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Attracting divisors on projective algebraic varieties

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Dedicated to the memory of Piotr Kubowicz

Abstract. We obtain sufficient and necessary conditions (in terms of positive singular metrics on an associated line bundle) for a positive divisor D on a projective algebraic variety X to be attracting for a holomorphic map $f : X \to X$.

1. Preliminaries. Attracting sets play an important part in the study of dynamical systems. In recent years much attention has been devoted to attracting periodic points for maps in several complex variables ([FS], [Ga], [Ue] and others) and, to somewhat lesser extent, to attracting hypersurfaces ([BD], [BDM], [St1], [St2]). It would be interesting to find a unifying framework for dealing with attracting periodic points on Riemann surfaces and hypersurfaces in higher dimensional complex manifolds as well as to undertake a study of algebraic attracting sets of codimension greater than 1. We hope that initial steps towards both goals can be made by considering attracting divisors for holomorphic maps on projective algebraic varieties, which are the subject of the present paper. We characterize an attracting divisor D for a holomorphic map $f : X \to X$ by the behavior of a suitable positive singular hermitian metric on the line bundle $[D]$ and of an associated class of ω-plurisubharmonic functions. We prove our main result in Section 3, having gathered theoretical tools in Sections 1 and 2.

Throughout the paper, X will be an n-dimensional complex algebraic variety (we will make more assumptions about X later). $\mathcal{O}_X$ will denote the sheaf of germs of holomorphic functions on X, and $\mathcal{M}$ the sheaf of germs of meromorphic functions on X. For detailed definitions and more information we refer the reader to [GR], [GH], [Fu] or [Ha].

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Definition 1 ([Fu, B.4.1, B.4.2, B.4.5]). A (*Cartier*) *divisor* D on X is defined by data (U_ι, f_ι), where the U_ι form an open covering of X and the f_ι are non-zero meromorphic functions such that f_ι/f_κ is a unit (i.e., a holomorphic, nowhere vanishing function) on $U_{\iota\kappa} = U_\iota \cap U_\kappa$. The rational functions f_ι are called *local equations* for D (they are determined up to multiplication by units on U_ι). The *support* of D is the set of all points $x \in X$ such that a local equation for D is not in $\mathcal{O}^*_{x,X}$ (i.e., it is not a unit). A divisor D is *effective* if the local equations f_ι are sections of $\mathcal{O}$ on U_ι.

Writing the local equation of D on U_ι as $f_\iota = a_\iota/b_\iota$, with a_ι, b_ι holomorphic on U_ι, we see that the support of D in U_ι consists of components of the set $a_\iota b_\iota = 0$. Hence (see [Ł, II.5.3]), the support of $D \neq 0$ in X is an analytic subset of X of pure codimension one.

We will use interchangeably the languages of divisors, line bundles and invertible sheaves, because of the following one-to-one correspondences (cf. [Fu, Appendix B.4.4] and [Ha, Ch. II]:

A divisor D on X determines a line bundle on X, denoted by $[D]$. The sheaf of sections of $[D]$ may be identified with the $\mathcal{O}_X$-subsheaf of $\mathcal{M}$ (i.e., the $\mathcal{O}_X$-submodule of $\mathcal{M}$) generated on the open cover U_ι by $1/f_\iota$. Equivalently, the transition functions for $[D]$ with respect to the covering U_ι are $g_{\iota\kappa} = f_\iota/f_\kappa$. And a section $\sigma = \{s_\iota\}$, $s_\iota = g_{\iota\kappa} s_\kappa$, of a line bundle L on X determines a divisor $D = \{(U_\iota, s_\iota)\}$.

Let X, Y be non-singular compact complex varieties, and $f : X \to Y$ be a proper holomorphic map. Recall that the pushforward operator on (p,q)-currents on X, $f_* : \mathcal{D}'^{(p,q)}(X) \to \mathcal{D}'^{(p,q)}(Y)$, is defined by $\langle f_*T, \phi\rangle = \langle T, f^*\phi\rangle$ for all $T \in \mathcal{D}'^{(p,q)}(X)$ and all $\phi \in \mathcal{D}^{(p,q)}(Y)$. This operator commutes with the differential operators d and d^c, hence $dd^c f_* = f_* dd^c$. If $T = dd^c u$, where u is a plurisubharmonic function on X and f has finite fibers, then $f_*u(y) = \sum_{x\in f^{-1}(y)} u(x)$, $y \in Y$.

For X, Y and f as above it is possible to define (up to isomorphism) the pushforward f_*L of a line bundle L over X corresponding to a sheaf $\mathcal{F}$ as the line bundle corresponding to the sheaf $f_*\mathcal{F}$. The pushforward of an invertible sheaf is constructed as follows ([GR, Theorem 2.3.4 and Appendix A]): Let $y \in Y$ and let $x_1, \ldots, x_t$ be different points in the fiber $f^{-1}(y)$, $U'_1, \ldots, U'_t$ pairwise disjoint open neighborhoods of $x_1, \ldots, x_t$ and V' an open neighborhood of y. Then there exists an open neighborhood $V \subset V'$ of y such that $f^{-1}(V) = \bigcup_{j=1}^t U_j$, where $U_j = f^{-1}(U_j) \cap U'_j$ are pairwise disjoint neighborhoods of the points x_j, $j = 1, \ldots, t$ (in particular, $f(U_j) \subset V$, $j = 1, \ldots, t$). For a sheaf $\mathcal{F}$ on X these give canonical bijections $(f_{U_j,V})_*(\mathcal{F}(U_j)) \rightleftarrows \mathcal{F}({f_{U_j,V}}^{-1}(V)) = \mathcal{F}(U_j)$, $j = 1, \ldots, t$ (where $f_{U_j,V} = f|_{U_j} : U_j \to V$ and $\mathcal{F}(f^{-1}(V)) \rightleftarrows \prod_{j=1}^t \mathcal{F}(U_j)$ ($\prod$ denotes here the Cartesian product). These bijections allow us to define $f_*(\mathcal{F})(V) = \mathcal{F}(f^{-1}(V))$.

For pushforward of divisors and associated line bundles, the following proposition holds:

PROPOSITION 1. *Let X, Y be non-singular compact complex varieties, $f : X \to Y$ be a proper holomorphic surjection, and D be a divisor on X. Then*:

(a) *the current f_*D is a divisor on Y*;
(b) *$f_*[D] = [f_*D]$, where $[D]$ is the line bundle associated with the divisor D.*

Proof. Part (a) is Lemma 3.1 from [Ji]. From its proof it can be deduced (cf. also [Fu, Section 1.4]) that if $g_{\iota\kappa}$ are transition functions in $[D]$, then $f_*(g_{\iota\kappa})$ are transition functions in $[f_*D]$, which implies (b). ∎

2. Singular hermitian metrics on line bundles and ω-plurisubharmonic functions. In the treatment of singular hermitian metrics on line bundles we will follow the approach of Demailly (cf. [De2], [De3]).

DEFINITION 2 ([De3, Definition 3.12]). Let L be a complex line bundle over a complex manifold X. A *singular* (*hermitian*) *metric* on L is a metric which is given in any trivialization $\theta : L|_U \to U \times \mathbb{C}$ by

$$\|\xi\| = |\theta(\xi)| \exp(-\psi(x)), \qquad x \in U,\ \xi \in L_x,$$

where $\psi \in L^1_{\mathrm{loc}}(U)$ is a function called the *weight* of the metric with respect to the trivialization θ.

A singular metric can thus be given by a collection of functions $\psi = \{\psi_\iota\}$, $\psi_\iota \in L^1_{\mathrm{loc}}(U_\iota)$, satisfying $\psi_\iota = \psi_\kappa + \log|g_{\iota\kappa}|$ in $U_{\iota\kappa}$, where U_ι is a trivializing cover for L and $g_{\iota\kappa}$ are the transition functions. The metric is called *positive* if the functions ψ_ι are plurisubharmonic.

The following proposition is simple but useful:

PROPOSITION 2. *If the collection $\{\psi_\iota\}$ defines a positive singular metric in a line bundle L on X and $f : X \to Y$ is a proper holomorphic surjective map, then $\{f_*(\psi_\iota)\}$ defines a positive singular metric in the line bundle f_*L on Y.*

Proof. If f is holomorphic and proper, the functions $f_*(\psi_\iota)$ are plurisubharmonic if ψ_ι are (see e.g. [De1, Proposition 1.13]). Further, $\psi_\kappa = \psi_\iota + \log|g_{\iota\kappa}|$ gives $f_*\psi_\kappa(z) = f_*\psi_\iota(z) + \log\prod_{x\in f^{-1}(z)} |g_{\iota\kappa}(x)|$ in $U_{\iota\kappa}$. The latter term is the logarithm of the modulus of a transition function in the bundle f_*L (cf. Proposition 1), so the compatibility conditions also hold. ∎

Let now D be an effective divisor on X and $[D]$ its associated line bundle. One can define a positive singular metric on $[D]$ as in Example (3.14) in [De3]. Assume that $\sigma_0, \sigma_1, \ldots, \sigma_N$ are non-zero holomorphic sections of $[D]$. They define a singular hermitian metric on $[D]$ by

$$\|\xi\|_{\sim}^2 = \frac{|\theta(\xi)|^2}{|\theta(\sigma_0(x))|^2 + \cdots + |\theta(\sigma_N(x))|^2}$$

with respect to a trivialization θ. The weight function for this metric is $\psi(x) = \log(\sum_{j=0}^N |\theta(\sigma_j(x))|^2)^{1/2}$, which is a plurisubharmonic function. This metric can be viewed as introduced by a metric on $H^0(X,[D])$ as follows: Let $\sigma_0, \sigma_1, \ldots, \sigma_N$ be a basis for the linear system $|D|$ of all divisors linearly equivalent to D and $B_{|D|} = \bigcap \sigma_j{}^{-1}(0)$ its base locus. There is a meromorphic map $\Phi_{|D|} : X \setminus B_{|D|} \to \mathbb{P}^N$, $\Phi_{|D|}(x) = (\sigma_0(x) : \ldots : \sigma_N(x))$. Then the positive closed $(1,1)$-current $\frac{i}{2\pi}\Theta(F) = dd^c\psi$ is equal to the pullback over $X \setminus B_{|D|}$ of the Fubini–Study metric $\omega_{FS} = \frac{1}{2\pi} dd^c \log(|z_0|^2 + \cdots + |z_N|^2)$ of $\mathbb{P}^N$ by $\Phi_{|D|}$.

In what follows we will assume that D is very ample, i.e., that the map $\Phi_{|D|} : X \to \mathbb{P}^N$ associated with the linear system $|D| = \mathbb{P}(H^0(X,[D]))$ is a regular embedding. Then in particular $B_{|D|} = \emptyset$. According to Theorem II.7.1 in [Ha], $\Phi^*(\mathcal{O}(1))$ is an invertible sheaf on X, which is generated by the global sections $\sigma_0, \ldots, \sigma_N$, $\sigma_j = \Phi^*(z_j)$, $j = 0, \ldots, N$. Moreover, as Φ is an embedding, each open set $X_j := X \setminus \sigma_j{}^{-1}(0)$, $j = 0, \ldots, N$, is affine ([Ha, Proposition II.7.2]).

We will also assume that X is normal, i.e., the linear system $|D|$ on X giving the embedding $\Phi : X \to \mathbb{P}^N$ is complete. This means (cf. [GH, p. 177]) that the restriction map $H^0(\mathbb{P}^N, \mathcal{O}(H)) \to H^0(X, \mathcal{O}(H))$ is surjective, where H denotes the hyperplane bundle.

It will be convenient to consider another positive singular metric on the line bundle $[D]$ associated with a divisor D, which can be introduced using an isomorphism between $H^0(X,[D])$ and $\{u \in \mathcal{M}(X)^* : \operatorname{div} u \geq -D\}$. Let $\sigma_0, \sigma_1, \ldots, \sigma_N$ be a basis for $H^0(X,[D])$. We can assume that $D = \operatorname{div} \sigma_0$, so $\operatorname{supp} D = \{x \in X : \sigma_0(x) = 0\}$. Then, for $u = \tau/\sigma_0$, one just takes $\|u\| = |u| = |\tau| \exp(-\log|\sigma_0|)$ ([De3, Example 3.13]). Hence the weight for this metric coincides with the logarithm of the local equation of D in the trivialization θ_ι for $[D]$. The following relation holds:

PROPOSITION 3. *The metrics* $\|\cdot\|_{\sim}$ *and* $\|\cdot\|$ *are equivalent, i.e.,* $\exists c > 0$ $\forall \xi : c^{-1}\|\xi\| \leq \|\xi\|_{\sim} \leq c\|\xi\|$.

Proof. One uses the Rudin–Sadullaev estimates on X in exactly the same way as in Lemma 2 in [St2]. ■

The metric $\|\cdot\|$ is particularly useful in measuring the distance from a point in X to the support of D. Consider the subspace $Y = \operatorname{span}(\sigma_1, \ldots, \sigma_N)$ of $H^0(X,[D])$. Then $\mathbb{C}\sigma_0 \times \{(0, \ldots, 0)\}$ is the linear complement of $\{0\} \times Y$ in $H^0(X,[D])$. Taking the norm $|||(\tau\sigma_0, z)||| = |\tau| + \|(0,z)\|$ in $\mathbb{C}\sigma_0 \times Y$, where $\|\cdot\|$ is the metric in $H^0(X,[D])$ introduced above, one can construct the following neighborhood base for the set $Y_\infty = \mathbb{P}(0 \times Y)$ (which equals the support of D):

$$\Omega_K = \{\lambda \in \mathbb{P}(\mathbb{C}\sigma_0 \times Y) : \lambda \subset \{(\tau, y) : |\tau| < (1/K)\|y\|\},\ K > 0\},$$

which is the same as $\{z \in \mathbb{C}^N : |z| \geq K\} \cup Y_\infty$ ([Ł, VII.3.4]).

Let H be a hyperplane in the linear space Y such that $0 \notin H$. There is a unique linear form $\lambda_H \in Y^*$ such that $H = \{z : \lambda_H(z) = 1\}$. Let $H_* = \ker \lambda_H$. On the space $\mathbb{P}(\mathbb{C} \times Y)$ we have a chart $\beta_H : \mathbb{P}(\mathbb{C} \times Y) \setminus \mathbb{P}(\mathbb{C} \times H_*) \ni \mathbb{C}z \mapsto z/\lambda_H(z) \in \mathbb{C} \times H$. The domains of the charts β_H cover the set $Y_\infty = \mathbb{P}(\{0\} \times Y)$. Having chosen a norm in Y and the norm $|||(t, z)||| = |t| + \|z\|$ in $\mathbb{C} \times H_*$, we can take $\varrho_{\beta_H}(z, Y_\infty \setminus \mathbb{P}(\mathbb{C} \times H_*)) = |\lambda_H(z)|^{-1}$, $z \in Y \setminus H_*$, as the distance between Y_∞ and $z \in Y$. In the set $H_i = X_i \setminus (X_i \cap \{\sigma_0 = 0\})$, $\lambda_{H_i} = (\sigma_i/\sigma_0)$ is the reciprocal of the local equation of D, $i = 1, \ldots, N$.

The neighborhoods Ω_K have an alternative interpretation:

PROPOSITION 4. *$\Omega_K = \{(x, \xi) : |\beta_{H_i}(\xi)| \exp(-\psi(x)) < 1/K\}$ is a strongly pseudoconvex $1/K$-tube around A.*

Proof. Note that the existence of a strongly pseudoconvex neighborhood of the zero section is equivalent to the negativity of the holomorphic line bundle (see [FG, Propositions VI.6.1 and VI.6.2]). We will use $\mathcal{O}(-1)$, the tautological line bundle over the projective space $\mathbb{P}(\mathbb{C} \times \mathbb{C}^N) = \mathbb{P}(H^0(X, [D]))$. A point $\lambda \in \mathbb{P}(\mathbb{C} \times \mathbb{C}^N)$ can be identified with the fiber of $\mathcal{O}(-1)$. Under this identification, the set $Y_\infty = A$ corresponds to the zero section of $\mathcal{O}(-1)$ and Ω_K is a strongly pseudoconvex $1/K$-tube around A for all $K > 0$. ∎

We will also need the notion of ω-plurisubharmonic functions on a compact connected Kähler manifold X with respect to a closed real current ω of bidegree $(1,1)$ on X. (On complex projective spaces such functions were first considered in [BT].) Under the assumptions we made, X is a projective algebraic manifold. The class of ω-plurisubharmonic functions is defined as follows (see [GZ]):

$$\mathrm{PSH}(\omega, X) = \{\phi \in L^1(X, \mathbb{R} \cup \{-\infty\}) : dd^c\phi \geq -\omega,\ \phi \text{ is upper semicontinuous}\}.$$

Let L be a holomorphic line bundle on X and $h = \{h_\iota\}$ a smooth metric in L. The curvature $\omega = dd^c h_\iota$ in U_ι defines a real closed current globally on X, because of the compatibility conditions for h_ι. It was observed in [GZ, beginning of §5] that the class $\mathrm{PSH}(X, \omega)$ is in one-to-one correspondence with the set of positive singular metrics in L on X (that is, with the set of their weights). Namely, for a metric ψ the function $\phi = \psi - h$ satisfies $dd^c\phi \geq -\omega$, and conversely, for a $\phi \in \mathrm{PSH}(X, \omega)$, the collection $\psi = \{\psi_\iota = \phi + h_\iota\}$ defines a positive singular metric in L on X.

3. Attracting divisors. We will use the following definition of an attracting set, which is of topological character:

DEFINITION 3 (see [St1] or [St2] and the references there). Let (X,d) be a metric space and let $f : X \to X$ be a continuous map. A closed set $A \subset X$ is *attracting* for f if $f(A) = A$ and there exists a neighborhood U of A, of the form $U = \{x \in X : d(x,A) \le \varepsilon\}$, such that $f(U) \subset \operatorname{int} U$ and $\bigcap_{n\ge 0} f^n(U) = A$.

Recall our working setting: X is a normal projective algebraic manifold, D is a very ample divisor on X given by a holomorphic section $\{\sigma_0 = 0\}$ (by the Kodaira embedding theorem, D is positive), A is the support of D, and $f : X \to X$ is a proper holomorphic map with finite fibers, satisfying $f(A) = A$. Then a neighborhood base for A is given by $\{x : \psi(x) \le \varepsilon\}$, where ψ is the weight of a positive singular metric on the line bundle $[D]$. We will say that D is attracting for f if its support A is an attracting set for f in the sense of the above definition.

We can now prove our main result:

THEOREM. *The following are equivalent*:

(i) *D is attracting for f*;

(ii) *there is a positive singular metric with weight ψ on the line bundle $[D]$ and a neighborhood V of A such that*

$$\exists 0 < \beta < 1 : \quad f_*\psi \ge \beta\psi \text{ in } V;$$

(iii) *there is a positive singular metric with weight ϕ on the line bundle $[D]$ such that*

$$\exists 0 < \beta < 1 : \quad f_*(\mathrm{PSH}(X,\omega)) \subset \beta\mathrm{PSH}(X,\omega), \text{ where } \omega = dd^c\phi.$$

Proof. (i)⇔(ii): Observe that $-\psi(x) = -\log|\sigma_0(x)| = -\log \operatorname{dist}(x,A)$ is a plurisubharmonic exhaustion on $X \setminus A$. The equivalence is the same as in Lemma 1 of [St2].

(ii)⇒(iii): Let ψ be a metric as in (ii) and let $\omega = dd^c\psi$. As in Example 1.2 in [GZ], one can establish the following one-to-one correspondence between $\mathrm{PSH}(X,\omega)$ and $\{u \in \mathrm{PSH}(X\setminus A) : u - \psi \le c_u\}$: if $u \in \mathrm{PSH}(X\setminus A)$ satisfies $u \le \psi + c_u$ with some constant c_u depending only on u, then

$$v(x) = \begin{cases} u(x) - \psi(x), & x \notin A, \\ \limsup\limits_{X\setminus A \ni y \to x} (u(y) - \psi(y)), & x \in A, \end{cases}$$

is in $\mathrm{PSH}(X,\omega)$. A similar correspondence exists for $\mathrm{PSH}(X, f_*\omega)$, involving the weight $f_*\psi$. Let $v \in \mathrm{PSH}(X,\omega)$. By (ii), $f_*v - \psi$ has a well-defined $\limsup_{X\setminus A\ni y\to x}(f_*v(y) - \psi(y)) \le \limsup_{X\setminus A\ni y\to x}(f_*v(y) - f_*\psi(y))$ for $x \in A$, hence $f_*v \in \mathrm{PSH}(X,\omega)$.

(iii)⇒(ii): By (iii), $f_*(-\phi) \in \mathrm{PSH}(X, \beta\omega)$, so $f_*(-\phi)+\beta\phi$ defines a metric in $[D]$. On $\operatorname{supp} D$ this metric is equal to 0, hence $f_*\phi - \beta\phi \to \infty$ as $x \to A$, which implies that close enough to $A = \operatorname{supp} D$, $f_*\phi - \beta\phi > 0$. Thus $\psi = \phi$ is a metric as in condition (ii). ∎

REMARKS. (1) Even though trivializations were used to formulate the definition of a singular metrics, the Theorem is independent of the trivializations defining the line bundle $L = [D]$. Indeed, two sets of trivializations $\theta_\iota, \theta'_\iota$ defining L in the open cover U_ι are related by $\theta'_\iota = \eta_\iota \cdot \theta_\iota$ with $\eta_\iota \in \mathcal{O}^*(U_\iota)$, so in passing from θ to θ', the form ω in the Theorem will be replaced by a cohomologous form $\omega' = \omega + dd^c\chi$, where χ is a function on X which is integrable with respect to any smooth volume form. Accordingly, the metric ψ will be replaced by $\psi - \chi$ and the class $\mathrm{PSH}(X, \omega)$ by $\mathrm{PSH}(X, \omega') = \mathrm{PSH}(X, \omega) + \chi$ (the last equality is Proposition 1.3.3 in [GZ]).

(2) It was observed in [DS, Section 6] that the classes of ω-plurisubharmonic functions are in general not stable under the operator f_*. Condition (iii) of our Theorem gives an example of a situation in which stability does occur, since $\beta\mathrm{PSH}(X, \omega) \subset \mathrm{PSH}(X, \omega)$ for $0 < \beta < 1$.

(3) If A is a regular hypersurface in X, then the first adjunction formula (see e.g. [FG, IV.5.10], [GH, p. 145]) says that on A the line bundle $[A]$ coincides with the normal bundle $N_X(A)$ of A in X. The neighborhoods in the definition of the attracting set and in the Theorem can then be chosen as tubular neighborhoods of A (cf. Proposition 4).

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