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CONTROL-THEORETIC PROPERTIES OF STRUCTURAL ACOUSTIC MODELS WITH THERMAL EFFECTS, II. TRACE REGULARITY RESULTS

Abstract. We consider a structural acoustic problem with the flexible wall modeled by a thermoelastic plate, subject to *Dirichlet* boundary control in the thermal component. We establish sharp regularity results for the traces of the thermal variable on the boundary in case the system is supplemented with *clamped* mechanical boundary conditions. These regularity estimates are most crucial for validity of the optimal control theory developed by Acquistapace et al. [Adv. Differential Equations, 2005], which ensures well-posedness of the corresponding differential Riccati equations. The proof takes full advantage of the exceptional boundary regularity of the mechanical component of the clamped thermoelastic system as well as of the sharp trace theory pertaining to wave equations with Neumann boundary data.

1. Introduction. This paper continues—and concludes—the study initiated in [7], focused on a class of boundary control problems for a system of partial differential equations (PDE) describing fluid-structure interactions (*structural acoustic model*), which also include thermal effects. Our primary goal is to discuss the question of solvability of the associated quadratic optimal control problems over a finite time interval, along with well-posedness of the corresponding differential Riccati equations (DRE). As it is known and will become clearer later, this naturally leads us to undertake a preliminary investigation of the regularity properties of the solution to a dual (homogeneous) boundary value problem.

The structural acoustic model under investigation is the same as in [7], except for the boundary conditions. More precisely, the PDE system (that

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is, system (2.1) in the next section) consists of a wave equation in a bounded domain Ω in $\mathbb{R}^3$ which is strongly coupled, at a portion Γ_0 in its boundary, with a thermoelastic system. The dynamics of the active wall is described by means of a Kirchhoff thermoelastic plate model, which is known to be of "predominantly *hyperbolic* character" ([18]). The PDE system is subject to boundary control acting on the thermal component. The distinctive feature of the present boundary value problem (2.1) is the combination of *clamped* mechanical boundary condition (B.C.) and *Dirichlet* (thermal) boundary control (b.c.).

Let us recall that while well-posedness of DRE generally holds true in the case of parabolic-like dynamics, it is not expected in boundary control problems for evolutionary systems of hyperbolic type; see [16], [19], [6]. Moreover, in the latter case, optimal regularity theory is a key prerequisite for the study of the corresponding linear quadratic (LQ) control problem. This regularity analysis is particularly challenging in the case of composite systems of PDE, because of the different character of the elements of the system and/or the fact that the equations are set on manifolds of different dimensions. Yet, the interaction between the various components of the system, and in particular the influence—through the coupling—of the parabolic component may improve the regularity properties of the overall dynamics, resulting in a number of remarkable consequences for the associated optimal control problem.

Indeed, it was shown in [7] that under three different sets of coupled (mechanical/thermal) boundary conditions, namely in the case of Neumann boundary control, with hinged or clamped B.C., and in the case of hinged B.C./Dirichlet b.c., the abstract control system $y' = Ay + Bu$ corresponding to the structural acoustic model allows certain estimates of the operator $e^{At}B$—known as *singular estimates*—which ensure well-posedness of both the algebraic and differential Riccati equations corresponding to the associated optimal control problems. This follows from the theory developed in [4, 20, 21] (see also [11, 12]). It should be recalled that singular estimates establish, in addition—in the absence of analyticity of the C_0-semigroup generated by A—the existence of solutions to the semilinear initial/boundary value problem under *nonlinear* boundary conditions; see [7, Section 5]. It is important to emphasize that to achieve singular estimates for $e^{At}B$ one needs to prove suitable *interior* regularity estimates for the solutions to the *uncontrolled* PDE problem. (Singular estimates have been established in the case of diverse composite PDE systems with boundary/point control; see, e.g., [9], [12, 21]—which provide several illustrations—and [8]).

In contrast, in the present case when the PDE model is supplemented with clamped B.C./Dirichlet b.c., singular estimates do not hold. Nevertheless, on the basis of the results previously obtained in [1] for the thermoelastic system (alone), we intend to establish specific control-theoretic properties of the corresponding abstract dynamics which enable us to invoke the more recent optimal control theory developed in [2]. This guarantees that the *gain operator* $B^*P(t)$ is well defined and *bounded* on a dense set, and moreover that the Riccati operator $P(t)$ satisfies the DRE on $\mathcal{D}(A)$. The main result of this work is the proof of the sharp boundary regularity estimates of the thermal component of the (dual) PDE problem which are needed to verify the abstract conditions characterizing the class of systems studied in [2].

2. Statement of the problem and main result

The PDE model. Let Ω be an open bounded domain in $\mathbb{R}^3$, with boundary $\Gamma = \overline{\Gamma_0 \cup \Gamma_1}$, where Γ_0 and Γ_1 are open, connected, disjoint parts; the surface Γ_0 is *flat* ([1]). It is assumed that Ω is either smooth (i.e. Γ is of class C^2) or convex. The structural acoustic model under investigation consists of a wave equation (within the "chamber" Ω) and a thermoelastic system (acting on the elastic wall Γ_0 of the chamber) which are strongly coupled at a common interface. The dynamics of the plate is influenced by a boundary control (b.c.) u acting on $\partial\Gamma_0$. If the variable z denotes the velocity potential of the acoustic medium, while w and θ denote the vertical displacement of the plate and the temperature, respectively, the PDE system is given by:

$$
(2.1)\qquad
\begin{cases}
z_{tt} = \Delta z & \text{in } (0,T] \times \Omega =: Q, \\
\dfrac{\partial z}{\partial \tilde{\nu}} + d_1 z = 0 & \text{in } (0,T] \times \Gamma_1 =: \Sigma_1, \\
\dfrac{\partial z}{\partial \tilde{\nu}} = w_t & \text{in } (0,T] \times \Gamma_0 =: \Sigma_0, \\
w_{tt} - \varrho \Delta w_{tt} + \Delta^2 w + \Delta\theta + z_t = 0 & \text{in } \Sigma_0, \\
\theta_t - \Delta\theta = \Delta w_t & \text{in } \Sigma_0, \\
w = \dfrac{\partial w}{\partial \nu} = 0, \quad \theta = u & \text{on } (0,T] \times \partial\Gamma_0,
\end{cases}
$$

to be supplemented with the initial conditions

$$
(2.2)\qquad
\begin{aligned}
& z(0,\cdot) = z^0, \quad z_t(0,\cdot) = z^1 \quad \text{in } \Omega; \\
& w(0,\cdot) = w^0, \quad w_t(0,\cdot) = w^1, \quad \theta(0,\cdot) = \theta^0 \quad \text{in } \Gamma_0.
\end{aligned}
$$

The constant ϱ, which is proportional to the thickness of the plate, is taken to be small and positive; it is also assumed that the constant d_1 is positive. In the description of the boundary conditions (B.C.) associated with the

([1]) The LQ-problem for a structural acoustic model with *curved*—rather than flat—flexible wall Γ_0 has been explored in [10]; for more on modeling and control of shell-like structures, see the same authors' work in the references therein.

wave equation, $\tilde{\nu}$ denotes the unit outward normal to $\Gamma := \partial\Omega$; while in the description of the B.C. associated with the thermoelastic system, ν denotes the unit outward normal to the curve $\partial\Gamma_0$. As already pointed out in the introduction, a challenging feature of the present boundary value problem is the combination of mechanical *clamped* B.C. and thermal *Dirichlet* b.c.

Well-posedness. It is convenient to recast the (controlled) PDE system (2.1) as an abstract evolution equation in a specific Hilbert space. It was shown in [7] that the PDE model (2.1) can be written as a linear control system

$$y' = Ay + Bu \quad \text{in } [\mathcal{D}(A^*)]', \tag{2.3}$$

in the natural state space

$$Y = H^1(\Omega) \times L_2(\Omega) \times H_0^2(\Gamma_0) \times H_0^1(\Gamma_0) \times L_2(\Gamma_0), \tag{2.4}$$

where (i) the free dynamics operator A is the generator of a C_0-semigroup e^{At} in Y, and (ii) the *control* operator B has a degree of unboundedness "up to that of A", i.e. $A^{-1}B$ is a *bounded* operator from the control space $U = L_2(\Gamma)$ into Y. The full statement of these basic properties is found in [7, Proposition 2.1], along with the computations showing the validity of (ii). The explicit proof of item (i), that is, well-posedness of the *uncontrolled* model, was given in [22, Theorem 1.1]. (For the actual expressions of A and B see formulas (2.13) and (2.15) in [7, Section 2].)

For the reader's convenience and since it will be used later, let us recall the second order abstract system—corresponding to (2.1)—which eventually gives rise to (2.3), that is (utilizing the same notation of [7]),

$$\begin{cases} z_{tt} + \tilde{A}_N z - \tilde{A}_N N_0 w_t = 0, \\ \mathcal{M} w_{tt} + \mathcal{A} w - A_D\theta + N_0^* \tilde{A}_N z_t = -A_D D u, \\ \theta_t + A_D\theta + A_D w_t = A_D D u. \end{cases} \tag{2.5}$$

We now recall the meaning of the various abstract (linear) operators occurring in (2.5), which the reader may wish to postpone reading until the need arises; more detailed information on these operators is found in [7, Section 2]. The operator $-\tilde{A}_N$ is the realization of the laplacian Δ in Ω with Neumann/Robin (on Γ_0 and Γ_1, respectively) boundary conditions; N_0 represents the Neumann map from $L^2(\Gamma_0)$ into $L^2(\Omega)$. The operator $\mathcal{A}$ is the bilaplacian Δ^2 in Γ_0 with clamped B.C., while $-A_D$ is the laplacian Δ in Γ_0 with Dirichlet boundary conditions; $\mathcal{M} := I + \varrho A_D$ is known as the *stiffness* operator. D denotes, as usual, the Dirichlet map from $L^2(\partial\Gamma_0)$ to $L^2(\Gamma_0)$. It is useful to record explicitly the well known trace result

$$D^* A_D\, \varphi = \frac{\partial\varphi}{\partial\nu}\Big|_{\partial\Gamma_0} \qquad \forall\varphi \in H^{3/2+\delta}(\Gamma_0) \cap H_0^1(\Gamma_0),\ \delta > 0, \tag{2.6}$$

since it will be applied throughout the paper; the proof is found, e.g., in [19, Lemma 3.1.1, p. 181]. It is also important to recall that

$$\text{(2.7)}\quad \mathcal{D}(A) = \Big\{(z_1, z_2, w_1, w_2, \theta) \in H^2(\Omega) \times H^1(\Omega) \times \mathcal{D}(\mathcal{A}^{3/4}) \times H_0^2(\Gamma_0) \times [H^2(\Gamma_0) \cap H_0^1(\Gamma_0)] : \frac{\partial z_1}{\partial \tilde\nu}\Big|_{\Gamma_0} = w_2,\ \Big[\frac{\partial z_1}{\partial \tilde\nu} + d_1 z_1\Big]\Big|_{\Gamma_1} = 0\Big\},$$

and that $\mathcal{D}(A) = \mathcal{D}(A^*)$.

The linear quadratic optimal control problem. Aiming to ascertain well-posedness of the differential Riccati equations arising in the LQ-problem associated with the PDE system (2.1), a more in-depth analysis of the distinctive features of the evolution described by the couple (A, B) is needed. Indeed, our eventual goal is to prove that the abstract counterpart (2.3) of the PDE problem (2.1) is covered by the theory developed in [2]. This theory, which generalizes the one of [21], was in fact motivated by composite PDE systems comprising a parabolic component, yet with an overall hyperbolic character.

First, let us briefly recall the abstract formulation of the classical LQ-problem, as well as the properties which characterize the class of systems introduced in [2]. With the abstract control system (2.3), we associate a quadratic cost functional over a given time interval $[0, T]$:

$$\text{(2.8)}\qquad J(u) = \int_0^T (\|Ry(t)\|_Z^2 + \|u(t)\|_U^2)\, dt.$$

Above, Z is a third Hilbert space and $R \in \mathcal{L}(Y, Z)$ is known as the *observation operator*. The optimal control problem is to minimize the functional (2.8) over all $u \in L^2(0, T; U)$, where y is the solution of (2.3) corresponding to the control u.

We will show that the first-order system (2.3) fits in the novel class of abstract dynamics introduced in [2]. The corresponding theory guarantees the key property that the operator $B^*P(\cdot)$ which occurs in the feedback synthesis of the optimal pair $(u^*(\cdot), y^*(\cdot))$ of the control problem (2.3)–(2.8) is in fact *bounded* on a dense subspace of the state space. For the sake of clarity and the reader's convenience, the specific abstract conditions which characterize the class of control systems studied in [2] are recorded below.

HYPOTHESES 2.1 ([2]). *For each $t \in [0, T]$, the operator $B^* e^{A^* t}$ can be represented as*

$$\text{(2.9)}\qquad B^* e^{A^* t} y = F(t)y + G(t)y, \qquad t \ge 0,\ y \in \mathcal{D}(A^*),$$

where $F(t) : Y \to U$ and $G(t) : \mathcal{D}(A^*) \to U$, $t > 0$, are bounded linear operators satisfying the following assumptions:

(i) *there is* $\gamma \in (1/2, 1)$ *such that*

$$\|F(t)\|_{\mathcal{L}(Y,U)} \le ct^{-\gamma} \quad \forall t \in (0,T]; \tag{2.10}$$

(ii) *the operator* $G(\cdot)$ *belongs to* $\mathcal{L}(Y, L^p(0,T;U))$ *for all* $p \in [1,\infty)$, *with*

$$\|G(\cdot)\|_{\mathcal{L}(Y,L^p(0,T;U))} \le c_p < \infty \quad \forall p \in [1,\infty); \tag{2.11}$$

(iii) *there is* $\varepsilon > 0$ *such that:*

(a) *the operator* $G(\cdot)A^{*-\varepsilon}$ *belongs to* $\mathcal{L}(Y, C([0,T];U))$, *and in particular*

$$\|A^{-\varepsilon}G(t)^*\|_{\mathcal{L}(U,Y)} \le c < \infty \quad \forall t \in [0,T]; \tag{2.12}$$

(b) *the operator* R^*R *belongs to* $\mathcal{L}(\mathcal{D}(A^\varepsilon), \mathcal{D}(A^{*\varepsilon}))$, *i.e.*

$$\|A^{*\varepsilon}R^*RA^{-\varepsilon}\|_{\mathcal{L}(Y)} \le c < \infty; \tag{2.13}$$

(c) *there is* $q \in (1,2)$ (*depending, in general, on* ε) *such that the operator* $B^*e^{A^*\cdot}R^*RA^\varepsilon$ *has an extension in* $\mathcal{L}(Y, L^q(0,T;U))$.

Remark 2.2. It should be recalled that in order to check the key Hypothesis 2.1(iii)(c) it suffices to show that there exist $q \in (1,2)$ and $K \ge 0$ such that

$$\|B^*e^{A^*\cdot}A^{*\varepsilon}y\|_{L^q(0,T;U)} \le K\|y\|_Y \quad \forall y \in \mathcal{D}(A^{*\varepsilon}). \tag{2.14}$$

Indeed, if (2.14) holds, then using (2.13) one has, *a fortiori*,

$$\begin{aligned}\|B^*e^{A^*\cdot}R^*RA^\varepsilon y\|_{L^q(0,T;U)} &= \|B^*e^{A^*\cdot}A^{*\varepsilon}\,A^{*-\varepsilon}R^*RA^\varepsilon y\|_{L^q(0,T;U)}\\ &\le K\|A^{*-\varepsilon}R^*RA^\varepsilon y\|_Y \le K\,\|R^*R\|_{\mathcal{L}(\mathcal{D}(A^\varepsilon),\mathcal{D}(A^{*\varepsilon}))}\|y\|_Y \quad \forall y \in \mathcal{D}(A^{*\varepsilon}),\end{aligned}$$

i.e. Hypothesis 2.1(iii)(c) is satisfied.

In order to proceed to the verification of all the assumptions required in Hypotheses 2.1, we now provide a meaning to the abstract conditions involving the operators $B^*e^{A^*t}$ and (in view of the above remark) $B^*e^{A^*t}A^{*\varepsilon}$.

The PDE interpretation of the abstract conditions. 1. Consider first $B^*e^{A^*t}y^0$, with y^0 initially in $\mathcal{D}(A^*)$. The argument is by now standard: one observes that $e^{A^*t}y^0$ is nothing but the solution of the Cauchy problem $y' = A^*y$, $y(0) = y^0$. Computing the adjoint operator A^* (see, e.g., (13) in [22]) it is easily seen that

$$e^{A^*t}y^0 =: y(t) = (z(t), -z_t(t), w(t), -w_t(t), \theta(t)), \tag{2.15}$$

where (z, w, θ) solves the composite system

$$z_{tt} + \tilde{A}_N z + \tilde{A}_N N_0 w_t = 0, \tag{2.16a}$$

$$\mathcal{M}w_{tt} + \mathcal{A}w - A_D\theta - N_0^*\tilde{A}_N z_t = 0, \tag{2.16b}$$

$$\theta_t + A_D\theta + A_D w_t = 0, \tag{2.16c}$$

which is just a bit different from (2.5) with $u \equiv 0$. Indeed, (2.16) is the abstract formulation of the *uncontrolled* (dual) boundary value problem

$$
\begin{cases}
z_{tt} = \Delta z & \text{in } Q, \\
\dfrac{\partial z}{\partial \tilde{\nu}} + d_1 z = 0 & \text{in } \Sigma_1, \\
\dfrac{\partial z}{\partial \tilde{\nu}} = -w_t & \text{in } \Sigma_0, \\
w_{tt} - \varrho \Delta w_{tt} + \Delta^2 w + \Delta\theta - z_t = 0 & \text{in } \Sigma_0, \\
\theta_t - \Delta\theta = \Delta w_t & \text{in } \Sigma_0, \\
w = \dfrac{\partial w}{\partial \nu} = 0, \quad \theta = 0 & \text{on } (0,T] \times \partial\Gamma_0.
\end{cases}
\tag{2.17}
$$

On the other hand, by the definition ((2.15) in [7]) of the control operator B it is elementary to deduce that its adjoint B^* acts as follows:

$$
\varphi = (\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5) \in \mathcal{D}(A^*) \ \Rightarrow\ B^*\varphi = D^* A_D(\varphi_5 - \varphi_4). \tag{2.18}
$$

Thus, (2.18) combined with (2.15) gives, in view of the trace result (2.6),

$$
B^* e^{A^* t} y^0 = \frac{\partial}{\partial \nu}(\theta + w_t) = \left.\frac{\partial \theta}{\partial \nu}\right|_{\partial\Gamma_0}, \tag{2.19}
$$

where in the last equality we have taken into account the clamped boundary conditions for the mechanical component of the system.

2. As for condition (2.14) (related to Hypothesis 2.1(iii)(c)), we first claim that the required estimate for $B^* e^{A^* t} A^{*\varepsilon} \overline{y}$ with $\overline{y} \in Y$ is equivalent to the same estimate for $B^* \frac{d}{dt} e^{A^* t} y^0$ with $y^0 \in \mathcal{D}(A^{*1-\varepsilon})$; indeed, this is shown in [1, §2.2]. Thus, using once again (2.15), (2.18) and the clamped boundary conditions (which yield, as well, $\frac{\partial}{\partial \nu} w_{tt} = 0$ on $\partial\Gamma_0$), we similarly find

$$
B^* \frac{d}{dt} e^{A^* t} y^0 = \left.\frac{\partial \theta_t}{\partial \nu}\right|_{\partial\Gamma_0}. \tag{2.20}
$$

Consequently, in view of (2.19) and (2.20), we see that in order to check the conditions listed in Hypotheses 2.1 we must explore the regularity of the outer normal derivatives on $\partial\Gamma_0$ of both the thermal component and its velocity. The respective results are stated explicitly in the next section.

2.1. *The main result.* We have recalled and interpreted the abstract conditions which describe the class of control systems introduced in [2]. Thus, the assertions in the following theorem show that the PDE model under investigation is covered by the LQ control theory of [2].

THEOREM 2.3. *Let* (z, w, θ) *be the solution to the dual* (*uncontrolled*) *PDE problem* (2.17), *with initial condition* (2.2), *and let* $y^0 = (z^0, -z^1, w^0, -w^1, \theta^0)$. *Then the following statements are valid.*

1. *The thermal outer normal derivative* $\frac{\partial\theta}{\partial\nu}\big|_{\partial\Gamma_0}$ *splits into*

$$\frac{\partial\theta}{\partial\nu}\bigg|_{\partial\Gamma_0} = F(t)y^0 + G(t)y^0, \qquad t \ge 0,\ y^0 \in \mathcal{D}(A^*), \tag{2.21}$$

where $F(t) : Y \to U$ *and* $G(t) : \mathcal{D}(A^*) \to U$, $t > 0$, *are bounded linear operators with the following regularity properties:*

(i) $\|F(t)\|_{\mathcal{L}(Y,U)} \le Ct^{-3/4-\sigma}$ *for all* $t \in (0,T]$ *and arbitrary small* $\sigma > 0$;

(ii) $G(\cdot) \in \mathcal{L}(Y, L^p(0,T;U))$ *for all* $p \in [1,\infty)$, *with*

$$\|G(\cdot)\|_{\mathcal{L}(Y,L^p(0,T;U))} \le C_p < \infty \qquad \forall p \in [1,\infty); \tag{2.22}$$

(iii) $G(\cdot) \in \mathcal{L}(\mathcal{D}(A^{*\varepsilon}), C([0,T]; L^2(U))$ *for any* $0 < \varepsilon < 1/2$, *with*

$$|G(\cdot)y^0|_{C([0,T];U))} \le C|A^{*\varepsilon}y^0| \quad \forall y^0 \in \mathcal{D}(A^{*\varepsilon}); \tag{2.23}$$

2. *Assuming* $y^0 \in \mathcal{D}(A^{*1-\varepsilon})$, $\varepsilon \in (0,1/4)$, *the corresponding solution further satisfies*

$$\frac{\partial\theta_t}{\partial\nu}\bigg|_{\partial\Gamma_0} \in L^q(0,T;U) \tag{2.24}$$

continuously in y^0, *with*

$$1 < q < \min\left\{\frac{8}{7}, \frac{4}{3+4\varepsilon}\right\}. \tag{2.25}$$

3. Trace regularity results. This section is entirely devoted to show the statements of Theorem 2.3. First, we derive a sharp boundary regularity result pertaining to the elastic component of the system (Proposition 3.2), which is critical in the proof of our main result. Proposition 3.2 specifically asserts that the component w of the solution $(z(t), w(t), \theta(t))$ to the (dual, uncontrolled) structural acoustic system (2.17) satisfies $\Delta w \in L^2(0,T;L^2(\partial\Gamma_0))$. This regularity result is an analogue for the structural acoustic model of the sharp trace estimate established for the clamped thermoelastic system in [5].

3.1. *Trace estimates for the mechanical component.* Consider first the (uncoupled) thermoelastic problem corresponding to (2.16b)–(2.16c), namely

$$\begin{cases} w_{tt} - \varrho\Delta w_{tt} + \Delta^2 w + \Delta\theta = f & \text{in } \Sigma_0, \\ \theta_t - \Delta\theta - \Delta w_t = 0 & \text{in } \Sigma_0, \\ w = \dfrac{\partial w}{\partial\nu} = 0, \quad \theta = 0 & \text{on } (0,T] \times \partial\Gamma_0, \\ w(0,\cdot) = w^0, \quad w_t(0,\cdot) = w^1, \quad \theta(0,\cdot) = \theta^0 & \text{in } \Gamma_0, \end{cases} \tag{3.1}$$

where we have introduced a generic nonhomogeneous term f in place of $z_t|_{\Gamma_0}$. The following assertion generalizes the trace estimate established in

[5, Lemma 2] by allowing $f \not\equiv 0$; this kind of generalization has been introduced already in the context of stability analysis for structural acoustic interactions in [13, Lemma 2.4] and [23, Lemma 2.3]. Since the proof of the aforementioned extensions is omitted, a proof of the lemma recorded below is outlined for the sake of completeness and reader's convenience.

LEMMA 3.1. *Assume $f \in L^1(0,T;H^{-1}(\Gamma_0))$. Then the component w of the solution (w, w_t, θ) to the thermoelastic system* (3.1) *satisfies $\Delta w \in L^2(0,T;L^2(\partial\Gamma_0))$, with the estimate*

$$\int_0^T \|\Delta w\|^2_{L_2(\partial\Gamma_0)}\,dt \le C\Big\{\int_0^T E_w(t)\,dt + \int_0^T \|\nabla\theta\|^2_{L_2(\Gamma_0)}\,dt + \Big(\int_0^T \|f\|_{H^{-1}(\Gamma_0)}\,dt\Big)^2 + E_{w,\theta}(T) + E_{w,\theta}(0)\Big\}, \tag{3.2}$$

where $E_{w,\theta}(t) = E_w(t) + E_\theta(t)$ is the energy of the system at time t, with

$$E_w(t) := \|\Delta w\|^2_{0,\Gamma_0} + \|w_t\|^2_{0,\Gamma_0} + \varrho\|\nabla w_t\|^2_{0,\Gamma_0},$$
$$E_\theta(t) := \|\theta\|^2_{0,\Gamma_0}.$$

Proof. We follow the proof of [5, Lemma 2] verbatim. Let h be a $C^2(\Gamma_0)$ vector field such that $h|_{\partial\Gamma_0} \equiv \nu$. It is assumed, as usual, that initial data are smooth enough to justify the foregoing computations: the achieved estimate will eventually be extended to all initial data by continuity. We multiply the plate equation of system (3.1) by $h \cdot \nabla w$, and integrate between 0 and T, thus obtaining

$$\int_0^T (w_{tt} - \varrho\Delta w_{tt} + \Delta^2 w + \Delta\theta - f, h\cdot\nabla w)\,dt = 0. \tag{3.3}$$

In view of the computations performed in [5, Lemma 2], we know that the equality (3.3) yields the following one:

$$\int_0^T \|\Delta w\|^2_{0,\partial\Gamma_0}\,dt = 2(w_t + \varrho\nabla w_t, h\cdot\nabla w) + \mathcal{O}\Big(\int_0^T \|w_t\|^2_{0,\Gamma_0}\,dt\Big) + \mathcal{O}\Big(\int_0^T \|\nabla w_t\|^2_{0,\Gamma_0}\,dt\Big) + \mathcal{O}\Big(\int_0^T \|\Delta w\|^2_{0,\Gamma_0}\,dt\Big) - 2\int_0^T (\nabla\theta, \nabla(h\cdot\nabla w))\,dt + 2\int_0^T (f, h\cdot\nabla w)\,dt \tag{3.4}$$

($\mathcal{O}$ denotes the Landau "big O" symbol). To obtain (3.2), one needs to make pretty simple estimates of the various summands on the right hand side of (3.4). We write explicitly just the bound for the latter term:

$$2\int_0^T |(f, h\cdot\nabla w)|\,dt \le C(h)\sup_{[0,T]}\|w(t)\|_{2,\Gamma_0}\int_0^T \|f\|_{-1,\Gamma_0}\,dt$$
$$\le C[E_w(0)]^{1/2}\,\|f\|_{L^1(0,T;H^{-1}(\Gamma_0))}. \ \blacksquare$$

The previous result enables us to show the following proposition.

PROPOSITION 3.2. *Let* $(z(t), w(t), \theta(t))$ *be a solution to the (uncontrolled) structural acoustic model* (2.17), *with initial datum* $y^0 = (z^0, z^1, w^0, w^1, \theta^0) \in Y$. *Then the component* w *satisfies* $\Delta w|_{\partial\Gamma_0} \in L^2(0,T;L^2(\partial\Gamma_0))$, *and there exists* $C_T > 0$ *such that*

$$\int_0^T \|\Delta w\|^2_{L^2(\partial\Gamma_0)}\,dt \le C_T\|y^0\|^2_Y, \qquad y^0 \in Y. \tag{3.5}$$

Consequently,

$$\int_0^T \|\Delta w_t\|^2_{L^2(\partial\Gamma_0)}\,dt \le C_T\|A^*y^0\|^2_Y, \qquad y^0 \in \mathcal{D}(A^*). \tag{3.6}$$

Proof. Our starting point is the inequality (3.2), here with $f \equiv z_t|_{\Gamma_0}$. We seek to bring forward the bounds for the terms on the right hand side of (3.2). To accomplish this, let us first write the energy identity for the structural acoustic model (2.17). Indeed, by combining the identities pertaining to the energy functional associated with the wave equation, namely $E_z(t) = \|\nabla z\|^2_{0,\Omega} + \|z_t\|^2_{0,\Omega} + d_1\|z\|^2_{0,\Gamma_1}$, with the one associated with the thermoelastic system (i.e. $E_{w,\theta}(t)$ recalled in Lemma 3.1), one obtains for the total energy of the system $E(t) = E_z(t) + E_{w,\theta}(t)$ the equality

$$E(T) + 2\int_0^T \|\nabla\theta\|^2_{0,\Gamma_0}\,dt = E(0) \tag{3.7}$$

(see, e.g., [22, Proposition 2.1]). Notice that the above equality implies that $\int_0^T \|\nabla\theta\|^2_{0,\Gamma_0}\,dt \le E(0)$ and $E(T) \le E(0)$, or more generally $E(t) \le E(s)$ for $s \le t$, that is, the energy is nonincreasing. This yields as well

$$\int_0^T E_{w,\theta}(t)\,dt \le \int_0^T E(t)\,dt \le TE(0). \tag{3.8}$$

In order to complete the estimate of the right hand side of (3.2), we exploit the regularity theory of hyperbolic equations with nonhomogeneous Neumann boundary conditions; see [14, 15, 17], [24] and their references. In particular, we may use the following sharp estimate of the trace of z_t on Γ_0 (valid for a general Ω which is either smooth or convex):

$$\int_0^T \|z_t\|^2_{-1/3,\Gamma_0}\,dt \le C_T\Big(\|z^0\|^2_{1,\Omega}+\|z^1\|^2_{0,\Omega}+\int_0^T \|w_t\|^2_{1/3,\Gamma_0}\,dt\Big) \tag{3.9}$$

(a useful sketch of the proof is given in [9, Proposition 3.8]). Thus, substituting the bounds stemming from the energy identity (3.7), the inequality (3.8) and the estimate (3.9) into (3.2), we conclude that there exists a constant $C_T>0$ such that

$$\int_0^T \|\Delta w\|^2_{0,\partial\Gamma_0}\,dt \le C_T\Big(E(0)+\int_0^T \|w_t\|^2_{1,\Gamma_0}\,dt\Big), \tag{3.10}$$

which finally implies (3.5), since $\|w_t\|_{C([0,T];H^1(\Gamma_0))}\le C\|y^0\|_Y$. If $y^0\in\mathcal{D}(A^*)$, the inequality (3.6) follows from (3.5) by C_0-semigroup theory. ∎

3.2. *Proof of Theorem 2.3: boundary regularity results for the thermal component*

Proof of Theorem 2.3. 1. Let $(z(t),w(t),\theta(t))$ be the solution to system (2.17)—equivalently, (2.16)—corresponding to an initial datum $y^0\in\mathcal{D}(A^*)$. By (2.16c), the thermal component θ is given by

$$\begin{aligned}\theta(t) &= e^{-A_D t}\theta^0 - \int_0^t e^{-A_D(t-s)}A_D w_t(s)\,ds \\ &= e^{-A_D t}\theta^0 + [e^{-A_D(t-s)}w_t(s)]|_0^t - \int_0^t e^{-A_D(t-s)}w_{tt}(s)\,ds \\ &= e^{-A_D t}\theta^0 + w_t(t) + e^{-A_D t}w^1 - \int_0^t e^{-A_D(t-s)}w_{tt}(s)\,ds.\end{aligned} \tag{3.11}$$

In view of (2.19), we seek a decomposition of $\frac{\partial\theta(t)}{\partial\nu}\big|_{\partial\Gamma_0}$ by means of two linear operators $F(t)$ and $G(t)$ which further satisfy the regularity properties listed in Hypotheses 2.1. The start is analogous to the one of the proof of Theorem 3.3 in [8, Section 5]: we apply (2.6) to (3.11), taking into account the clamped boundary conditions for the elastic component; next use the basic singular estimates pertaining to analytic semigroups, thus obtaining the first bound

$$\Big|\frac{\partial\theta}{\partial\nu}\Big|_{0,\partial\Gamma_0}$$

$$= \Big|D^*A_D e^{-A_D t}(\theta^0+w^1) - D^*A_D\int_0^t e^{-A_D(t-s)}w_{tt}(s)\,ds\Big|_{0,\partial\Gamma_0} \tag{3.12}$$

$$\le \frac{C}{t^{3/4+\varepsilon}}|\theta^0|_{0,\Gamma_0} + \frac{C}{t^{1/4+\varepsilon}}|w^1|_{1,\Gamma_0} + \Big|D^*A_D\int_0^t e^{-A_D(t-s)}w_{tt}(s)\,ds\Big|_{0,\partial\Gamma_0}. \tag{3.13}$$

Let us denote the last summand on the right hand side of (3.13) by $|b(t;y^0)|$. Replacing w_{tt} by its expression derived from (2.16b) and using once again the trace result (2.6), we decompose and bound $b(t;y^0)$ as follows:

$$
\begin{aligned}
(3.14)\quad |b(t;y^0)| \le & \Big|\underbrace{D^*A_D\int_0^t e^{-A_D(t-s)}\mathcal{M}^{-1}\mathcal{A}w(s)\,ds}_{\phi(t)y^0}\Big| \\
& + \Big|\underbrace{D^*A_D\int_0^t e^{-A_D(t-s)}\mathcal{M}^{-1}A_D\theta(s)\,ds}_{-F_2(t)y^0}\Big| + \Big|\underbrace{D^*A_D\int_0^t e^{-A_D(t-s)}\mathcal{M}^{-1}z_t|_{\Gamma_0}\,ds}_{-F_3(t)y^0}\Big|.
\end{aligned}
$$

The analysis of $\phi(t)y^0$ and $F_2(t)y^0$ in (3.14) requires essentially the same arguments as the ones used in [8, Section 5]. Indeed, it is rather straightforward to deduce the pointwise estimate

$$
(3.15)\qquad |F_2(t)y^0| \le \frac{c}{t^{1/4-\varepsilon}}\,|y^0|_Y
$$

near $t=0$. Instead, $\phi(t)y^0$ requires a further splitting:

$$
\begin{aligned}
(3.16)\qquad \phi(t)y^0 = & \underbrace{D^*A_D\int_0^t e^{-A_D(t-s)}\mathcal{M}^{-1}A_D\,\Delta w(s)\,ds}_{F_1(t)y^0} \\
& \underbrace{-\,D^*A_D\int_0^t e^{-A_D(t-s)}\mathcal{M}^{-1}A_D\,D(\Delta w|_\Gamma)(s)\,ds}_{G(t)y^0},
\end{aligned}
$$

where the first summand in (3.16) readily satisfies

$$
(3.17)\qquad |F_1(t)y^0| \le c\int_0^t \frac{1}{(t-s)^{3/4+\varepsilon}}\,|w(s)|_{H^2(\Gamma_0)}\,ds \le Ct^{1/4-\varepsilon}|y^0|_Y.
$$

To pinpoint the regularity of the convolution $G(t)y^0$ in (3.16) we follow the lines of [8, Section 5, Step 4]. Again, combining the classical analytic estimates with the sharp trace estimate (3.5) established for the elastic component in Subsection 3.1 and using the Young inequality, we obtain

$$
(3.18)\qquad
\begin{aligned}
& G(\cdot)y^0 \in L_p(0,T;L_2(\partial\Gamma_0)) \quad \forall p\in[1,\infty); \\
& |G(\cdot)y^0|_{L_p(0,T;L_2(\partial\Gamma_0))} \le C|y^0|,
\end{aligned}
$$

exactly as in the case of the uncoupled thermoelastic system; see [8, Section 5, Step 4] for details. Finally, the third summand $F_3(t)y^0$ in (3.14) satisfies

$$(3.19)\quad |F_3(t)y^0| = \Big|D^*\int_0^t e^{-A_D(t-s)}[A_D\mathcal{M}^{-1}]z_t|_{\Gamma_0}(s)\,ds\Big|$$
$$\le C\int_0^t |z_t(s)|_{0,\Gamma_0}\,ds \le C\sqrt{t}\Big(\int_0^t |z_t(s)|^2_{0,\Gamma_0}\,ds\Big)^{1/2}$$
$$\le C\sqrt{t}\Big[|z^0|+|z^1|+\Big(\int_0^t |w_t(s)|^2_{1,\Gamma_0}\,ds\Big)^{1/2}\Big] \le C_T|y^0|,$$

where we appealed to the trace regularity result (3.9) (pertaining to the wave component) recalled in Subsection 3.1.

Let us summarize the results obtained so far. In view of (3.12), (3.14) and (3.16) we have proved that

$$(3.20)\qquad \frac{\partial\theta(t)}{\partial\nu}\Big|_{\partial\Gamma_0} = F_0(t)y^0 - b(t;y^0),$$

with

$$(3.21)\quad F_0(t) = D^*A_De^{-A_Dt}(\theta^0+w^1),\qquad b(t;y^0) = -\sum_{i=1}^3 F_i(t)y^0 - G(t)y^0,$$

where $F_i(t)$, $i=1,2,3$, and $G(t)$ are introduced in (3.14) and (3.16). Consequently, by (3.20) and (3.21) and in view of the estimates established for each F_i (i.e. (3.13), (3.17), (3.15), (3.19)), we see that the sought-after decomposition (2.21) holds true with $F(t)y^0 := \sum_{i=0}^3 F_i(t)y^0$, along with the (singular) estimate claimed in (i). Therefore, Hypothesis 2.1(i) is satisfied, with $\gamma = 3/4+\sigma$. Moreover, notice that (3.18) is nothing but condition (2.22), or Hypothesis 2.1(ii).

Now, to prove the additional regularity property (2.23) of G we may proceed as in the conclusion of [8, Section 5]. We just observe that a key step in showing the above regularity is establishing the membership $\Delta w|_{\partial\Gamma_0} \in L_{2/(1-2\varepsilon)}(0,T;L_2(\partial\Gamma_0))$ when $y^0\in\mathcal{D}(A^{*\varepsilon})$, $0<\varepsilon<1/2$. This is achieved by means of interpolation, in view of (3.5) and (3.6); the reader is referred to [8, Section 5] for more details.

2. *(Step 0) Introduction.* Let $[z(t),w(t),\theta(t)]$ be the solution of system (2.17)—equivalently, (2.16)—with initial datum y^0, which is initially assumed to belong to $\mathcal{D}(A^*)$. This enables us to justify the foregoing computations. According to (2.16c), θ_t satisfies the evolution equation $\theta_{tt}+A_D\theta_t+A_Dw_{tt}=0$, and therefore is given by

$$(3.22)\qquad \theta_t(t) = e^{-A_Dt}\theta_t(0) - \int_0^t e^{-A_D(t-s)}A_Dw_{tt}(s)\,ds.$$

Owing to (2.6), to compute the outer normal derivative of θ_t on $\partial\Gamma_0$ we apply D^*A_D on both sides of (3.22). Then, using (2.16b) to rewrite w_{tt}, we compute

$$
\begin{aligned}
(3.23)\qquad \frac{\partial}{\partial\nu}\theta_t(t) &= D^*A_D\, e^{-A_D t}[\theta_t(0)+w_{tt}(0)] \\
&\quad + D^*A_D \int_0^t e^{-A_D(t-s)}\mathcal{M}^{-1}A_D\,[\theta_t(s)+\Delta w_t(s)]\,ds \\
&\quad - D^*A_D \int_0^t e^{-A_D(t-s)}\mathcal{M}^{-1}A_D\,D(\Delta w_t(s)|_\Gamma)\,ds \\
&\quad - D^*A_D\, e^{-A_D t}\mathcal{M}^{-1}z_t(0)|_{\Gamma_0} + D^*A_D\,\mathcal{M}^{-1}z_t(t)|_{\Gamma_0} \\
&\quad - D^*A_D \int_0^t A_D\, e^{-A_D(t-s)}\mathcal{M}^{-1}z_t(s)|_{\Gamma_0}\,ds \\
&=: T_1(t)+T_2(t)+T_3(t)+T_4(t)+T_5(t)+T_6(t).
\end{aligned}
$$

Now an analysis of each term T_i is called for.

Step 1. To investigate the regularity of the terms $T_i(t)$, $i=1,2,3$, we can mimic the proof of Theorem 1.1 in [1]. This is accomplished in the present and the next steps. We first observe that the solution corresponding to $y^0 \in \mathcal{D}(A^{*1-\varepsilon})$ yields—by interpolation—the following interior regularity:

$$(3.24)\qquad w \in C([0,T]; H^{3-\varepsilon}(\Omega)\cap H_0^2(\Omega)),$$

$$(3.25)\qquad w_t \in C([0,T]; H_0^{2-\varepsilon}(\Omega)) \subset C([0,T]; \mathcal{D}(A_D^{1-\varepsilon/2})),$$

$$(3.26)\qquad \theta \in C([0,T]; H^{2-2\varepsilon}(\Omega)\cap H_0^1(\Omega)) \equiv C([0,T]; \mathcal{D}(A_D^{1-\varepsilon})).$$

Therefore, according to (3.25) and (3.26) we rewrite (2.16c) as

$$\theta_t = -A_D^{\varepsilon}(A_D^{1-\varepsilon}\theta) - A_D^{\varepsilon/2}(A_D^{1-\varepsilon/2}w_t),$$

and easily conclude that

$$(3.27a)\qquad \theta_t \in C([0,T]; [\mathcal{D}(A_D^{\varepsilon})]'),$$

$$(3.27b)\qquad \|\theta_t\|_{C([0,T];[\mathcal{D}(A_D^{\varepsilon})]')} \le C\|z\|_{D(A^{*1-\varepsilon})}, \quad 0<\varepsilon<1/2.$$

We next claim

$$(3.28a)\qquad w_{tt} \in C([0,T]; H^{\varepsilon}(\Omega)) \subseteq C([0,T]; \mathcal{D}(A_D^{\varepsilon/2})),$$

$$(3.28b)\qquad \|w_{tt}\|_{C([0,T];H^{\varepsilon}(\Omega))} \le C\|z\|_{D(A^{*1-\varepsilon})}, \quad 0<\varepsilon<1/2.$$

To show this, following [1, Lemma 3.2] we multiply the equation for w ((2.16b)) by $\varphi \in L^2(\partial\Gamma_0)$ and integrate by parts, using Green's formulas.

Thus, as $\mathcal{M}^{-1}\varphi \equiv 0$ on $\partial\Gamma_0$, we find

$$(3.29)\quad (w_{tt}(t), \varphi)_{0,\Gamma_0} = \left(\Delta w(t), \frac{\partial}{\partial\nu}\mathcal{M}^{-1}\varphi\right)_{0,\partial\Gamma_0} - (\Delta w(t), \Delta\mathcal{M}^{-1}\varphi)_{0,\Gamma_0} + (\nabla\theta(t), \nabla\mathcal{M}^{-1}\varphi)_{0,\Gamma_0} + (z_t, \mathcal{M}^{-1}\phi)_{0,\Gamma_0}.$$

Notice that since $y^0 \in \mathcal{D}(A^{*1-\varepsilon})$, by interpolation we know that $z_t \in H^{1-\varepsilon}(\Omega)$ so that $z_t|_{\Gamma_0} \in H^{1/2-\varepsilon}(\Gamma_0)$ $(0 < \varepsilon < 1/2)$. Utilizing the memberships (3.24) and (3.26) we see that $w_{tt}(t)$ can be actually extended from $L^2(\Gamma_0)$ to the dual space $[H^{-\varepsilon}(\Gamma_0)]'$, that is, $H^{\varepsilon}(\Omega)$. This confirms the regularity (3.28a), which holds continuously with respect to initial data, i.e. the bound (3.28b) follows as well. Thus, the basic regularity of (3.25) and (3.27), combined with the more subtle one (3.28), enables us to establish the following result; see [1, Proposition 3.3] for details.

LEMMA 3.3. *The terms* T_i, $i = 1, 2$, *in* (3.23) *satisfy the following estimates* (*the first for arbitrarily small* $\delta > 0$):

$$\|T_1(t)\|_{0,\partial\Gamma_0} \le \frac{C}{t^{3/4+\varepsilon+\delta}}\|z\|_{D(A^{*1-\varepsilon})},$$

so that

$$(3.30)\qquad \forall\varepsilon < 1/4,\ \exists q \in (1,2) : \|T_1\|_{L^q(0,T;L^2(\partial\Gamma_0))} \le C_\varepsilon\,\|z\|_{D(A^{*1-\varepsilon})},$$

and

$$(3.31)\qquad \|T_2\|_{L^\infty(0,T;L^2(\partial\Gamma_0))} \le C_\varepsilon\,\|z\|_{D(A^{*1-\varepsilon})}\quad \forall\varepsilon \in (0, 1/4).$$

In particular, *given* $\varepsilon \in (0, 1/4)$, *the regularity in* (3.30) *is valid with any exponent* q *such that*

$$1 < q < \frac{4}{3+4\varepsilon}.$$

Step 2. The regularity of T_3 depends critically upon the exceptional boundary regularity of the elastic component established in Proposition 3.2. Indeed, owing to the assertions (3.5) and (3.6), we can repeat the arguments used in the tricky step 4 of the proof of [1, Theorem 1.1] to achieve the following.

LEMMA 3.4. *Assume* $\varepsilon \in (0, 1/4)$, *and let* $z \in \mathcal{D}(A^{*1-\varepsilon})$. *Then* $T_3 \in L^q(0,T;L^2(\Gamma))$ *for all* $q \in [1, 8/7)$, *with* q *independent of* ε, *and*

$$(3.32)\qquad \|T_3\|_{L^q(0,T;L^2(\partial\Gamma_0))} \le C\,\|z\|_{D(A^{*1-\varepsilon})}.$$

Step 3. To deal with the terms T_i, $i = 4, 5, 6$, we observe once again that $y^0 \in \mathcal{D}(A^{*1-\varepsilon})$ yields $z_t \in C([0,T]; H^{1-\varepsilon}(\Omega))$, so that by trace theory $z_t|_{\Gamma_0} \in C([0,T]; H^{1/2-\varepsilon}(\Gamma_0))$ for all $0 < \varepsilon < 1/2$. Consequently, we readily have

$$(3.33)\qquad \|T_i\|_{L^\infty(0,T;L^2(\partial\Gamma_0))} \le C\,\|z\|_{D(A^{*1-\varepsilon})},\qquad i = 4, 5.$$

Finally, by rewriting $T_6(t)$ as

(3.34) $T_6(t) =$

$$-(D^* A_D^{1/4-\delta}) \int_0^t A_D^{(1+\varepsilon)/2+\delta} e^{-A_D(t-s)} (A_D \mathcal{M}^{-1}) (A_D^{1/4-\varepsilon/2} z_t(s)|_{\Gamma_0})\, ds,$$

it is not difficult to obtain the pointwise estimate

$$\|T_6(t)\|_{0,\partial\Gamma_0} \le C \int_0^t \frac{1}{(t-s)^{(1+\varepsilon)/2+\delta}}\, ds\, \|A^{*1-\varepsilon} y^0\|,$$

which implies

$$\|T_6\|_{L^\infty(0,T;L^2(\partial\Gamma_0))} \le C\, \|y^0\|_{D(A^{*1-\varepsilon})}, \tag{3.35}$$

with no constraints on ε (except for $0 < \varepsilon < 1$), since δ can be chosen arbitrarily small. We sum the estimates obtained in Lemmas 3.3 and 3.4, along with the inequalities (3.33) and (3.35), to find the conclusion (2.24), with q as in (2.25). ∎

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