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Tautness of locally taut domains in complex spaces

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Abstract. A necessary and sufficient condition for tautness of locally taut domains in a weakly Brody hyperbolic complex space is given. Moreover, some results of Kobayashi and Gaussier are deduced as corollaries.

1. Introduction. Characterizing tautness in the sense of Wu (see [W] or [K]) is one of the most important problems of hyperbolic complex analysis. Much attention has been given to this problem, and the results obtained can be applied to understanding the local geometry of an unbounded domain in $\mathbb{C}^n$ near a boundary point.

Recall that an open complex subspace Y of a complex space X is said to be *locally taut* if each point $a \in \partial Y$ has a neighbourhood V in X such that $V \cap Y$ is taut. S. Kobayashi [K, Lemma 5.2.8 and Theorem 5.2.11] and H. Gaussier [G, Proposition 2] gave sufficient conditions for tautness of locally taut domains in $\mathbb{C}^n$.

Our main aim in this article is to give a necessary and sufficient condition for tautness of locally taut domains in a weakly Brody hyperbolic complex space. The results of Kobayashi and Gaussier follow immediately from our main theorem.

2. Main results. First of all we recall the following

2.1. Definition. A complex space X is said to be *taut* if whenever Y is a complex space and $\{f_j : Y \to X\}$ is a sequence of holomorphic mappings, then it has either a subsequence which is compactly divergent or a subsequence which converges uniformly on compact subsets to a holomorphic mapping $f : Y \to X$. It suffices that this condition should hold when $Y = \varDelta$ (see [Ba]), where $\varDelta$ is the open unit disc in $\mathbb{C}$.

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2.2. Definition (see [T]). Let X be a complex space and h_X a hermitian metric on the tangent space TX of X. Assume that ϱ_X is the distance function (or integrated metric) associated with h_X. The space X is said to have the *Schottky property for* h_X if for each $p \in X$, each relatively compact open set W in a coordinate neighbourhood of p, and each $r \in (0,1)$, there exists a positive constant $S = S(W,r)$ such that every map $f \in \mathrm{Hol}(\Delta, X)$ with $f(0) \in W$ satisfies $\varrho_X(p, f(z)) \leq S$ for $|z| \leq r$, where $\mathrm{Hol}(\Delta, X)$ denotes the space of holomorphic mappings from Δ into X equipped with the compact-open topology.

In [T, Remark 1.3] we showed that the Schottky property of a complex space can depend on the choice of the hermitian metric.

Modifying the above definition we give the following

2.3. Definition. Let X be a complex space, h_X a hermitian metric on the tangent space TX, and ϱ_X the distance function associated with h_X. Let M be a domain in X, i.e. M is a nonempty connected open subset of X. We say that M has the *Schottky property for* h_X if for each $p \in X$, each relatively compact open set W in a coordinate neighbourhood of p, and each $r \in (0,1)$, there exists a positive constant $S = S(W,r)$ such that every map $f \in \mathrm{Hol}(\Delta, M)$ with $f(0) \in W$ satisfies $\varrho_X(p, f(z)) \leq S$ for $|z| \leq r$.

2.4. Definition. A complex space X is said to be *weakly Brody hyperbolic* if each holomorphic mapping $f : \mathbb{C} \to X$ with $f(\mathbb{C}) \Subset X$ is constant.

By Liouville's theorem, $\mathbb{C}^n$ is weakly Brody hyperbolic for each $n \geq 1$.

We now prove the main theorem of this article.

2.5. Theorem. *Let X be a weakly Brody hyperbolic complex space and h_X a complete hermitian metric on TX. Let M be a locally taut domain in X. Then the domain M is taut if and only if M has the Schottky property for h_X.*

In order to prove Theorem 2.5 we need the following lemmas:

2.6. Lemma. *Let Ω be a domain in $\mathbb{C}^m$, let M be a complete hermitian complex space, and let $\mathcal{F} \subset \mathrm{Hol}(\Omega, M)$, where $\mathrm{Hol}(\Omega, M)$ is equipped with the open-compact topology. If the family $\mathcal{F}$ is not relatively compact in $\mathrm{Hol}(\Omega, M)$ then there exist sequences $\{p_j\} \subset \Omega$ with $p_j \to p_0 \in \Omega$, $\{f_j\} \subset \mathcal{F}$, $\{\varrho_j\} \subset \mathbb{R}$ with $\varrho_j > 0$ and $\varrho_j \to 0$ such that the functions*

$$g_j(\xi) = f_j(p_j + \varrho_j \xi), \qquad \xi \in \mathbb{C}^m,$$

satisfy one of the following two assertions:

(i) *The sequence $\{g_j\}_{j \geq 1}$ is compactly divergent on $\mathbb{C}^m$.*

(ii) *The sequence $\{g_j\}_{j \geq 1}$ converges uniformly on compact subsets of $\mathbb{C}^m$ to a nonconstant holomorphic map $g : \mathbb{C}^m \to M$.*

Proof. The proof of this lemma is taken from [TTH, Thm. 2.5]. However, since we will be using some technical details of the proof and also for the reader's convenience, we repeat the details.

We consider two cases:

CASE 1: *The family $\mathcal{F}$ is compactly divergent.* Then there is a sequence $\{f_j\} \subset \mathcal{F}$ which is compactly divergent. Take $p_0 \in \Omega$ and $r_0 > 0$ such that $B(p_0, r_0) \Subset \Omega$. Take $p_j = p_0$ for all $j \geq 1$, $\varrho_j > 0$ for all $j \geq 1$ such that $\varrho_j \to 0^+$, and

$$g_j(\xi) = f_j(p_j + \varrho_j \xi) \quad \text{for all } j \geq 1.$$

Note that g_j is defined on

$$\left\{\xi \in \mathbb{C}^m : \|\xi\| \leq R_j = \frac{1}{\varrho_j} \operatorname{dist}(p_0, \partial\Omega)\right\}.$$

Assume that K is a compact subset of $\mathbb{C}^m$ and L is a compact subset of M. Then there exists $j_0 \geq 1$ such that $p_j + \varrho_j K \subset B(p_0, r_0)$ for all $j \geq j_0$. This implies that $g_j(K) \subset f_j(\overline{B}(p_0, r_0))$ for each $j \geq j_0$.

Since the sequence $\{f_j\}$ is compactly divergent, there exists $j_1 > j_0$ such that $f_j(\overline{B}(p_0, r_0)) \cap L = \emptyset$ for all $j \geq j_1$. Thus $g_j(K) \cap L = \emptyset$ for all $j \geq j_1$. This means that the family $\{g_j\}$ is compactly divergent.

CASE 2: *The family $\mathcal{F}$ is not compactly divergent.* By [TTH, Lemma 2.6], there exists a length function E on M and a compact subset K_0 of Ω and sequences $\{f_j\} \subset \mathcal{F}$, $\{\widetilde{p}_j\} \subset K_0$, $\{t_j\} \subset \mathbb{C}^m$ with $\|t_j\| = 1$ for all $j \geq 1$ such that

$$E\big(f_j(\widetilde{p}_j), df_j(\widetilde{p}_j)(t_j)\big) \geq j^2 \qquad \text{for each } j \geq 1.$$

Without loss of generality, we may assume that $\widetilde{p}_j \to \widetilde{p}_0 \in K_0$ and that $\|\widetilde{p}_j - \widetilde{p}_0\| < 1/(2j)$. We may also assume that $\widetilde{p}_0 = 0$. Put

$$M_j = \max_{\substack{\|t\|=1 \\ \|z\| \leq 1/j}} (1 - \|z\|^2 \cdot j^2) E(f_j(z), df_j(z)(t)).$$

Take p_j and t_j^0 which maximize this last expression. Then

$$\begin{aligned} M_j &\geq (1 - \|\widetilde{p}_j\| \cdot j^2) E(f_j(\widetilde{p}_j), df_j(\widetilde{p}_j)(t_j)) \\ &\geq \left(1 - \frac{1}{(2j)^2} \cdot j^2\right) \cdot j^2 = \frac{3}{4} j^2 \to \infty \qquad \text{as } j \to \infty. \end{aligned}$$

Put

$$\varrho_j := \frac{1}{E(f_j(p_j), df_j(p_j)(t_j^0))} = \frac{1 - \|p_j\|^2 \cdot j^2}{M_j} \leq \frac{1}{M_j}$$

for each $j \geq 1$. It is easy to see that $\varrho_j > 0$ and $\varrho_j \to 0^+$. Put

$$R_j := \frac{1/j - \|p_j\|}{\varrho_j}.$$

Then

$$\|p_j + \varrho_j\xi\| \le \|p_j\| + \|\varrho_j\xi\| \le \|p_j\| + \varrho_j \cdot \frac{1}{\varrho_j}\left(\frac{1}{j} - \|p_j\|\right) = \frac{1}{j}$$

for each $\|\xi\| < R_j$, and

$$\begin{aligned} R_j &= \frac{1 - j\|p_j\|}{j\varrho_j} = \frac{1 - \|p_j\|^2 \cdot j^2}{j\varrho_j(1 + \|p_j\| \cdot j)} \\ &= \frac{M_j}{j(1 + \|p_j\| \cdot j)} \ge \frac{\frac{3}{4}j^2}{2j} = \frac{3}{8}j \to \infty \quad \text{as } j \to \infty. \end{aligned}$$

Define $g_j(\xi) = f_j(p_j + \varrho_j\xi)$ for $j \ge 1$. Since $\|p_j + \varrho_j\xi\| \le 1/j$ for each $\|\xi\| < R_j$, it follows that

$$M_j \ge (1 - \|p_j + \varrho_j\xi\|^2 \cdot j^2) \cdot E(g_j(\xi), df_j(p_j + \varrho_j\xi)(t))$$

for each $t \in \mathbb{C}^m$ with $\|t\| = 1$.

Fix $R > 0$ and j large such that $R < R_j$. If $\|\xi\| \le R$ then, for every $t \in \mathbb{C}^m$ with $\|t\| = 1$, we have

$$\begin{aligned} E(g_j(\xi), dg_j(\xi)(t)) &= \varrho_j E(g_j(\xi), df_j(p_j + \varrho_j\xi)(t)) \\ &\le \varrho_j \cdot \frac{M_j}{1 - \|p_j + \varrho_j\xi\|^2 \cdot j^2} = \frac{1 - \|p_j\|^2 \cdot j^2}{1 - \|p_j + \varrho_j\xi\|^2 \cdot j^2} \\ &= \frac{1 + \|p_j\| \cdot j}{1 + \|p_j + \varrho_j\xi\| \cdot j} \cdot \frac{1 - \|p_j\| \cdot j}{1 - \|p_j + \varrho_j\xi\| \cdot j} \\ &\le 2 \cdot \frac{1 - \|p_j\| \cdot j}{1 - \|p_j\| \cdot j - \varrho_j \cdot \|\xi\| \cdot j} = \frac{2}{1 - \dfrac{\varrho_j\|\xi\| \cdot j}{1 - \|p_j\|j}} \\ &= \frac{2}{1 - \|\xi\|/R_j} \le \frac{2}{1 - R/R_j} \to 2 \quad \text{as } j \to \infty. \end{aligned}$$

So the g_j are holomorphic on larger and larger discs in $\mathbb{C}^m$ and they have bounded derivatives (on compact subsets). Thus the family $\{g_j\}$ is equicontinuous. If $\{g_j\}$ is not compactly divergent, then by a result of Wu [Wu, Lemma 1.1(iii)], there exists a subsequence $\{g_{j_k}\} \subset \{g_j\}$ such that $g_{j_k} \to g$. It is easy to see that

$$E(g_j(0), dg_j(0)(t_j^0)) = \varrho_j E(g_j(0), df_j(p_j)(t_j^0)) = \varrho_j E(f_j(p_j), df_j(p_j)(t_j^0)) = 1.$$

This implies $g'(0) \ne 0$, hence g is not a constant function. ∎

2.7. Lemma. *Let X be a weakly Brody hyperbolic complex space and h_X a complete hermitian metric on TX. Let M be a domain in X such that M has the Schottky property for h_X. Assume that a sequence $\{f_n\} \subset \mathrm{Hol}(\Delta, M)$ is not compactly divergent. Then it has a subsequence which converges uniformly on compact subsets of Δ to a map $f \in \mathrm{Hol}(\Delta, X)$.*

Proof. Since $\{f_n\}$ is not compactly divergent, without loss of generality we may assume that there exists a sequence $\{z_n\} \subset \Delta$ such that $z_n \to z_0 \in \Delta$ and $f_n(z_n) \to q \in M$. For each $n \geq 0$ consider the Möbius automorphism $\varphi_n : \Delta \to \Delta$ given by

$$\varphi_n(z) = \frac{z_n + z}{1 + \overline{z}_n \cdot z} \quad \text{for } z \in \Delta.$$

Then [L, p. 13] the sequence $\{\varphi_n\}$ converges uniformly on compact subsets of Δ to φ_0, and also $\{\varphi_n^{-1}\}$ converges uniformly on compact subsets of Δ to ${\varphi_0}^{-1}$.

Put $g_n = f_n \circ \varphi_n$ for all $n \geq 1$. Since the sequence $\{g_n(0) = f_n(z_n)\}$ converges to q, we can assume that there is a relatively compact open set W in a coordinate neighbourhood of q such that $g_n(0) \in W$ for all $n \geq 1$.

Since M has the Schottky property for h_X, there is a positive constant R such that $\varrho_X(q, g_n(z)) < R$ for $|z| \leq 1/2$ and for each $n \geq 1$. This means that $g_n(\overline{\Delta}_{1/2}) \subset B_{\varrho_X}(q, R)$ for each $n \geq 1$, where $\overline{\Delta}_{1/2} = \{z \in \mathbb{C} : |z| \leq 1/2\}$ and $B_{\varrho_X}(q, R) = \{x \in X : \varrho_X(q, x) < R\}$. Since h_X is a complete hermitian metric on TX, the ball $B_{\varrho_X}(q, R)$ is relatively compact in X. This implies that the family $\mathcal{F} = \{g_n\}_{n \geq 1} \subset \mathrm{Hol}(\Delta, X)$ is not compactly divergent.

Suppose that the family $\mathcal{F} \subset \mathrm{Hol}(\Delta, X)$ is not relatively compact in $\mathrm{Hol}(\Delta, X)$. Then, by Lemma 2.6, there exist sequences $\{p_j\} \subset \Delta$ with $p_j \to p_0 \in \Delta$, $\{g_j\} \subset \mathcal{F}$, $\{\varrho_j\} \subset \mathbb{R}$ with $\varrho_j > 0$ and $\varrho_j \to 0$ such that the functions

$$h_j(\xi) = g_j(p_j + \varrho_j \xi), \quad \xi \in \Delta_{R_j} := \{\xi \in \mathbb{C} : |\xi| < R_j\},$$

satisfy one of the following two assertions:

(i) The sequence $\{h_j\}$ is compactly divergent on $\mathbb{C}$.

(ii) The sequence $\{h_j\}$ converges uniformly on compact subsets of $\mathbb{C}$ to a nonconstant holomorphic map $h : \mathbb{C} \to X$.

From Case 2 in the proof of Lemma 2.6, we have $\mathrm{range}(h_j) \subset g_j(\overline{\Delta}_{1/2}) \subset B_{\varrho_X}(q, R) \Subset X$ for each $j \geq 2$. This implies that $\{h_j\}$ is not compactly divergent, and hence it converges uniformly on compact subsets of $\mathbb{C}$ to a nonconstant holomorphic map $h : \mathbb{C} \to X$.

It is easy to see that

$$h(\mathbb{C}) \subset \overline{B_{\varrho_X}(q, R)} \Subset X.$$

Since X is weakly Brody hyperbolic, it follows that h is constant. This is a contradiction. Thus the family $\mathcal{F}$ is relatively compact in $\mathrm{Hol}(\Delta, X)$, i.e there exists a subsequence $\{g_{n_k}\}_{k=1}^{\infty}$ which converges uniformly on compact subsets of Δ to a map $g \in \mathrm{Hol}(\Delta, X)$. Then $\{f_{n_k}\}_{k=1}^{\infty}$ converges uniformly on compact subsets of Δ to the map $g \circ \varphi_0^{-1}$ in $\mathrm{Hol}(\Delta, X)$, as claimed. ■

Proof of Theorem 2.5. Necessity. Assume that M does not have the Schottky property for h_X. Then there exist $q \in M$, a relatively compact open set W in a coordinate neighbourhood of q, $r \in (0,1)$, $\{f_n\} \subset \mathrm{Hol}(\varDelta, M)$ with $f_n(0) \in W$ for all $n \geq 1$, and $\{z_n\} \subset \varDelta_r$ such that

$$\varrho_X(q, f_n(z_n)) \to \infty \quad \text{as } n \to \infty.$$

Since M is taut, by a theorem of Abate (see [A] or [K, Theorem 5.1.6, p. 242]) it follows that $\mathrm{Hol}(\varDelta, M) \cup \{\infty\}$ is a compact subset of $\mathcal{C}(\varDelta, M^+)$, where $M^+ = M \cup \{\infty\}$ is the Aleksandrov one-point compactification of M. Since $\{f_n(0)\}_{n=1}^{\infty} \subset W \Subset X$, there exists a subsequence $\{f_{n_k}\}$ which converges uniformly on compact subsets of $\varDelta$ to a map $f \in \mathrm{Hol}(\varDelta, M)$. Without loss of generality we may assume that $z_n \to z_0 \in \varDelta$. Then $\varrho_X(q, f(z_0)) = \lim_{k\to\infty} \varrho_X(q, f_{n_k}(z_{n_k})) = \infty$. This is a contradiction.

Sufficiency. Assume that M has the Schottky property for h_X. Let $\{f_n\}$ be a sequence in $\mathrm{Hol}(\varDelta, M)$ which is not compactly divergent. We now prove that it contains a subsequence which converges uniformly on compact subsets of $\varDelta$ to a map $f \in \mathrm{Hol}(\varDelta, M)$.

Indeed, by Lemma 2.7 and by passing to a subsequence if necessary, we may assume that $\{f_n\}$ converges uniformly on compact subsets of $\varDelta$ to a map $f \in \mathrm{Hol}(\varDelta, X)$. Since $\{f_n\}$ is not compactly divergent, there exist compact sets $K \subset \varDelta$ and $L \subset X$ such that $f_n(K) \cap L \neq \emptyset$ for infinitely many n. We may assume that this is the case for all n. Since $f_n(K) \cap L \neq \emptyset$ for all n, we have $f(K) \cap L \neq \emptyset$. We now have to show that $f \in \mathrm{Hol}(\varDelta, M)$. Assume the contrary. Then the open subset $f^{-1}(M)$ of $\varDelta$ is distinct from $\varDelta$. It is nonempty since $f(K) \cap L \neq \emptyset$. Let a be a boundary point of $f^{-1}(M)$ in $\varDelta$, and set $p = f(a)$. Then $p \notin M$. Let V be a neighbourhood of p in X such that $V \cap M$ is taut. Let W be a neighbourhood of a in $\varDelta$ such that $f(\overline{W}) \subset V$. By taking a subsequence we may assume that $f_n(\overline{W}) \subset V$ for all $n \geq 1$. Since $V \cap M$ is taut, $\{f_n|_W\} \subset \mathrm{Hol}(W, V \cap M)$ is either compactly divergent or has a convergent subsequence. But since $\lim f_n(a) = f(a) = p \notin M$, it cannot have a convergent subsequence; and it cannot be compactly divergent either, because, for any $b \in W \cap f^{-1}(M)$, we have $\lim f_n(b) = f(b) \in V \cap M$.

This is a contradiction. ∎

Since every bounded domain in $\mathbb{C}^n$ has the Schottky property for the hermitian metric induced by the canonical metric of $\mathbb{C}^n$, the following result of S. Kobayashi can be deduced immediately from Theorem 2.5.

2.8. Corollary ([K, Lemma 5.2.8 and Theorem 5.2.11]). *If a bounded domain $M \subset \mathbb{C}^n$ is locally taut, then it is taut.*

We now recall the following

2.9. Definition. Let M be a domain in a complex space X. Let $X^+ = X \cup \{\infty\}$ be the Aleksandrov one-point compactification of X. Denote by $\overline{M}$ the closure of M in X^+. We say that M is *nonbounded* if $\infty \in \overline{M}$. If M

is nonbounded and φ is a function defined on M, we set $\varphi(\infty) = c \in \overline{\mathbb{R}}$ if $\lim_{z\to\infty} \varphi(z) = c$.

Let M be a nonbounded domain in a complex space X.

(i) A function φ is called a *local peak plurisubharmonic function* at p in $\partial M \cup \{\infty\}$ if there exists a neighbourhood U of p in X^+ such that φ is plurisubharmonic on $U \cap M$, continuous up to $U \cap \overline{M}$ and satisfies

$$\begin{cases} \varphi(p) = 0, \\ \varphi(z) < 0 \quad \text{for all } z \in (U \cap \overline{M}) \setminus \{p\}. \end{cases}$$

(ii) A function ψ is called a *local antipeak plurisubharmonic function* at p in $\partial M \cup \{\infty\}$ if there is a neighbourhood U of p in X^+ such that ψ is plurisubharmonic on $U \cap M$, continuous up to $U \cap \overline{M}$ and satisfies

$$\begin{cases} \psi(p) = -\infty, \\ \psi(z) > -\infty \quad \text{for all } z \in (U \cap \overline{M}) \setminus \{p\}. \end{cases}$$

Note that an antipeak plurisubharmonic function exists at every finite point p.

We now prove the following.

2.10. Corollary. *Let M be a locally taut nonbounded domain in a weakly Brody hyperbolic complete hermitian complex space X. If there are local peak and antipeak plurisubharmonic functions φ and ψ at ∞, then M is taut.*

In particular, when $X = \mathbb{C}^n$ we get the result of Gaussier [G, Proposition 2].

Repeating the argument in [G, Lemma 2.1.1] we have the following

2.11. Lemma. *Let M be a nonbounded domain in a complex space X. Assume that there are local peak and antipeak plurisubharmonic functions φ and ψ at p in $\partial M \cup \{\infty\}$. Let $0 < r < 1$. Then for every neighbourhood W of p in X^+ there exists a neighbourhood W' of p in X^+ such that every holomorphic map $f : \Delta \to M$ satisfies*

$$f(0) \in W' \;\Rightarrow\; f(\Delta_r) \subset W.$$

Proof of Corollary 2.10. By Theorem 2.5, it suffices to prove that M has the Schottky property for h_X, where h_X is a complete hermitian metric on TX.

Assume the contrary. Then there exist $p \in M$, a relatively compact open set U in a coordinate neighbourhood of p, $r \in (0,1)$, $\{f_n\} \subset \mathrm{Hol}(\Delta, M)$ with $f_n(0) \in U$ for all $n \geq 1$, and $\{z_n\} \subset \Delta_r$ such that

$$\varrho_X(p, f_n(z_n)) \to \infty \quad \text{as } n \to \infty.$$

By taking a subsequence we may assume that $f_n(z_n) \to \infty$ in X^+. Take a neighbourhood W of ∞ in X^+ such that $W \cap \overline{U} = \emptyset$. By Lemma 2.11, there exists a neighbourhood W' of ∞ in X^+ such that every holomorphic map $f : \Delta \to M$ satisfies

$$f(0) \in W' \ \Rightarrow \ f(\Delta_r) \subset W.$$

For each $n \geq 1$ consider the Möbius automorphism $\varphi_n : \Delta \to \Delta$ given by

$$\varphi_n(z) = \frac{z + z_n}{1 + \overline{z}_n \cdot z} \quad \text{for } z \in \Delta.$$

Put $g_n = f_n \circ \varphi_n$ for all $n \geq 1$. Since the sequence $\{g_n(0) = f_n(z_n)\}$ converges to ∞, we can assume that $g_n(0) \in W'$ for each $n \geq 1$. Then $g_n(\Delta_r) \subset W$ for all $n \geq 1$. This implies that $g_n(-z_n) = f_n(\varphi_n(-z_n)) = f_n(0) \in W$ for all $n \geq 1$. This is a contradiction. ∎

We now give a generalization of a result of D. D. Thai [T, Proposition 1.4].

2.12. COROLLARY. *Let X be a weakly Brody hyperbolic complex space and h_X a complete hermitian metric on TX. Then X is taut if and only if X has the Schottky property for h_X.*

Proof. The necessity is deduced immediately from Theorem 2.5, and the sufficiency follows from Lemma 2.7. ∎

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