ANNALES
POLONICI MATHEMATICI
99.1 (2010)

On “special” fibred coordinates for general and classical connections

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Abstract. Using a general connection Γ on a fibred manifold $p : Y \to M$ and a torsion free classical linear connection ∇ on M, we distinguish some “special” fibred coordinate systems on Y, and then we construct a general connection $\widetilde{\mathcal{F}}(\Gamma, \nabla)$ on $Fp : FY \to FM$ for any vector bundle functor $F : \mathcal{M}f \to \mathcal{VB}$ of finite order.

1. Introduction. A *general connection* on a fibred manifold $Y \to M$ is a section $\Gamma : Y \to J^1Y$ of the first jet prolongation $J^1Y \to Y$ of $Y \to M$, which can be (equivalently) considered as the corresponding lifting map $\Gamma : Y \times_M TM \to TY$. If $p : Y \to M$ is a vector bundle and $\Gamma : Y \to J^1Y$ is a vector bundle map, then Γ is called a *linear general connection* on $p : Y \to M$. A linear general connection $\Gamma : TM \to J^1TM$ on the tangent bundle $p : TM \to M$ of M is called a *classical linear connection* on M, which can be (equivalently) considered as its corresponding covariant derivative ∇. A classical linear connection ∇ on M is called *torsion free* if its torsion tensor is zero. More information on connections can be found in the fundamental monograph [KMS].

The present short note is devoted to studying prolongation of connections. In Section 2, given a general connection Γ on a fibred manifold $p : Y \to M$ and a torsion free classical linear connection ∇ on M, we distinguish some “special” so called (Γ, ∇)-quasi-normal fibred coordinate systems on Y. In fact, we essentially strengthen [M3, Lemma 2]. In Section 3, applying these (Γ, ∇)-quasi-normal fibred coordinate systems, we construct a general connection $\widetilde{\mathcal{F}}(\Gamma, \nabla)$ on $Fp : FY \to FM$ for any vector bundle functor $F : \mathcal{M}f \to \mathcal{VB}$ (the concept of bundle functors can be found in [KMS]).

We recall that in [S], J. Slovák constructed a general connection $\mathcal{F}(\Gamma)$ on $Fp : FY \to FM$ from a general connection Γ on a fibred manifold $p : Y \to M$ for any product-preserving bundle functor $F : \mathcal{M}f \to \mathcal{FM}$. In

2010 *Mathematics Subject Classification*: 58A05, 58A20, 58A32.
Key words and phrases: jets, general connections, natural operators.

DOI: 10.4064/ap99-1-9

[M1] (see also [M4]), we proved that if a vector bundle functor $F : \mathcal{M}f \to \mathcal{VB}$ (or a bundle functor $F : \mathcal{M}f \to \mathcal{FM}$ with the point property) is not product-preserving then there is no canonical construction of a general connection on $Fp : FY \to FM$ from a general connection Γ on a fibred manifold $p : Y \to M$. Consequently, we see that an auxiliary torsion free classical linear connection ∇ on M is unavoidable to construct a general connection on $Fp : FY \to FM$ from a general connection on $p : Y \to M$. We also recall that in [M2], given a general connection Γ on a fibred manifold $p : Y \to M$ and a p-projectable torsion free classical linear connection $\widetilde{\nabla}$ on Y, we constructed a general connection $A^F(\Gamma, \widetilde{\nabla})$ on $Fp : FY \to FM$ for any vector bundle functor $F : \mathcal{M}f \to \mathcal{VB}$ of finite order. Of course, torsion free classical linear connections on M are "simpler" objects than p-projectable torsion free classical linear connections on Y. So, the construction $\widetilde{\mathcal{F}}(\Gamma, \nabla)$ (presented in this note) is "more economic" than $A^F(\Gamma, \widetilde{\nabla})$ from [M2].

All manifolds and maps we consider are assumed to be ($\mathcal{C}^\infty$) smooth.

2. On quasi-normal fibred coordinate systems. Just as in [M3], let

$$\Phi_r : J^{r-1}_0(T^*\mathbb{R}^m \otimes \mathbb{R}^n) \to J^r_0(\mathbb{R}^m, \mathbb{R}^n)_0$$

be the usual symmetrization

$$\bigoplus_{q=0}^{r-1} S^q T^*_0\mathbb{R}^m \otimes T^*_0\mathbb{R}^m \otimes \mathbb{R}^n \to \bigoplus_{q=0}^{r-1} S^{q+1} T^*_0\mathbb{R}^m \otimes \mathbb{R}^n$$

modulo the obvious ($\mathrm{GL}(m)$-invariant) identifications

$$J^{r-1}_0(T^*\mathbb{R}^m \otimes \mathbb{R}^n) = \bigoplus_{q=0}^{r-1} S^q T^*_0\mathbb{R}^m \otimes T^*_0\mathbb{R}^m \otimes \mathbb{R}^n$$

and

$$\bigoplus_{q=0}^{r-1} S^{q+1} T^*_0\mathbb{R}^m \otimes \mathbb{R}^n = J^r_0(\mathbb{R}^m, \mathbb{R}^n)_0.$$

In other words, $\Phi_r : J^{r-1}_0(T^*\mathbb{R}^m \otimes \mathbb{R}^n) \to J^r_0(\mathbb{R}^m, \mathbb{R}^n)_0$ is the linear map such that

$$\Phi_r(j^{r-1}_0((x^{i_1} \dots x^{i_q} dx^j)e_k)) = \frac{1}{q+1}\, j^r_0(x^{i_1} \dots x^{i_q} x^j e_k)$$

for any $i_1, \dots, i_q, j = 1, \dots, m$, $q = 0, \dots, r-1$ and $k = 1, \dots, n$, where e_k is the usual canonical basis in $\mathbb{R}^n$ and $x^1, \dots, x^m$ are the usual coordinates on $\mathbb{R}^m$. Then

$$\Phi_r(j^{r-1}_0(d\sigma)) = j^r_0(\sigma)$$

for any $\sigma : \mathbb{R}^m \to \mathbb{R}^n$ with $\sigma(0) = 0$. Clearly, Φ_r is $\mathrm{GL}(m)$-invariant and linear.

Let m, n, r be positive integers, $\Gamma : Y \to J^1 Y$ be a general connection on a fibred manifold $p : Y \to M$ with $\dim(m) = M$ and $\dim(Y) = m + n$, ∇ be a torsion free classical linear connection on M and $y_0 \in Y$ be a point with $x_0 = p(y_0) \in M$.

DEFINITION 2.1. A (Γ, ∇, y_0, r)-*quasi-normal coordinate system* on Y is a fibred chart ψ on Y with $\psi(y_0) = (0, 0) \in \mathbb{R}^m \times \mathbb{R}^n$ covering a ∇-normal coordinate system $\underline{\psi}$ on M with centre x_0 such that

$$\Phi_r\Big(j_0^{r-1}\Big(\sum_{|\alpha|+|\beta|\le r-1}\sum_{j=1}^{m}\sum_{k=1}^{n}\Gamma^k_{j\alpha\beta}x^\alpha dx^j \otimes e_k\Big)\Big) = 0 \tag{2.1}$$

for any $\beta \in (\mathbb{N} \cup \{0\})^n$ with $|\beta| \le r - 1$, where

$$j_0^{r-1}\Big(\sum_{i=1}^{m} dx^i \otimes \frac{\partial}{\partial x^i} + \sum_{|\alpha|+|\beta|\le r-1}\sum_{j=1}^{m}\sum_{k=1}^{n}\Gamma^k_{j\alpha\beta}x^\alpha y^\beta dx^j \otimes \frac{\partial}{\partial y^k}\Big) \tag{2.2}$$

is the coordinate expression of $j^{r-1}_{(0,0)}(\psi_*\Gamma)$ and $x^1, \ldots, x^m, y^1, \ldots, y^n$ are the usual coordinates on $\mathbb{R}^m \times \mathbb{R}^n$.

We prove the following result, which is essentially stronger than the one in [M3, Lemma 2].

PROPOSITION 2.2. *Let m, n, r, $p : Y \to M$, $\Gamma : Y \to J^1Y$, ∇, $y_0 \in Y$, $x_0 = p(y_0) \in M$ be as above.*

(a) *There exists a (Γ, ∇, y_0, r)-quasi-normal fibred coordinate system ψ.*

(b) *If ψ^1 is another (Γ, ∇, y_0, r)-quasi-normal coordinate system on Y then*

$$j^r_{y_0}\psi^1 = j^r_{y_0}((A \times H) \circ \psi) \tag{2.3}$$

for a map $A \in GL(m)$ and a diffeomorphism $H : \mathbb{R}^n \to \mathbb{R}^n$ preserving 0.

Proof. (a) Because of the existence of ∇-normal coordinate systems and the fact that J^1Y is the orbit of $j^1_0(0)$ under the action of the pseudogroup of local fibred diffeomorphisms, we may assume that $Y = \mathbb{R}^m \times \mathbb{R}^n$, $M = \mathbb{R}^m$, $p = \mathrm{pr}_1 : \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}^m$ is the obvious projection, $y_0 = (0, 0)$, $\mathrm{id}_{\mathbb{R}^m}$ is a ∇-normal coordinate system with centre 0, formula (2.2) is the coordinate expression of $j^{r-1}_{(0,0)}\Gamma$ for any r, and $\Gamma^k_{j(0)(0)} = 0$ for $k = 1, \ldots, n$ and $j = 1, \ldots, m$.

We will proceed by induction on r.

The case $r = 1$ is trivial because $\Gamma^k_{j(0)(0)} = 0$.

Now, we assume that there exists a $(\Gamma, \nabla, (0, 0), r-1)$-quasi-normal fibred coordinate system, $r \ge 2$. Replacing Γ by the image of Γ under this fibred chart (this $(\Gamma, \nabla(0, 0), r-1)$-quasi-normal fibred coordinate system), we can assume that $\mathrm{id}_{\mathbb{R}^m\times\mathbb{R}^n}$ is a $(\Gamma, \nabla, (0, 0), r - 1)$-quasi-normal fibred coordinate system. Next, for any $\beta \in (\mathbb{N} \cup \{0\})^n$ with $|\beta| \le r-1$, let $\sigma_\beta = (\sigma^k_\beta)^n_{k=1} : \mathbb{R}^m \to \mathbb{R}^n$ be a map with $\sigma_\beta(0) = 0$ such that

$$j^r_0(\sigma_\beta) = \Phi_r\Big(j^{r-1}_0\Big(\sum_{|\alpha|+|\beta|=r-1}\sum_{j=1}^m\sum_{k=1}^n \Gamma^k_{j\alpha\beta}x^\alpha dx^j \otimes e_k\Big)\Big).$$

Define $\psi : \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}^m \times \mathbb{R}^n$ by

$$\psi(x,y) = \Big(x, y - \sum_{|\beta|\le r-1} y^\beta \sigma_\beta(x)\Big).$$

It is easy to see that $j^{r-1}_{(0,0)}\psi = \mathrm{id}$. We prove that ψ is a $(\Gamma, \nabla, (0,0), r)$-quasi-normal fibred coordinate system. We see that ψ preserves

$$j^{r-1}_{(0,0)}\Big(\sum_{|\alpha|+|\beta|\le r-1}\sum_{j=1}^m\sum_{k=1}^n \Gamma^k_{j\alpha\beta}x^\alpha y^\beta\, dx^j \otimes \frac{\partial}{\partial y^k}\Big)$$

because $j^{r-1}_{(0,0)}\psi = \mathrm{id}$, $\underline{\psi} = \mathrm{id}$ and $\Gamma^k_{j(0)(0)} = 0$. Moreover, ψ sends $j^{r-1}_{(0,0)}\big(\sum_{i=1}^m dx^i \otimes \frac{\partial}{\partial x^i}\big)$ to

$$j^{r-1}_{(0,0)}\Big(\sum_{i=1}^m dx^i \otimes \frac{\partial}{\partial x^i} - \sum_{|\beta|\le r-1}\sum_{k=1}^n\sum_{i=1}^m \frac{\partial \sigma^k_\beta}{\partial x^i}(x) y^\beta dx^i \otimes \frac{\partial}{\partial y^k}\Big).$$

Then expressing $j^{r-1}_{(0,0)}(\psi_*\Gamma)$ by (2.2) with $\tilde{\Gamma}^k_{j\alpha\beta}$ instead of $\Gamma^k_{j\alpha\beta}$ we see that

$$\begin{aligned}
&\Phi_r\Big(j^{r-1}_0\Big(\sum_{|\alpha|+|\beta|\le r-1}\sum_{j=1}^m\sum_{k=1}^n \tilde{\Gamma}^k_{j\alpha\beta}x^\alpha dx^j \otimes e_k\Big)\Big)\\
&\quad= \Phi_r\Big(j^{r-1}_0\Big(\sum_{|\alpha|+|\beta|\le r-1}\sum_{j=1}^m\sum_{k=1}^n \Gamma^k_{j\alpha\beta}x^\alpha dx^j \otimes e_k\Big)\Big) - \Phi_r(j^{r-1}_{(0,0)}(d\sigma_\beta))\\
&\quad= j^r_0(\sigma_\beta) - j^r_0(\sigma_\beta) = 0.
\end{aligned}$$

for any $\beta \in (\mathbb{N}\cup\{0\})^n$ with $|\beta| \le r-1$ (as $\Phi_r(j^{r-1}_0(\sum_{|\alpha|+|\beta|\le r-2}\Gamma^k_{j\alpha\beta}x^\alpha dx^j \otimes e_k)) = 0$ since $\mathrm{id}_{\mathbb{R}^m\times\mathbb{R}^n}$ is $(\Gamma, \nabla, (0,0), r-1)$-adapted).

(b) Replacing Γ by $\psi_*\Gamma$ we may assume that $\mathrm{id}_{\mathbb{R}^m\times\mathbb{R}^n}$ is a $(\Gamma, \nabla, (0,0), r)$-quasi-normal fibred coordinate system. Next, we will proceed by induction on r.

The case $r = 1$ is clear.

Now, let ψ^1 be a $(\Gamma, \nabla, (0,0), r)$-quasi-normal fibred coordinate system. Then (clearly) it is a $(\Gamma, \nabla, (0,0), r-1)$-quasi-normal fibred coordinate system. Hence by the inductive assumption, $j^{r-1}_{(0,0)}\psi^1 = j^{r-1}_{(0,0)}(A \times \tilde{H})$ for some $A \in \mathrm{GL}(m)$ and some $\tilde{H} : \mathbb{R}^n \to \mathbb{R}^n$. Clearly, $A\times\tilde{H}$ is a $(\Gamma, \nabla, (0,0), r)$-quasi-normal coordinate system. Then replacing Γ by $(A \times \tilde{H})_*\Gamma$ we may assume

$j^{r-1}_{(0,0)}\psi^1 = \mathrm{id}$. Next, replacing Γ by $(\mathrm{id}_{\mathbb{R}^m}\times\overline{H})_*\Gamma$, where $\overline{H}(y)=\tilde\psi^1(0,y)$, $\psi^1(x,y)=(x,\tilde\psi^1(x,y))$, we may assume $j^{r-1}_{(0,0)}\psi^1=\mathrm{id}$ and

$$\psi^1(x,y)=\Big(x,y-\sum_{|\beta|\le r-1} y^\beta\sigma_\beta(x)\Big),$$

where $\sigma_\beta:\mathbb{R}^m\to\mathbb{R}^n$, $\sigma_\beta(0)=0$. Then (quite similarly to the inductive step in the proof of (a)) expressing $j^{r-1}_{(0,0)}(\psi^1_*\Gamma)$ by (2.2) with $\tilde\Gamma^k_{j\alpha\beta}$ instead of $\Gamma^k_{j\alpha\beta}$ we get

$$0=\Phi_r\Big(j_0^{r-1}\Big(\sum_{|\alpha|+|\beta|\le r-1}\sum_{j=1}^m\sum_{k=1}^n\tilde\Gamma^k_{j\alpha\beta}x^\alpha dx^j\otimes e_k\Big)\Big)=0-j_0^r(\sigma_\beta)$$

for any $\beta\in(\mathbb{N}\cup\{0\})^n$ with $|\beta|\le r-1$. So, $j^r_{(0,0)}\psi^1=\mathrm{id}$. ∎

3. An application to prolongation of connections. We are going to apply the result from the previous section to prolongation of connections. We start with the following lemma.

LEMMA 3.1. *Let $F:\mathcal{M}f\to\mathcal{VB}$ be a vector bundle functor. Let $\mathrm{pr}_1:\mathbb{R}^m\times\mathbb{R}^n\to\mathbb{R}^m$ be the trivial bundle (the obvious projection).*

(a) *The fibred manifold $F\,\mathrm{pr}_1:F(\mathbb{R}^m\times\mathbb{R}^n)\to F\mathbb{R}^m$ is isomorphic to the trivial bundle $\mathrm{Pr}_1:(\mathbb{R}^m\times F_0\mathbb{R}^m)\times(\mathbb{R}^n\times\ker(F_{(0,0)}\,\mathrm{pr}_1))\to\mathbb{R}^m\times F_0\mathbb{R}^m$ (the obvious projection). An isomorphism $\Phi:(\mathbb{R}^m\times F_0\mathbb{R}^m)\times(\mathbb{R}^n\times\ker(F_{(0,0)}\,\mathrm{pr}_1))\to F(\mathbb{R}^m\times\mathbb{R}^n)$ is given by*

$$\Phi((x,X),(y,Y))=F\tau_{(x,y)}(F_0i(X)+Y)$$

for any $x\in\mathbb{R}^m$, $X\in F_0\mathbb{R}^m$, $y\in\mathbb{R}^n$, $Y\in\ker(F_{(0,0)}\,\mathrm{pr}_1)$, where $i:\mathbb{R}^m\to\mathbb{R}^m\times\mathbb{R}^n$ is given by $i(x)=(x,0)$ and $\tau_{(x,y)}:\mathbb{R}^m\times\mathbb{R}^n\to\mathbb{R}^m\times\mathbb{R}^n$ is the translation by (x,y). The inverse isomorphism is given by

$$\Phi^{-1}(v)=((x,X),(y,Y))$$

for $v\in F_{(x,y)}(\mathbb{R}^m\times\mathbb{R}^n)$, $(x,y)\in\mathbb{R}^m\times\mathbb{R}^n$, where $X=F\tau_{-x}(F\,\mathrm{pr}_1(v))\in F_0\mathbb{R}^m$ and $Y=F\tau_{-(x,y)}(v)-F_0i(X)\in\ker(F_{(0,0)}\,\mathrm{pr}_1)$.

(b) *Let $H:\mathbb{R}^n\to\mathbb{R}^n$ be a local diffeomorphism. Then (under the above isomorphism)*

$$F(\mathrm{id}_{\mathbb{R}^m}\times H)((x,X),(y,Y))=((x,X),\tilde H(y,Y))$$

for any $((x,X),(y,Y))\in F(\mathbb{R}^m\times\mathbb{R}^n)$, where $\tilde H:\mathbb{R}^n\times\ker(F_{(0,0)}\,\mathrm{pr}_1)\to\mathbb{R}^n\times\ker(F_{(0,0)}\,\mathrm{pr}_1)$ is given by

$$\tilde H(y,Y)=(H(y),F_{(0,0)}(\mathrm{id}_{\mathbb{R}^m}\times(\tau_{-H(y)}\circ H\circ\tau_y))(Y)).$$

(c) *Let $A\in\mathrm{GL}(m)$. Then*

$$F(A\times\mathrm{id}_{\mathbb{R}^n})((x,X),(y,Y))=(A_1(x,X),A_2(y,Y))$$

for any $((x,X),(y,Y)) \in F(\mathbb{R}^m \times \mathbb{R}^n)$, *where* $A_1 : \mathbb{R}^m \times F_0\mathbb{R}^m \to \mathbb{R}^m \times F_0\mathbb{R}^n$ *is given by* $A_1(x,X) = (A(x), F_0A(X))$ *and* $A_2 : \mathbb{R}^n \times \ker(F_{(0,0)}\,\mathrm{pr}_1) \to \mathbb{R}^n \times \ker(F_{(0,0)}\,\mathrm{pr}_1)$ *is given by* $A_2(y,Y) = (y, F_{(0,0)}(A \times \mathrm{id}_{\mathbb{R}^n})(Y))$.

Proof. The proof is standard. ∎

We now present the following example of prolongation of connections.

Example 3.2. Let $F : \mathcal{M}f \to \mathcal{VB}$ be a vector bundle functor of order r. Consider a general connection Γ on a fibred manifold $p : Y \to M$ and a torsion free classical linear connection ∇ on M. Write $\dim(M) = m$ and $\dim(Y) = m+n$. We construct a general connection $\widetilde{\mathcal{F}}(\Gamma,\nabla)$ on $Fp : FY \to FM$ as follows. Let $u_0 \in F_{y_0}Y$, $y_0 \in Y$. Let ψ be a $(\Gamma,\nabla,y_0,r+1)$-quasi-normal fibred coordinate system (see Proposition 2.2(a)). We put

$$\widetilde{\mathcal{F}}(\Gamma,\nabla)(u_0) := J^1(F\psi^{-1})(\Theta(F\psi(u_0))),$$

where Θ denotes the trivial general connection on the trivial bundle $F\,\mathrm{pr}_1 : F(\mathbb{R}^m \times \mathbb{R}^n) \to F\mathbb{R}^m$ (modulo the isomorphism from Lemma 3.1(a)). Suppose ψ^1 is another $(\Gamma,\nabla,y_0,r+1)$-quasi-normal fibred coordinate system. Then (by Proposition 2.2(b)) $j^{r+1}_{y_0}(\psi^1) = j^{r+1}_{y_0}((A \times H) \circ \psi)$ for some $A \in \mathrm{GL}(m)$ and some local diffeomorphism $H : \mathbb{R}^n \to \mathbb{R}^n$. But Θ is invariant with respect to $F(A \times H)$ because of Lemma 3.1(b)–(c). Hence

$$J^1(F((\psi^1)^{-1}))(\Theta(F(\psi^1)(u_0))) = J^1(F\psi^{-1})(\Theta(F\psi(u_0))),$$

i.e. $\widetilde{\mathcal{F}}(\Gamma,\nabla)$ is correctly defined. As we can choose a family of $(\Gamma,\nabla,y_0,r+1)$-quasi-normal fibred coordinate systems to be smooth in y_0 (it is sufficient to analyse the proof of Proposition 2.2(a)), $\widetilde{\mathcal{F}}(\Gamma,\nabla)$ is a smooth general connection on $Fp : FY \to FM$.

4. A final remark. It seems that our quasi-normal fibred coordinate systems from Proposition 2.2 may be used in the classification problems of natural operators $A(\Gamma,\nabla)$ on pairs (Γ,∇) of connections. Indeed, according to the general theorem on natural operators [KMS], such natural operators are in bijection with $G^r_{m,n}$-invariant maps of respective type. Now, passing to quasi-normal fibred coordinate systems, we see that natural operators $A(\Gamma,\nabla)$ are in bijection with $\mathrm{GL}(m)\times G^r_n$-invariant maps. So, Proposition 2.2 gives a very strict reduction for natural operators on pairs (Γ,∇) of connections. We hope that (for example) we may benefit from the above reduction and find all natural operators A constructing general connections $A(\Gamma,\nabla)$ on $J^2Y \to M$ from general connections Γ on $Y \to M$ by means of torsion free classical linear connections ∇ on M. We note that a reduction for gauge natural operators on pairs (Γ,∇) of principal and classical linear connections has been described in [DM], [JV]. Some "special" principal coordinate systems for principal and classical connections have been described in [H], [DM], [K].

References

[DM] M. Doupovec and W. M. Mikulski, *Reduction theorems for principal and classical connections*, Acta Math. Sinica (Engl. Ser.) 26 (2010), 169–184.

[H] G. W. Horndeski, *Replacement theorems for concomitants of gauge fields*, Utilitas Math. 19 (1981), 215–246.

[K] I. Kolář, *On the gauge version of exponential map*, Rep. Math. Phys. 65 (2010), 241–246.

[KMS] I. Kolář, P. W. Michor and J. Slovák, *Natural Operations in Differential Geometry*, Springer, Berlin, 1993.

[JV] J. Janyška and J. Vondra, *Natural principal connections on the principal gauge prolongation of a principal bundle*, Rep. Math. Phys. 64 (2009), 395–415.

[M1] W. M. Mikulski, *Non-existence of some canonical constructions on connections*, Comment. Math. Univ. Carolin. 44 (2003), 691–695.

[M2] —, *On the existence of prolongations of connections by bundle functors*, Extracta Math. 22 (2007), 297–314.

[M3] —, *On prolongation of connections*, Ann. Polon. Math. 97 (2010), 101–121.

[M4] —, *Bundle functors with the point property which admit prolongation of connections*, ibid. 97 (2010), 253–256.

[S] J. Slovák, *Prolongations of connections and sprays with respect to Weil functors*, Rend Circ. Mat. Palermo (2) Suppl. 14 (1987), 143–155.

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Received 30.11.2009
and in final form 15.12.2009 (2120)