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Canonical tensor fields of type $(p, 0)$ on Weil bundles

by Jacek Dębecki (Kraków)

Abstract. We give a classification of canonical tensor fields of type $(p, 0)$ on an arbitrary Weil bundle over n-dimensional manifolds under the condition that $n \geq p$. Roughly speaking, the result we obtain says that each such canonical tensor field is a sum of tensor products of canonical vector fields on the Weil bundle.

Let A be a Weil algebra and T^A the Weil functor corresponding to A, which is a product preserving bundle functor (see [2]). Fix non-negative integers n and p. A *canonical tensor field of type* $(p, 0)$ *on* T^A is, by definition, a family of tensor fields V_M of type $(p, 0)$ on $T^A M$ indexed by n-dimensional manifolds and satisfying for all such manifolds M, N and every embedding $f : M \to N$ the condition

$$\bigotimes^p TT^A f \circ V_M = V_N \circ T^A f. \tag{1}$$

Recall that a *derivation* of the algebra A is a linear map $D : A \to A$ such that $D(ab) = aD(b) + bD(a)$ for all $a, b \in A$. The vector space of derivations of A will be denoted by $\operatorname{Der} A$. It is well known (see [1]) that if $n \geq 1$, then there is a one-to-one correspondence between the canonical tensor fields of type $(1, 0)$ (in other words, canonical vector fields) on T^A and the derivations of A. Namely, every $D \in \operatorname{Der} A$ induces a unique canonical vector field $\widetilde{D}$ on T^A such that

$$\widetilde{D}_{\mathbb{R}^n}(X) = (D(X^1), \ldots, D(X^n)) \tag{2}$$

for every $X \in A^n$, and conversely, for every canonical vector field V on T^A there is a unique $D \in \operatorname{Der} A$ such that $V = \widetilde{D}$. Our purpose is to generalize this result to all p.

Consider the tensor product $\bigotimes^p A$ of the vector spaces A. For every $r \in \{1, \ldots, p\}$ and every $a \in A$ we have the linear map $Z_a^r : \bigotimes^p A \to \bigotimes^p A$ such that

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$$Z^r_a(b_1 \otimes \cdots \otimes b_p) = b_1 \otimes \cdots \otimes b_{r-1} \otimes ab_r \otimes b_{r+1} \otimes \cdots \otimes b_p$$

for all $b_1, \ldots, b_p \in A$.

Definition. Let $\operatorname{Der}^p A$ denote the vector space of p-linear maps $D : A \times \cdots \times A \to \bigotimes^p A$ with the property that

$$(3)\qquad D(a_1, \ldots, a_{r-1}, bc, a_{r+1}, \ldots, a_p) = Z^r_b(D(a_1, \ldots, a_{r-1}, c, a_{r+1}, \ldots, a_p)) + Z^r_c(D(a_1, \ldots, a_{r-1}, b, a_{r+1}, \ldots, a_p))$$

for every $r \in \{1, \ldots, p\}$ and all $a_1, \ldots, a_{r-1}, a_{r+1}, \ldots, a_p, b, c \in A$.

Note that $\operatorname{Der}^0 A = \mathbb{R}$ and $\operatorname{Der}^1 A = \operatorname{Der} A$.

Consider the tensor product $\bigotimes^p \operatorname{Der} A$ of the vector spaces $\operatorname{Der} A$. We have the linear map $I^p : \bigotimes^p \operatorname{Der} A \to \operatorname{Der}^p A$ such that

$$I^p(D_1 \otimes \cdots \otimes D_p)(a_1, \ldots, a_p) = D_1(a_1) \otimes \cdots \otimes D_p(a_p)$$

for all $D_1, \ldots, D_p \in \operatorname{Der} A$ and $a_1, \ldots, a_p \in A$.

Lemma. $I^p : \bigotimes^p \operatorname{Der} A \to \operatorname{Der}^p A$ *is an isomorphism of vector spaces.*

Proof. The proof is by induction on p. For $p = 0$ and $p = 1$ there is nothing to prove. Suppose $p \geq 2$ and the assertion of the Lemma is true for $p - 1$. Let $D \in \operatorname{Der}^p A$.

Fix a basis $\varepsilon_1, \ldots, \varepsilon_a$ of the vector space A. For any $a_1, \ldots, a_{p-1}, b \in A$ there are unique $E^i_b(a_1, \ldots, a_{p-1}) \in \bigotimes^{p-1} A$ indexed by $i \in \{1, \ldots, a\}$ such that

$$D(a_1, \ldots, a_{p-1}, b) = \sum_{i=1}^{a} E^i_b(a_1, \ldots, a_{p-1}) \otimes \varepsilon_i.$$

From the uniqueness of $E^i_b(a_1, \ldots, a_{p-1})$ we easily deduce that $E^i_b \in \operatorname{Der}^{p-1} A$ for every $i \in \{1, \ldots, a\}$ and every $b \in A$.

Fix a basis $\partial_1, \ldots, \partial_d$ of the vector space $\operatorname{Der} A$. By assumption, I^{p-1} is an isomorphism, and so $I^{p-1}(\partial_{l_1} \otimes \cdots \otimes \partial_{l_{p-1}})$ for $l_1, \ldots, l_{p-1} \in \{1, \ldots, d\}$ form a basis of the vector space $\operatorname{Der}^{p-1} A$. Therefore for every $i \in \{1, \ldots, a\}$ and every $b \in A$ there are unique $F^{l_1 \ldots l_{p-1} i}(b) \in \mathbb{R}$ indexed by $l_1, \ldots, l_{p-1} \in \{1, \ldots, d\}$ such that

$$E^i_b = \sum_{l_1=1}^{d} \cdots \sum_{l_{p-1}=1}^{d} F^{l_1 \ldots l_{p-1} i}(b) I^{p-1}(\partial_{l_1} \otimes \cdots \otimes \partial_{l_{p-1}}),$$

which is equivalent to

$$E^i_b(a_1, \ldots, a_{p-1}) = \sum_{l_1=1}^{d} \cdots \sum_{l_{p-1}=1}^{d} F^{l_1 \ldots l_{p-1} i}(b) \partial_{l_1}(a_1) \otimes \cdots \otimes \partial_{l_{p-1}}(a_{p-1})$$

for all $a_1, \ldots, a_{p-1} \in A$.

Taking, for every $b \in A$,

$$G^{l_1 \dots l_{p-1}}(b) = \sum_{i=1}^{a} F^{l_1 \dots l_{p-1} i}(b) \varepsilon_i,$$

we get a family of elements of A indexed by $l_1, \dots, l_{p-1} \in \{1, \dots, d\}$ such that

$$D(a_1, \dots, a_{p-1}, b) = \sum_{l_1=1}^{d} \dots \sum_{l_{p-1}=1}^{d} \partial_{l_1}(a_1) \otimes \dots \otimes \partial_{l_{p-1}}(a_{p-1}) \otimes G^{l_1 \dots l_{p-1}}(b)$$

for all $a_1, \dots, a_{p-1} \in A$. Moreover, $G^{l_1, \dots, l_{p-1}}(b)$ in this formula are uniquely determined, because of the uniqueness of $E^i_b(a_1, \dots, a_{p-1})$ and $F^{l_1 \dots l_{p-1} i}(b)$. From the uniqueness of $G^{l_1 \dots l_{p-1}}(b)$ we easily deduce that $G^{l_1 \dots l_{p-1}} \in \operatorname{Der} A$ for all $l_1, \dots, l_{p-1} \in \{1, \dots, d\}$.

For any $l_1, \dots, l_{p-1} \in \{1, \dots, d\}$ there are unique $H^{l_1 \dots l_{p-1} m} \in \mathbb{R}$ indexed by $m \in \{1, \dots, d\}$ such that

$$G^{l_1 \dots l_{p-1}} = \sum_{m=1}^{d} H^{l_1 \dots l_{p-1} m} \partial_m.$$

Hence

$$D(a_1, \dots, a_p) = \sum_{l_1=1}^{d} \dots \sum_{l_p=1}^{d} H^{l_1 \dots l_p} \partial_{l_1}(a_1) \otimes \dots \otimes \partial_{l_p}(a_p) \tag{4}$$

for all $a_1, \dots, a_p \in A$, which is equivalent to

$$D = \sum_{l_1=1}^{d} \dots \sum_{l_p=1}^{d} H^{l_1 \dots l_p} I^p(\partial_{l_1} \otimes \dots \otimes \partial_{l_p}).$$

Moreover, $H^{l_1 \dots l_p}$ in these formulas are uniquely determined, because of the uniqueness of $G^{l_1 \dots l_{p-1}}(b)$ and $H^{l_1 \dots l_{p-1} m}$ in the previous formulas. This means that $I^p(\partial_{l_1} \otimes \dots \otimes \partial_{l_p})$ for $l_1, \dots, l_p \in \{1, \dots, d\}$ form a basis of $\operatorname{Der}^p A$, which implies that I^p is an isomorphism. This completes the proof.

Let $e_1, \dots, e_n$ denote the standard basis of $\mathbb{R}^n$. We will identify A^n with $A \otimes \mathbb{R}^n$, and consequently $\bigotimes^p A^n$ with $\bigotimes^p A \otimes \bigotimes^p \mathbb{R}^n$. Hence (2) may be written as

$$\widetilde{D}_{\mathbb{R}^n}(X) = \sum_{s=1}^{n} D(X^s) \otimes e_s$$

for every $D \in \operatorname{Der} A$ and every $X \in A^n$.

We can now formulate our main result.

THEOREM. *If $n \geq p$, then there is a one-to-one correspondence between the canonical tensor fields of type $(p, 0)$ on T^A and the elements of* $\operatorname{Der}^p A$.

Namely, every $D \in \operatorname{Der}^p A$ *induces a unique canonical tensor field* $\widetilde{D}$ *of type* $(p,0)$ *on* T^A *such that*

$$\widetilde{D}_{\mathbb{R}^n}(X) = \sum_{s_1=1}^{n} \cdots \sum_{s_p=1}^{n} D(X^{s_1}, \ldots, X^{s_p}) \otimes e_{s_1} \otimes \cdots \otimes e_{s_p} \tag{5}$$

for every $X \in A^n$, *and conversely, for every canonical tensor field* V *of type* $(p,0)$ *on* T^A *there is a unique* $D \in \operatorname{Der}^p A$ *such that* $V = \widetilde{D}$.

Proof. Let $D \in \operatorname{Der}^p A$. Our first task is to construct $\widetilde{D}$. Since we can use any chart on an n-dimensional manifold as f in (1), we see that if V, W are two canonical tensor fields of type $(p,0)$ on T^A such that $V_{\mathbb{R}^n} = W_{\mathbb{R}^n}$, then $V = W$. Hence (5) guarantees the uniqueness of $\widetilde{D}$. Let $\partial_1, \ldots, \partial_d$ be a basis of the vector space $\operatorname{Der} A$. By the lemma, there are $H^{l_1 \ldots l_p} \in \mathbb{R}$ for $l_1, \ldots, l_p \in \{1, \ldots, d\}$ such that (4) holds. Put

$$\widetilde{D}_M = \sum_{l_1=1}^{d} \cdots \sum_{l_p=1}^{d} H^{l_1 \ldots l_p} \widetilde{\partial_{l_1}}_M \otimes \cdots \otimes \widetilde{\partial_{l_p}}_M$$

for every n-dimensional manifold M. It is evident that this $\widetilde{D}$ is a canonical tensor field of type $(p,0)$ on T^A. From (4) we conclude that it satisfies (5), and the construction of $\widetilde{D}$ is complete.

We now turn to the second part of the theorem. Fix a canonical tensor field V of type $(p,0)$ on T^A.

Since for every $X \in A^n$ we have $V_{\mathbb{R}^n}(X) \in \bigotimes^p A \otimes \bigotimes^p \mathbb{R}^n$, there are unique smooth $B^{s_1 \ldots s_p} : A^n \to \bigotimes^p A$ for $s_1, \ldots, s_p \in \{1, \ldots, n\}$ such that

$$V_{\mathbb{R}^n}(X) = \sum_{s_1=1}^{n} \cdots \sum_{s_p=1}^{n} B^{s_1 \ldots s_p}(X) \otimes e_{s_1} \otimes \cdots \otimes e_{s_p} \tag{6}$$

for every $X \in A^n$. Hence for every open subset U of $\mathbb{R}^n$ and every embedding $f : U \to \mathbb{R}^n$ we can write (1) in the form

$$\sum_{t_1=1}^{n} \cdots \sum_{t_p=1}^{n} \Big(Z^1_{T^A \frac{\partial f^{s_1}}{\partial x^{t_1}}(X)} \circ \cdots \circ Z^p_{T^A \frac{\partial f^{s_p}}{\partial x^{t_p}}(X)}\Big)(B^{t_1 \ldots t_p}(X)) = B^{s_1 \ldots s_p}(T^A f(X)) \tag{7}$$

for all $s_1, \ldots, s_p \in \{1, \ldots, n\}$ and every $X \in T^A U$.

From (7) with

$$f : \mathbb{R}^n \ni x \mapsto (\lambda^1 x^1, \ldots, \lambda^n x^n) \in \mathbb{R}^n,$$

where $\lambda^1, \ldots, \lambda^n \in \mathbb{R} \setminus \{0\}$, we have

$$\lambda^{s_1} \cdots \lambda^{s_p} B^{s_1 \ldots s_p}(X) = B^{s_1 \ldots s_p}(\lambda^1 X^1, \ldots, \lambda^n X^n)$$

for all $s_1, \dots, s_p \in \{1, \dots, n\}$ and every $X \in A^n$. By continuity, the same is true for all $\lambda^1, \dots, \lambda^n \in \mathbb{R}$. The homogeneous function theorem (see [2]) now shows that for any $s_1, \dots, s_p \in \{1, \dots, n\}$ there is a p-linear $C^{s_1 \dots s_p} : A \times \dots \times A \to \bigotimes^p A$ such that

$$B^{s_1 \dots s_p}(X) = C^{s_1 \dots s_p}(X^{s_1}, \dots, X^{s_p})$$

for every $X \in A^n$.

From now on we use the assumption $n \geq p$. Put

$$D = C^{1 \dots p}.$$

Let $r \in \{1, \dots, p\}$. From (7) with $s_1 = 1, \dots, s_p = p$, $U = \{x \in \mathbb{R}^n : x^r > 0\}$ and

$$f : U \ni x \mapsto (x^1, \dots, x^{r-1}, (x^r)^2, x^{r+1}, \dots, x^n) \in \mathbb{R}^n$$

we have

$$2Z^r_{X^r}(B^{1 \dots p}(X)) = B^{1 \dots p}(X^1, \dots, X^{r-1}, (X^r)^2, X^{r+1}, \dots, X^n)$$

for every $X \in T^A U$. This may be written as

$$(8) \quad 2Z^r_{X_r}(D(X^1, \dots, X^p)) = D(X^1, \dots, X^{r-1}, (X^r)^2, X^{r+1}, \dots, X^p).$$

In the same manner, with U replaced by $\{x \in \mathbb{R}^n : x^r < 0\}$, we can see that (8) also holds for $X \in T^A\{x \in \mathbb{R}^n : x^r < 0\}$, and so, by continuity, for every $X \in A^n$. Now the polarization of (8) with respect to X^r shows that D satisfies (3), and consequently $D \in \mathrm{Der}^p A$.

Our next goal is to show that

$$(9) \qquad C^{u_1 \dots u_p}(X^{u_1}, \dots, X^{u_p}) = D(X^{u_1}, \dots, X^{u_p})$$

for all $u_1, \dots, u_p \in \{1, \dots, n\}$ and every $X \in A^n$. We will identify any sequence $u_1, \dots, u_p \in \{1, \dots, n\}$ with $u : \{1, \dots, p\} \to \{1, \dots, n\}$ given by $u(1) = u_1, \dots, u(p) = u_p$ and denote by $N(u)$ the number of elements of the set $u(\{1, \dots, p\})$ for every such function u. The proof of (9) is by induction on $N(u)$. Fix $v : \{1, \dots, p\} \to \{1, \dots, n\}$ and suppose (9) holds whenever $N(u) \in \{N(v)+1, \dots, p\}$. Choose a subset R of $\{1, \dots, p\}$ such that for each $r \in v(\{1, \dots, p\})$ the set $v^{-1}(\{r\}) \cap R$ has one element. There is a bijective $w : \{1, \dots, n\} \to \{1, \dots, n\}$ such that $w|R = v|R$. Put

$$S_r = \begin{cases} \{w_r\} & \text{if } r \in R \cup \{p+1, \dots, n\}, \\ \{v_r, w_r\} & \text{if } r \in \{1, \dots, p\} \setminus R. \end{cases}$$

From (7) with $s_1 = 1, \dots, s_p = p$ and

$$f : \mathbb{R}^n \ni x \mapsto \Big(\sum_{u_1 \in S_1} x^{u_1}, \dots, \sum_{u_n \in S_n} x^{u_n} \Big) \in \mathbb{R}^n$$

we have

$$\sum_{u_1 \in S_1} \dots \sum_{u_p \in S_p} B^{u_1 \dots u_p}(X) = B^{1 \dots p}\Big(\sum_{u_1 \in S_1} X^{u_1}, \dots, \sum_{u_n \in S_n} X^{u_n} \Big)$$

for every $X \in A^n$. This may be written as

$$(10)\qquad \sum_{u_1\in S_1}\cdots\sum_{u_p\in S_p} C^{u_1\ldots u_p}(X^{u_1},\ldots,X^{u_p}) = \sum_{u_1\in S_1}\cdots\sum_{u_p\in S_p} D(X^{u_1},\ldots,X^{u_p}).$$

But if $u_1 \in S_1,\ldots,u_p \in S_p$ are such that there is $r \in \{1,\ldots,p\}\setminus R$ with the property that $u_r = w_r$, then $C^{u_1\ldots u_p}(X^{u_1},\ldots,X^{u_p}) = D(X^{u_1},\ldots,X^{u_p})$, on account of our assumption, because $N(u) \in \{N(v)+1,\ldots,p\}$ in this case. Subtracting all terms with such indices $u_1,\ldots,u_p$ from each side of (10) gives $C^{v_1\ldots v_p}(X^{v_1},\ldots,X^{v_p}) = D(X^{v_1},\ldots,X^{v_p})$, which is due to the fact that $w|R = v|R$. This completes the proof of (9).

Applying (9) we can rewrite (6) as

$$V_{\mathbb{R}^n}(X) = \sum_{s_1=1}^{n}\cdots\sum_{s_p=1}^{n} D(X^{s_1},\ldots,X^{s_p})\otimes e_{s_1}\otimes\cdots\otimes e_{s_p}$$

for every $X \in A^n$. Hence $V_{\mathbb{R}^n} = \widetilde{D}_{\mathbb{R}^n}$, which yields $V = \widetilde{D}$ on account of the above remark. The uniqueness of D is obvious. This completes the proof.

Combining the lemma with the theorem we obtain the following corollary.

COROLLARY. *If $n \geq p$, then every canonical tensor field of type $(p,0)$ on T^A is a sum of tensor products of canonical vector fields on T^A. More precisely, if $\partial_1,\ldots,\partial_d$ is a basis of the vector space* $\operatorname{Der} A$, *then for every canonical tensor field V of type $(p,0)$ on T^A there are unique $H^{l_1\ldots l_p} \in \mathbb{R}$ for $l_1,\ldots,l_p \in \{1,\ldots,d\}$ such that*

$$V_M = \sum_{l_1=1}^{d}\cdots\sum_{l_p=1}^{d} H^{l_1\ldots l_p}\widetilde{\partial_{l_1}}_M\otimes\cdots\otimes\widetilde{\partial_{l_p}}_M$$

for every n-dimensional manifold M.

The remainder of the paper is devoted to the following example.

EXAMPLE. Consider the Weil algebra $\mathbb{D}_k^r$ of r-jets at 0 of smooth functions $\mathbb{R}^k \to \mathbb{R}$, where r and k are non-negative integers.

We will denote by x^i for $i \in \{1,\ldots,k\}$ the r-jet at 0 of the ith projection $\mathbb{R}^k \to \mathbb{R}$ and write $x^\alpha = (x^1)^{\alpha^1}\cdots(x^k)^{\alpha^k}$ and $|\alpha| = \alpha^1+\cdots+\alpha^k$ for $\alpha \in \mathbb{N}^k$, where $\mathbb{N}$ stands for the set of non-negative integers. In addition, let $e_1,\ldots,e_k$ denote the standard basis of the module $\mathbb{Z}^k$.

Let $D \in \operatorname{Der}\mathbb{D}_k^r$. For every $\alpha \in \mathbb{N}^k$,

$$(11)\qquad D(x^\alpha) = \sum_{i\in\{l\in\{1,\ldots,k\}\,:\,\alpha^l>0\}} \alpha^i x^{\alpha-e_i}D(x^i),$$

as is easy to check by induction on $|\alpha|$. Of course, for every $i \in \{1,\dots,k\}$ there are unique $K^i_\alpha \in \mathbb{R}$ indexed by $\alpha \in \mathbb{N}^k$ such that $|\alpha| \leq r$, for which

$$D(x^i) = \sum_{\alpha \in \{\beta \in \mathbb{N}^k \,:\, |\beta| \leq r\}} K^i_\alpha x^\alpha. \tag{12}$$

Since $(x^i)^{r+1} = 0$, from (11) it follows that $0 = (r+1)(x^i)^r D(x^i)$. Combining this with (12) yields $(r+1)K^i_0(x^i)^r = 0$, and so $K^i_0 = 0$. Hence (12) can be rewritten as

$$D(x^i) = \sum_{\alpha \in \{\beta \in \mathbb{N}^k \,:\, 1 \leq |\beta| \leq r\}} K^i_\alpha x^\alpha. \tag{13}$$

Conversely, let $K^i_\alpha \in \mathbb{R}$ for $(i,\alpha) \in \{1,\dots,k\} \times \{\beta \in \mathbb{N}^k : 1 \leq |\beta| \leq r\}$. We prove that there is a unique $D \in \operatorname{Der} \mathbb{D}^r_k$ such that (13) holds. Obviously, we define $D(x^i)$ for $i \in \{1,\dots,k\}$ by (13), and next $D(x^\alpha)$ for $\alpha \in \mathbb{N}^k$ such that $|\alpha| \leq r$ by (11). Thus we have defined a linear $D : \mathbb{D}^r_k \to \mathbb{D}^r_k$, because the x^α for $\alpha \in \mathbb{N}^k$ such that $|\alpha| \leq r$ form a basis of the vector space $\mathbb{D}^r_k$. We only need to show that

$$D(x^\gamma x^\delta) = x^\gamma D(x^\delta) + x^\delta D(x^\gamma) \tag{14}$$

for all $\gamma, \delta \in \mathbb{N}^k$ such that $|\gamma| \leq r$, $|\delta| \leq r$. If $|\gamma + \delta| \leq r$, then $x^\gamma x^\delta = x^{\gamma+\delta}$ is an element of the basis in question, so we may use (11) three times to verify (14) in this case. Clearly, the left hand side of (14) vanishes whenever $|\gamma + \delta| \geq r + 1$. If $|\gamma + \delta| \geq r + 2$, then the right hand side vanishes on account of (11). If $|\gamma + \delta| = r + 1$, then the right hand side also vanishes on account of (11) and the fact that the term K^i_0 is excluded from (13) for every $i \in \{1,\dots,k\}$. This completes the proof.

Summing up, the map $J : \operatorname{Der} \mathbb{D}^r_k \to \mathbb{R}^{\{1,\dots,k\} \times \{\beta \in \mathbb{N}^k \,:\, 1 \leq |\beta| \leq r\}}$ defined by the requirement that

$$D(x^i) = \sum_{\alpha \in \{\beta \in \mathbb{N}^k : 1 \leq |\beta| \leq r\}} J(D)(i,\alpha) x^\alpha$$

for all $D \in \operatorname{Der} \mathbb{D}^r_k$ and $i \in \{1,\dots,k\}$ is an isomorphism of vector spaces. Counting elements of the set $\{1,\dots,k\} \times \{\beta \in \mathbb{N}^k : 1 \leq |\beta| \leq r\}$ we obtain

$$\dim \operatorname{Der} \mathbb{D}^r_k = k\left(\binom{r+k}{k} - 1\right).$$

The above corollary now asserts that if $n \geq p$, then the dimension of the vector space of canonical tensor fields of type $(p,0)$ on $T^{\mathbb{D}^r_k}$ equals

$$\left(k\left(\binom{r+k}{k} - 1\right)\right)^p.$$

Of course, a canonical tensor field V of type $(p,0)$ on T^A is called symmetric or skew-symmetric if the tensor field V_M is symmetric or skew-symmetric for every n-dimensional manifold. Therefore if $n \geq p$, then the dimensions of the vector spaces of symmetric and skew-symmetric canonical tensor fields of type $(p,0)$ on $T^{\mathbb{D}^r_k}$ equal

$$\binom{k\left(\binom{r+k}{k}-1\right)+p-1}{p} \quad \text{and} \quad \binom{k\left(\binom{r+k}{k}-1\right)}{p}$$

respectively.

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Institute of Mathematics
Jagiellonian University
Reymonta 4
30-059 Kraków, Poland
E-mail: debecki@im.uj.edu.pl

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