^{2,1}_p(\Omega^T)$ and $u(0), v(0) \in W^{2-2/p}_p(\Omega)$, $\Omega \subset \mathbb{R}^3$. Assume that $p \ge 7/3$. Then*

$$\|u\cdot\nabla v\|_{L_p(\Omega^T)} \le cT^{2/p}\sup_t \|u\|_{W_p^{2-2/p}(\Omega)} \sup \|v\|_{W_p^{2-2/p}(\Omega)}$$
$$\le cT^{2/p}(\|u\|_{W_p^{2,1}(\Omega^T)} + \|u(0)\|_{W_p^{2-2/p}(\Omega)})(\|v\|_{W_p^{2,1}(\Omega^T)} + \|v(0)\|_{W_p^{2-2/p}(\Omega)}),$$

where c does not depend on T.

3. Existence. To prove the local existence we utilize the following method of successive approximations:

$$(3.1)\qquad \begin{aligned} &\partial_t v_n - \nu\Delta v_n + \nabla q_n \\ &\qquad = f - v_{n-1}\cdot\nabla v_{n-1} + H_{n-1}\cdot\nabla H_{n-1} - \nabla(H_{n-1}^2/2), \\ &\operatorname{div} v_n = 0, \\ &\partial_t H_n - \nu_\sigma \Delta H_n = H_{n-1}\cdot\nabla v_{n-1} - v_{n-1}\cdot\nabla H_{n-1}, \\ &\operatorname{div} H_n = 0, \\ &\overline{n}\cdot D(v_n)\cdot\overline{\tau}_\alpha + \gamma v_n\cdot\overline{\tau}_\alpha = 0, \quad \alpha = 1,2, \\ &\overline{n}\cdot v_n|_S = 0, \\ &H_n|_S = 0, \\ &v_n|_{t=0} = v(0), \quad H_n|_{t=0} = H(0), \end{aligned}$$

and $v_0 = H_0 = 0$.

LEMMA 3.1. *Assume that $f\in L_p(\Omega^T)$, $v(0)\in W_p^{2-2/p}(\Omega)$, $H(0)\in W_p^{2-2/p}(\Omega)$, $p\ge 7/3$. Then there exists $T_0>0$ such that for all $T\le T_0$ system* (1.1) *has a unique solution $v\in W_p^{2,1}(\Omega^T)$, $H\in W_p^{2,1}(\Omega^T)$, $\nabla q\in L_p(\Omega^T)$, and the following estimate holds*:

$$(3.2)\qquad \begin{aligned} &\|v\|_{W_p^{2,1}(\Omega^T)} + \|H\|_{W_p^{2,1}(\Omega^T)} + \|\nabla q\|_{L_p(\Omega^T)} \\ &\qquad \le c(T)(\|f\|_{L_p(\Omega^T)} + \|v(0)\|_{W_p^{2-2/p}(\Omega)} + \|H(0)\|_{W_p^{2-2/p}(\Omega)}). \end{aligned}$$

Proof. Let

$$X_k(T) = \|v_k\|_{W_p^{2,1}(\Omega^T)} + \|H_k\|_{W_p^{2,1}(\Omega^T)},$$
$$d(T) = \|v(T)\|_{W_p^{2-2/p}(\Omega)} + \|H(T)\|_{W_p^{2-2/p}(\Omega)}.$$

In view of Lemmas 2.1, 2.2 and the imbeddings $W_p^{2,1}(\Omega^T)\subset L_{q_1}(\Omega^T)$, $\nabla W_p^{2,1}(\Omega^T)\subset L_{q_2}(\Omega^T)$, $W_p^{2-2/p}(\Omega)\subset L_{q_3}(\Omega)$, $\nabla W_p^{2-2/p}(\Omega)\subset L_{q_4}(\Omega)$ with $5/p-5/q_1\le 2$, $5/p-5/q_2\le 1$, $5/p-3/q_3\le 2$, $5/p-3/q_4\le 1$ (see [1, 3]) we have

$$(3.3)\qquad X_n(T) \le cT^{1/p}(X_{n-1}^2(T) + d^2(0)) + c(\|f\|_{L_p(\Omega^T)} + d(0)).$$

Suppose that

$$(3.4)\qquad X_{n-1}(T)\le A$$

and

$$cT^{1/p}(A^2 + d^2(0)) + c(\|f\|_{L_p(\Omega^T)} + d(0)) \leq A \tag{3.5}$$

Then we have the estimate

$$X_n(T) \leq A \tag{3.6}$$

for all $n \in \mathbb{N}$.

To satisfy condition (3.5) we assume

$$cT^{1/p}A \leq 1/2 \tag{3.7}$$

and

$$cT^{1/p}d^2(0) + c(\|f\|_{L_p(\Omega^T)} + d(0)) \leq \frac{1}{2}A. \tag{3.8}$$

Then for small A we have $T \leq (1/2cA)^p$, and then by (3.8), the data must be suitably small.

To show convergence we introduce the differences $\widetilde{v}_n = v_n - v_{n-1}$, $\widetilde{H}_n = H_n - H_{n-1}$, $\widetilde{q}_n = q_n - q_{n-1}$. They satisfy the following system of equations for $n \geq 2$:

$$\begin{aligned}
\widetilde{v}_n - \nu\Delta\widetilde{v}_n + \nabla\widetilde{q}_n = & -(\widetilde{v}_{n-1}\cdot\nabla v_{n-1} + v_{n-2}\cdot\nabla\widetilde{v}_{n-1}) \\
& -(\widetilde{H}_{n-1}\cdot\nabla H_{n-1} + H_{n-2}\cdot\nabla\widetilde{H}_{n-1}) \\
& -(\widetilde{H}_{n-1i}\nabla H_{n-1i} + H_{n-2i}\nabla\widetilde{H}_{n-1i}) \\
\operatorname{div}\widetilde{v}_n = 0, & \\
\partial_t\widetilde{H}_n - \nu_\sigma\Delta\widetilde{H}_n = & \widetilde{H}_{n-1}\cdot\nabla v_{n-1} + H_{n-2}\cdot\nabla\widetilde{v}_{n-1} \\
& -(\widetilde{v}_{n-2}\cdot\nabla H_{n-1} + v_{n-2}\cdot\nabla\widetilde{H}_{n-1}), \\
\operatorname{div}\widetilde{H}_n = 0, & \\
\widetilde{v}_n\cdot\overline{n} = 0, & \\
\overline{n}\cdot D(\widetilde{v}_n)\cdot\overline{\tau}_\alpha + \gamma\widetilde{v}_n\cdot\overline{\tau}_\alpha = 0, & \\
\widetilde{H}_n = 0, & \\
\widetilde{v}_n|_{t=0} = 0, \quad \widetilde{H}_n|_{t=0} = 0, &
\end{aligned} \tag{3.9}$$

where the summation over i is assumed.

Let us introduce

$$\Gamma_n(T) = \|\widetilde{v}_n\|_{W_p^{2,1}(\Omega^T)} + \|\widetilde{H}_n\|_{W_p^{2,1}(\Omega^T)}.$$

From (3.9) we obtain

$$\Gamma_n(T) \leq cT^{1/p}A\Gamma_{n-1}(T). \tag{3.10}$$

Hence for $cT^{2/p}\ A < 1$ we have convergence. This ends the proof.

To prove the global existence we have to control the initial data in order to be able to apply Lemma 3.1.

Lemma 3.2. *Assume that* $f \in L_p(\Omega^T)$, $f(0) \in L_2(\Omega)$, Ω *is a bounded domain, and let* $\|f\|_{L_2(\Omega)} \le \|f(0)\|_{L_2(\Omega)} e^{-\lambda t}$, $\lambda > 0$. *Assume that the Korn inequality* (3.13) *is valid. Then the following decay estimate holds*:

$$\|v\|^2_{L_2(\Omega)} + \|H\|^2_{L_2(\Omega)} \le ce^{-c_0 t} \quad \text{for } c_0 > 0. \tag{3.11}$$

Proof. Multiplying $(1.1)_1$ by v and $(1.1)_3$ by H, adding, integrating over Ω and using the boundary conditions we obtain

$$\frac{d}{dt}(\|v\|^2_{L_2(\Omega)} + \|H\|^2_{L_2(\Omega)}) + \nu_\sigma \|\nabla H\|^2_{L_2(\Omega)} + \nu\|D(v)\|^2_{L_2(\Omega)} + \gamma\|v \cdot \overline{\tau}\|^2_{L_2(S)} = \int_\Omega f \cdot v\, dx. \tag{3.12}$$

Assume that we have the Korn inequality

$$\|v\|^2_{H^1(\Omega)} \le c\|D(v)\|^2_{L_2(\Omega)}. \tag{3.13}$$

Then (3.12) implies

$$\frac{d}{dt}(\|v\|^2_{L_2(\Omega)} + \|H\|^2_{L_2(\Omega)}) + \nu'(\|v\|^2_{L_2(\Omega)} + \|H\|^2_{L_2(\Omega)}) \le c\|f\|_{L_2(\Omega)}\|v\|_{L_2(\Omega)}, \tag{3.14}$$

where $\nu' = \min\{\nu, \nu_\sigma\}$.

Let

$$\alpha(t) = \|v(t)\|^2_{L_2(\Omega)} + \|H(t)\|^2_{L_2(\Omega)}.$$

Then (3.14) implies

$$\frac{d}{dt}(\alpha(t)e^{\nu' t}) \le c\|f(t)\|^2_{L_2(\Omega)} e^{\nu' t}. \tag{3.15}$$

Integrating (3.15) with respect to time gives

$$\alpha(t) \le ce^{-\nu' t}\int_0^t \|f(t')\|^2_{L_2(\Omega)} e^{\nu' t'}\, dt' + e^{-\nu' t}\alpha(0). \tag{3.16}$$

Using the decay assumption

$$\|f(t)\|_{L_2(\Omega)} \le \|f(0)\|_{L_2(\Omega)} e^{-\lambda t}$$

we obtain

$$\alpha(t) \le ce^{-2\lambda t}\|f(0)\|^2_{L_2(\Omega)} + e^{-\nu' t}\alpha(0). \tag{3.17}$$

This ends the proof.

Remark 3.3. If (3.13) does not hold, we obtain from (3.12) the inequality

$$\frac{d}{dt}(\|v\|^2_{L_2(\Omega)} + \|H\|^2_{L_2(\Omega)})^{1/2} \le c\|f\|_{L_2(\Omega)}$$

so

$$(3.18)\quad \|v(t)\|_{L_2(\Omega)} + \|H(t)\|_{L_2(\Omega)} \le c\int_0^t \|f(t')\|_{L_2(\Omega)}\,dt' + \|v(0)\|_{L_2(\Omega)} + \|H(0)\|_{L_2(\Omega)}.$$

Proof of the Theorem. To prove global existence we introduce a smooth function

$$\zeta = \zeta(T_1,T_2,t) = \begin{cases} 1 & \text{for } t \ge T_1, \\ 0 & \text{for } t \le T_2, \end{cases} \qquad T_1 > T_2.$$

Let $\widetilde{v} = v\zeta$, $\widetilde{H} = H\zeta$, $\widetilde{q} = q\zeta$, $\widetilde{f} = f\zeta$. Then problem (1.1) takes the form

$$(3.19)\quad \begin{aligned} &\partial_t\widetilde{v} - \nu\Delta\widetilde{v} + \nabla\widetilde{q} = \widetilde{f} - v\cdot\nabla\widetilde{v} - H\cdot\nabla\widetilde{H} - H_i\nabla\widetilde{H}_i + v\dot{\zeta}, \\ &\operatorname{div}\widetilde{v} = 0, \\ &\partial_t\widetilde{H} - \nu_\sigma\Delta\widetilde{H} = H\cdot\nabla\widetilde{v} - v\cdot\nabla\widetilde{H} + H\dot{\zeta}, \\ &\operatorname{div}\widetilde{H} = 0, \\ &\widetilde{v}\cdot\overline{n}|_s = 0, \\ &\overline{n}\cdot D(\widetilde{v})\cdot\overline{\tau}_\alpha + \gamma\widetilde{v}\cdot\overline{\tau}_\alpha|_s = 0, \\ &\widetilde{H}|_s = 0, \\ &\widetilde{v}|_{t=0} = 0, \qquad \widetilde{H}|_{t=0} = 0, \end{aligned}$$

where $|\dot{\zeta}| \le c/(T_1 - T_2)$ and summation over repeated indices is assumed.

Assume that we have proved local existence up to time $T > T_1$. Then from (3.19) we have

$$\begin{aligned}(3.20)\quad d(T) &\equiv \|\widetilde{v}(T)\|_{W_p^{2-2/p}(\Omega)} + \|\widetilde{H}(T)\|_{W_p^{2-2/p}(\Omega)} \\ &\le c(\|\widetilde{v}\|_{W_p^{2,1}(\Omega^T)} + \|\widetilde{H}\|_{W_p^{2,1}(\Omega^T)}) \le c(\|f\|_{L_p(\Omega^T)} + T^{2/p}A^2) \\ &\quad + \frac{1}{(T_1-T_2)^2}\int_{T_2}^{T_1}(\|v(t')\|_{L_2(\Omega)} + \|H(t')\|_{L_2(\Omega)})\,dt' \\ &\le c(\|f\|_{L_p(\Omega\times(T_2,T_1))} + (T)^{2/p}A^2) \\ &\quad + \frac{1}{(T_1-T_2)}\sup_t(\|v(t)\|_{L_2(\Omega)} + \|H(t)\|_{L_2(\Omega)})\end{aligned}$$

Assuming that T is large, $T_1 - T_2$ small compared to T but still large, and using the decay estimate for f we can assume

$$\begin{aligned}(3.21)\quad &\|f\|_{L_p(\Omega\times(T_2,T_1))} + (T_1-T_2)^{2/p}A^2 \\ &\quad + \frac{1}{T_1-T_2}\sup_{t\in(T_2,T_1)}(\|v(t)\|_{L_2(\Omega)} + \|H(t)\|_{L_2(\Omega)}) \le d(0).\end{aligned}$$

This enables continuation of the local solution. For any $k \in \mathbb{N}$ and $T_0 = T_1 - T_2$ we have (see Lemma 3.2, Remark 3.3)

$$\|f\|_{L_p(\Omega_k)} + (T_1 - T_2)^{2/p} A^2 + \frac{1}{(T_1 - T_2)} \sup_{t \in (kT_0, (k+1)T_0)} (\|v(t)\|_{L_2(\Omega)} + \|H(t)\|_{L_2(\Omega)}) \le d(0), \tag{3.22}$$

where $\Omega_k = \Omega \times (kT_0, (k+1)T_0)$. Hence

$$\|v\|_{W_p^{2,1}(\Omega_k)} + \|H\|_{W_p^{2,1}(\Omega_k)} \le c(\|f\|_{L_p(\Omega_k)} + d(0)) \tag{3.23}$$

for sufficiently small initial data.

This ends the proof of existence of global solutions.

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