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Fuyi Xu (Zibo and Beijing)
Jia Yuan (Beijing)

GLOBAL MILD SOLUTIONS OF THE MICROPOLAR FLUID SYSTEM IN CRITICAL SPACES

Abstract. We prove the global in time existence of a small solution for the 3D micropolar fluid system in critical Fourier–Herz spaces by using the Fourier localization method and Littlewood–Paley theory.

1. Introduction. We consider the incompressible micropolar fluid system in $\mathbb{R}^+ \times \mathbb{R}^3$:

$$\begin{cases} \partial_t u - (\chi+\nu)\Delta u + u\cdot\nabla u + \nabla\pi - 2\chi\nabla\times\omega = 0, \\ \partial_t \omega - \mu\Delta\omega + u\cdot\nabla\omega + 4\chi\omega - \kappa\nabla\operatorname{div}\omega - 2\chi\nabla\times u = 0, \\ \operatorname{div} u = 0, \\ (u,\omega)|_{t=0} = (u_0,\omega_0). \end{cases} \tag{1.1}$$

Here $u(t,x)$ and $\omega(t,x)$ denote the linear velocity and the angular velocity field of the fluid respectively. The scalar $\pi(t,x)$ denotes the pressure of the fluid. The constants κ,χ,ν,μ are the viscosity coefficients. Here, for simplicity of exposition, we take $\chi=\nu=1/2$ and $\kappa=\mu=1$.

The modern theory of micropolar fluid dynamics began over 40 years ago, when Eringen [8] published his pioneering works on the micropolar fluid motion equations, a model which accounts for micro-rotation effects and micro-rotation inertia in a fluid motion system. In a physical sense, micropolar fluids represent fluids consisting of barlike elements. For example, some polymeric fluids and fluids containing certain additives in narrow films may be represented by a sort of fluid model as in (1.1) ([8, Secs. 1 and 6]). Moreover, experiments with fluids containing extremely small amounts of

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polymeric additives indicate that the skin friction near a rigid body in such fluids is considerably lower than that of the same fluids without additives (cf. [20]). In other words, micropolar fluid systems can describe phenomena which appear in a large number of complex fluids such as suspensions, animal blood, liquid crystals, which cannot be appropriately characterized by the Navier–Stokes system. For more background, we refer the reader to [18] and the references therein.

Due to their importance in mathematics and physics, there are a large number of works on the mathematical theory of micropolar fluid equations (1.1) (see, for example, [1, 8, 10, 12, 23]). First of all, we refer to [8] for the uniqueness and existence of local smooth solutions. The existence and uniqueness of global solutions were extensively studied by Lange [16], Yamaguchi [23], and Chen–Miao [6]. Recently, Ferreira and Villamizar-Roa [9] considered the existence and stability of solutions to micropolar fluid equations in exterior domains. Villamizar-Roa and Rodríguez-Bellido [21] studied the micropolar system in a bounded domain using the semigroup approach in L^p spaces, showing the global existence of strong solutions for small data as well as the asymptotic behavior and stability of solutions. Blow-up criteria for smooth solutions and regularity criteria for weak solutions can be found in [25, 24, 22] and in the references therein.

If the microstructure of the fluid is not taken into account, that is, the effect of the angular velocity fields of the particle rotation is omitted, i.e., $\omega = 0$, then equation (1.1) reduces to the classical Navier–Stokes equations

$$\begin{cases} \partial_t u - \nu \Delta u + u \cdot \nabla u + \nabla \pi = 0, \\ \operatorname{div} u = 0, \\ u(x,0) = u_0. \end{cases} \tag{1.2}$$

Fujita and Kato [11, 14] proved the local well-posedness for large initial data and the global well-posedness of this problem for small initial data in the homogeneous Sobolev space $\dot{H}^{1/2}$ and the Lebesgue space L^3 respectively. These spaces are critical ones, which is associated to the scaling of the Navier–Stokes equations: if (u, p) solves (1.2), then

$$(u_\lambda(t,x), p_\lambda(t,x)) := (\lambda u(\lambda^2 t, \lambda x), \lambda^2 p(\lambda^2 t, \lambda x)) \tag{1.3}$$

is also a solution of (1.2). The critical space is the one such that the associated norm is invariant under the scaling of (1.3). Recently, Lei and Lin [17] proved an interesting global well-posedness result for the initial data in the space $\mathcal{X}^{-1} := \{u \in \mathcal{D}'(\mathbb{R}^3);\, \int_{\mathbb{R}^3} |\xi|^{-1} |\hat{u}(\xi)|\, d\xi < \infty\}$. Cannone and Wu [5] improved that result by proving the global well-posedness for the Navier–Stokes equations in a family of critical Fourier–Herz spaces, which are larger than the space $\mathcal{X}^{-1}$ introduced by Lei and Lin. The Fourier–Herz spaces have also been used by Iwabuchi [13] to study the Keller–Segel system, and by Konieczny and Yoneda [15] to construct the stationary solution of the Navier–Stokes equations.

Since the micropolar fluid equations (1.1) have the same nonlinear convection structure as the incompressible Navier–Stokes equations, it is natural to expect analogous results on problem (1.1) in Fourier–Herz spaces. The purpose of this paper is to follow this way and to prove the global well-posedness for the three-dimensional micropolar fluid equations in critical Fourier–Herz spaces by means of Littlewood–Paley decomposition and Bony's paradifferential calculus.

2. Main results. The proofs of the results presented in this paper are based on a dyadic partition of unity in the Fourier variables, called homogeneous Littlewood–Paley decomposition. We briefly recall this construction below.

Let φ be a smooth function with values in $[0,1]$, supported in the shell $\{\xi \in \mathbb{R}^3;\, 3/4 \le |\xi| \le 8/3\}$ and satisfying

$$\sum_{j\in\mathbb{Z}} \varphi(2^{-j}\xi) = 1, \qquad \forall \xi \in \mathbb{R}^3 \setminus \{0\},$$

and denote $\varphi_j(\xi) = \varphi(2^{-j}\xi)$ and $h_j = \mathcal{F}^{-1}\varphi_j$, with $\mathcal{F}^{-1}$ the inverse Fourier transform.

Definition 2.1. For $s \in \mathbb{R}$, $1 \le q \le \infty$ and $u \in \mathcal{S}'$, we set

$$\|u\|_{\dot{\mathcal{B}}^s_q} := \Big(\sum_{j\in\mathbb{Z}} 2^{jsq}\|\varphi_j \hat{u}\|^q_{L^1}\Big)^{1/q} \qquad \text{if } q < \infty$$

and $\|u\|_{\dot{\mathcal{B}}^s_\infty} := \sup_{j\in\mathbb{Z}} 2^{js}\|\varphi_j\hat{u}\|_{L^1}$. Here $\hat{u}$ denotes the Fourier transform of u. Then, we define the *homogeneous Fourier–Herz space* as $\dot{\mathcal{B}}^s_q := \{u \in \mathcal{S}'/\mathcal{P};\, \|u\|_{\dot{\mathcal{B}}^s_q} < \infty\}$.

Remark 2.2. Using Hausdorff–Young's inequality, we immediately get the estimate

$$\|u\|_{\dot{B}^s_{\infty,q}} \le C\|u\|_{\dot{\mathcal{B}}^s_q},$$

where $\dot{B}^s_{\infty,q}$ is the usual homogeneous Besov space.

For mixed space-time spaces, we have the following definition.

Definition 2.3. Let $s \in \mathbb{R}$, $1 \le q, \rho \le \infty$ and $I = [0,T)$, with $T \in (0,\infty]$. We set

$$\|u(x,t)\|_{\mathcal{L}^\rho(I;\dot{\mathcal{B}}^s_q)} := \Big(\sum_{j\in\mathbb{Z}} 2^{jsq}\|\varphi_j\hat{u}\|^q_{L^\rho(I;L^1)}\Big)^{1/q},$$

and denote by $\mathcal{L}^\rho(I;\dot{\mathcal{B}}^s_q)$ the set of distributions in $\mathcal{S}'(I\times\mathbb{R}^3)/\mathcal{P}$ with finite $\|\cdot\|_{\mathcal{L}^\rho(I;\dot{\mathcal{B}}^s_q)}$ norm.

The following theorem is our main result.

THEOREM 2.1. *Let* $q\in[1,2]$, $(v_0,\omega_0)\in\dot{\mathcal{B}}^{-1}_q(\mathbb{R}^3)$, *and* $\operatorname{div} v_0=0$. *There exists a constant* c_q *depending only on* q *such that if*

$$\|(v_0,\omega_0)\|_{\dot{\mathcal{B}}^{-1}_q}<c_q, \tag{2.1}$$

then (1.1) *has a unique global solution* $(v,\omega)\in\mathcal{C}([0,\infty);\dot{\mathcal{B}}^{-1}_q)\cap\mathcal{L}^1(0,\infty;\dot{\mathcal{B}}^1_q)$ *satisfying*

$$\|(v(t),\omega(t))\|_{\mathcal{L}^\infty(0,\infty;\dot{\mathcal{B}}^{-1}_q)\cap\mathcal{L}^1(0,\infty;\dot{\mathcal{B}}^1_q)}\le C\|(v_0,\omega_0)\|_{\dot{\mathcal{B}}^{-1}_q}. \tag{2.2}$$

3. The linearized system and mild solutions. In this section, we consider the linearized system corresponding to (1.1) and we define mild solutions for (1.1).

Throughout this paper, for a matrix $M=(M_{ij})_{1\le i,j\le m}$ with elements from a Banach space X, we set

$$\|M\|_X:=\sum_{i,j}\|M_{ij}\|_X.$$

Let $y=(u,\omega)$. For $v\in\mathbb{R}^3$, we use the notation $v\otimes y=(v\otimes u,v\otimes\omega)$, where $\otimes$ denotes the tensor product in $\mathbb{R}^3$. Furthermore, we denote abusively $\mathbf{P}(v\otimes y)=(\mathbf{P}(v\otimes u),v\otimes\omega)$, where $\mathbf{P}$ is the Leray projection. Moreover, we define

$$\widehat{\mathbf{P}}(\widehat{v\otimes y})=(\mathbf{P}(\widehat{v\otimes u}),\widehat{v\otimes\omega})=(\widehat{\mathbf{P}}(\xi)(\widehat{v\otimes u}),\widehat{v\otimes\omega}).$$

Now, applying the Leray projection to the equations in (1.1), we obtain

$$\begin{cases}\partial_t u-\Delta u+\mathbf{P}(u\cdot\nabla u)-\nabla\times\omega=0,\\ \partial_t\omega-\Delta\omega+u\cdot\nabla\omega+2\omega-\nabla\operatorname{div}\omega-\nabla\times u=0,\\ \operatorname{div}u=0,\\ (u,\omega)|_{t=0}=(u_0,\omega_0).\end{cases} \tag{3.1}$$

Obviously, this system has no scaling property in contrast to the incompressible Navier–Stokes equations. In general there are two ways to achieve the global existence for small data in the critical space. The first one is Kato's semigroup approach extended by Cannone [4]; however, it turns out that the linear terms $\nabla\times\omega$ and $\nabla\times u$ do not allow one to obtain global in time solutions if they are regarded as perturbations. The second way is to use the energy method together with the Fourier localization technique, but the linear coupling effect of the system (3.1) is too strong to be controlled unless the coefficients of these two linear terms are sufficiently small, which is not a reasonable assumption.

To go around the troubles caused by the terms $\nabla \times \omega$ and $\nabla \times u$, we will view them as certain perturbations of the Laplace operator. More precisely, we will apply the idea developed in [7] for compressible Navier–Stokes equations, i.e., first, we investigate the following mixed linear system:

$$\begin{cases} \partial_t u - \Delta u - \nabla \times \omega = 0, \\ \partial_t \omega - \Delta \omega + 2\omega - \nabla \operatorname{div} \omega - \nabla \times u = 0, \end{cases} \tag{3.2}$$

and we study the properties of its Green matrix $G(x,t)$. From [10], we have

$$\widehat{Gf}(\xi,t) = e^{-A(\xi)t}\hat{f}(\xi), \tag{3.3}$$

where

$$A(\xi) = \begin{bmatrix} |\xi|^2 I & B(\xi) \\ B(\xi) & (|\xi|^2+2)I + C(\xi) \end{bmatrix}$$

with

$$B(\xi) = i\begin{bmatrix} 0 & -\xi_3 & \xi_2 \\ \xi_3 & 0 & -\xi_1 \\ -\xi_2 & \xi_1 & 0 \end{bmatrix} \quad \text{and} \quad C(\xi) = \begin{bmatrix} \xi_1^2 & \xi_1\xi_2 & \xi_1\xi_3 \\ \xi_1\xi_2 & \xi_2^2 & \xi_2\xi_3 \\ \xi_1\xi_3 & \xi_2\xi_3 & \xi_3^2 \end{bmatrix}.$$

It has been shown in [10] that $G(x,t)$ has properties similar to the heat kernel, namely,

$$|\widehat{G}(\xi,t)| \le e^{-c|\xi|^2 t}. \tag{3.4}$$

We are now in a position to introduce the notion of mild solutions for (3.1), with the help of Duhamel's principle.

Definition 3.1. A *mild solution* for (3.1) with initial data $y_0 = (u_0, \omega_0)$ with $\operatorname{div} u_0 = 0$ is a couple $y = (u, \omega)$ such that

$$y = G(x,t)y_0 - \int_0^t G(x,t-\tau)\nabla \cdot \mathbf{P}(u \otimes y)\, d\tau. \tag{3.5}$$

4. Proof of Theorem 2.1. Based on observations provided in Section 3, we are in a position to prove Theorem 2.1. First, we recall a classical result on the existence of fixed point solutions of equations with a bilinear continuous mapping. Its proof can be found e.g. in [3].

Theorem 4.1. *Let X be an abstract Banach space with norm $\|\cdot\|$ and $B : X \times X \to X$ a bilinear operator such that $\|B(x_1, x_2)\| \le \eta\|x_1\|\,\|x_2\|$ for any $x_1, x_2 \in X$. Then for any $y \in X$ such that $4\eta\|y\| < 1$ the equation $x = y + B(x,x)$ has a solution x in X. In particular, this solution satisfies $\|x\| \le 2\|y\|$ and it is the only one such that $\|x\| < 1/(2\eta)$.*

In order to use Theorem 4.1 in the proof of Theorem 2.1, we have to obtain the following linear and bilinear estimates.

Lemma 4.2. *There exists a constant C such that for every $y_0 = (u_0, \omega_0) \in \dot{\mathcal{B}}_q^{-1}(\mathbb{R}^3)$, we have*

$$\|G(\cdot,\cdot)y_0\|_{\mathcal{L}^\infty(I;\dot{\mathcal{B}}_q^{-1})\cap\mathcal{L}^1(I;\dot{\mathcal{B}}_q^1)} \le C\|y_0\|_{\dot{\mathcal{B}}_q^{-1}}. \tag{4.1}$$

Proof. Let us first observe that

$$\begin{aligned}\|G(x,t)y_0\|_{\mathcal{L}^\infty(I;\dot{\mathcal{B}}_q^{-1})} &= \Big\|2^{-j}\sup_{t\in I}\|\varphi_j\widehat{G(x,t)y_0}\|_{L^1}\Big\|_{l^q} \\ &= \Big\|2^{-j}\sup_{t\in I}\int|\varphi_j\widehat{G(x,t)y_0}|\,d\xi\Big\|_{l^q} = \Big\|2^{-j}\sup_{t\in I}\int|\varphi_j e^{-tA(\xi)}\widehat{y}_0|\,d\xi\Big\|_{l^q} \\ &\le \Big\|2^{-j}\sup_{t\in I}\int|e^{-ct|\xi|^2}\varphi_j\widehat{y}_0|\,d\xi\Big\|_{l^q} \le C\Big\|2^{-j}\int|\varphi_j\widehat{y}_0|\,d\xi\Big\|_{l^q} = C\|y_0\|_{\dot{\mathcal{B}}_q^{-1}},\end{aligned} \tag{4.2}$$

where we have used estimate (3.4).

Similarly, the second norm is estimated as follows:

$$\begin{aligned}\|G(x,t)y_0\|_{\mathcal{L}^1(I;\dot{\mathcal{B}}_q^1)} &= \|2^j\|\varphi_j\widehat{G(x,t)y_0}\|_{L^1(I;L^1)}\|_{l^q} \\ &= \Big\|2^j\iint_I|\varphi_j\widehat{G(x,t)y_0}|\,d\xi\,dt\Big\|_{l^q} = \Big\|2^j\iint_I|\varphi_j e^{-tA(\xi)}\widehat{y}_0|\,d\xi\,dt\Big\|_{l^q} \\ &\le \Big\|2^j\int_I e^{-ct2^{2j}}\,dt\int|\varphi_j\widehat{y}_0|\,d\xi\Big\|_{l^q} \le C\Big\|2^{-j}\int|\varphi_j\widehat{y}_0|\,d\xi\Big\|_{l^q} = C\|y_0\|_{\dot{\mathcal{B}}_q^{-1}}.\end{aligned} \tag{4.3}$$

From (4.2) and (4.3) we obtain (4.1).

To estimate the nonlinear term, we first recall the following bilinear estimate from [5].

Lemma 4.3. *Let $q \in [1,2]$ and*

$$X = \mathcal{L}^\infty(I;\dot{\mathcal{B}}_q^{-1}) \cap \mathcal{L}^1(I;\dot{\mathcal{B}}_q^1)$$

with the norm

$$\|u\|_X = \|u\|_{\mathcal{L}^\infty(I;\dot{\mathcal{B}}_q^{-1})} + \|u\|_{\mathcal{L}^1(I;\dot{\mathcal{B}}_q^1)}.$$

We have the following bilinear estimate:

$$\|\nabla\cdot(uv)\|_{\mathcal{L}^1(I;\dot{\mathcal{B}}_q^{-1})} \le C\|u\|_X\|v\|_X, \tag{4.4}$$

where C is a constant depending only on q.

Next we show the continuity of the bilinear form $B(y_1, y_2)$ with $y_1 = (u_1, \omega_1)$, $y_2 = (u_2, \omega_2)$ in the mild solution formulation (3.5), defined by

$$B(y_1,y_2) = -\int_0^t G(x,t-\tau)\nabla\cdot\mathbf{P}(u_1\otimes y_2)\,d\tau.$$

Lemma 4.4. *Let $X = \mathcal{L}^\infty(I;\dot{\mathcal{B}}_q^{-1}) \cap \mathcal{L}^1(I;\dot{\mathcal{B}}_q^1)$. There exists a constant η such that for every $y_1 = (u_1,\omega_1), y_2 = (u_2,\omega_2) \in X$ we have*

$$\|B(y_1,y_2)\|_X \le \eta\|y_1\|_X\|y_2\|_X. \tag{4.5}$$

Proof. Let us first estimate the norm in $\mathcal{L}^\infty(I;\dot{\mathcal{B}}_q^{-1})$. Using Young's inequality, (3.4) and (4.4), we obtain

$$
\begin{aligned}
(4.6)\qquad \|B(y_1,y_2)\|_{\mathcal{L}^\infty(I;\dot{\mathcal{B}}_q^{-1})} &= \Big\|2^{-j}\sup_{t\in I}\|\varphi_j\widehat{B(y_1,y_2)}\|_{L^1}\Big\|_{l^q}\\
&= \Big\|2^{-j}\sup_{t\in I}\int|\varphi_j\widehat{B(y_1,y_2)}|\,d\xi\Big\|_{l^q}\\
&= \Big\|2^{-j}\sup_{t\in I}\int\Big|\varphi_j\int_0^t e^{-(t-\tau)A(\xi)}\,\nabla\cdot\widehat{\mathbf{P}(u_1\otimes y_2)}\,d\tau\Big|\,d\xi\Big\|_{l^q}\\
&\le \Big\|2^{-j}\sup_{t\in I}\int_0^t\!\!\int e^{-c(t-\tau)|\xi|^2}|\varphi_j\,\nabla\cdot\widehat{\mathbf{P}(u_1\otimes y_2)}|\,d\xi\,d\tau\Big\|_{l^q}\\
&\le \Big\|2^{-j}\sup_{t\in I}\int_0^t e^{-c(t-\tau)2^{2j}}\int|\varphi_j\,\nabla\cdot\widehat{\mathbf{P}(u_1\otimes y_2)}|\,d\xi\,d\tau\Big\|_{l^q}\\
&\le c\big\|2^{-j}\|\varphi_j\,\nabla\cdot\widehat{\mathbf{P}(u_1\otimes y_2)}\|_{L^1(I;L^1)}\big\|_{l^q}\\
&= c\|\nabla\cdot\mathbf{P}(u_1\otimes y_2)\|_{\mathcal{L}^1(I;\dot{\mathcal{B}}_q^{-1})}\le K\|y_1\|_X\|y_2\|_X.
\end{aligned}
$$

The second part of the norm in X is estimated as follows

$$
\begin{aligned}
(4.7)\qquad \|B(y_1,y_2)\|_{\mathcal{L}^1(I;\dot{\mathcal{B}}_q^{1})} &= \big\|2^{j}\|\varphi_j\widehat{B(y_1,y_2)}\|_{L^1(I;L^1)}\big\|_{l^q}\\
&= \Big\|2^{j}\int_I\!\!\int|\varphi_j\widehat{B(y_1,y_2)}|\,d\xi dt\Big\|_{l^q}\\
&= \Big\|2^{j}\int_I\!\!\int\Big|\varphi_j\int_0^t e^{-(t-\tau)A(\xi)}\,\nabla\cdot\widehat{\mathbf{P}(u_1\otimes y_2)}\Big|\,d\tau\,d\xi\,dt\Big\|_{l^q}\\
&\le \Big\|2^{j}\int_I\int_0^t\!\!\int e^{-c(t-\tau)|\xi|^2}|\varphi_j\,\nabla\cdot\widehat{\mathbf{P}(u_1\otimes y_2)}|\,d\xi\,d\tau\,dt\Big\|_{l^q}\\
&\le \Big\|2^{j}\int_I\int_0^t e^{-c(t-\tau)2^{2j}}\int|\varphi_j\,\nabla\cdot\widehat{\mathbf{P}(u_1\otimes y_2)}|\,d\xi\,d\tau\,dt\Big\|_{l^q}\\
&\le c\big\|2^{-j}\|\varphi_j\,\nabla\cdot\widehat{\mathbf{P}(u_1\otimes y_2)}\|_{L^1(I;L^1)}\big\|_{l^q}\\
&= c\|\nabla\cdot\mathbf{P}(u_1\otimes y_2)\|_{\mathcal{L}^1(I;\dot{\mathcal{B}}_q^{-1})}\\
&\le K\|y_1\|_X\|y_2\|_X.
\end{aligned}
$$

Summing up, from (4.6) and (4.7) we obtain (4.5).

Proof of Theorem 2.1. It suffices to apply to equation (3.5) Lemmas 4.2 and 4.4 combined with Theorem 4.1. The proof of the continuity with respect to time is standard and omitted here.

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Fuyi Xu
School of Science
Shandong University of Technology
Zibo 255049, China
and
School of Mathematics and Systems Science
Beihang University
Beijing 100191, China
E-mail: zbxufuyi@163.com

Jia Yuan
School of Mathematics and Systems Science
Beihang University
Beijing 100191, China

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