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Natural maps depending on reductions of frame bundles

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Abstract. We clarify how the natural transformations of fiber product preserving bundle functors on $\mathcal{FM}_m$ can be constructed by using reductions of the rth order frame bundle of the base, $\mathcal{FM}_m$ being the category of fibered manifolds with m-dimensional bases and fiber preserving maps with local diffeomorphisms as base maps. The iteration of two general r-jet functors is discussed in detail.

Consider a fibered manifold $p\colon Y \to M$ and its iterated jet prolongation $J^sJ^rY = J^s(J^rY \to M)$. M. Modugno [12] constructed an involutive map $\mathrm{ex}_\Lambda\colon J^1J^1Y \to J^1J^1Y$ depending on a classical connection Λ on M. In [9], Modugno and the author proved that the only natural transformation $J^1J^1Y \to J^1J^1Y$ is the identity and the only two natural transformations $J^1J^1Y \to J^1J^1Y$ depending on a torsion-free Λ are $\mathrm{id}_{J^1J^1Y}$ and ex_Λ. Using the Weil algebra technique, M. Doupovec and the author deduced that the only natural transformation $J^rJ^sY \to J^rJ^sY$ is the identity (see [1]). In [2], Doupovec and W. M. Mikulski constructed a map $J^rJ^sY \to J^sJ^rY$ depending naturally on Λ. Mikulski [11] discussed the natural transformations of two fiber product preserving bundle (for short: f.p.p.b.) functors on $\mathcal{FM}_m$ depending on Λ.

We present a certain generalization of the last result by Mikulski. First of all, we point out that our main idea appears already in the case of classical natural bundles over m-manifolds. In Section 1, we consider two such bundles $F_1M = P^rM[S_1]$, $F_2M = P^rM[S_2]$ and a map $\varphi\colon S_1 \to S_2$ that is K-equivariant with respect to a subgroup $K \subset G^r_m$ only. Then every reduction Q of P^rM to K determines a map $\varphi_Q\colon F_1M \to F_2M$ that is natural in Q. In the most important cases, K is the classical injection of G^1_m into G^r_m. Here we find an interesting application of our result from [3] (see also [5]) that the reductions of P^rM to G^1_m are in bijection with the torsion-free connections on $P^{r-1}M$. In particular, every classical torsion-free connection Λ on M

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defines such a reduction by means of the exponential map. As a simple illustration, we determine all vector bundle morphisms $J^1TM \to TM \otimes T^*M$ depending naturally on Λ.

Section 2 is of auxiliary character. We consider a bundle functor E on the product category $\mathcal{M}f_m \times \mathcal{M}f$ preserving products in the second factor. According to [8], these functors are determined by a Weil algebra A and a group homomorphism $H\colon G^r_m \to \operatorname{Aut} A$, where $\operatorname{Aut} A$ is the group of all algebra automorphisms of A. Proposition 1 states that for two such functors (A_i, H_i), $i = 1, 2$, and $K \subset G^r_m$, an K-equivariant algebra homomorphism $\mu\colon A_1 \to A_2$ and a K-reduction Q of P^rM determine a map $\varphi_{Q,N}\colon (A_1, H_1)(M, N) \to (A_2, H_2)(M, N)$ that is natural in Q.

In Section 3 we consider the general case of a f.p.p.b. functor F of $\mathcal{FM}_m$. Here we take into account the injection of categories $i\colon \mathcal{M}f_m \times \mathcal{M}f \to \mathcal{FM}_m$ transforming (M, N) into the product fibered manifold $M \times N \to M$. According to [8], F is determined by $\bar{F} = F \circ i = (A, H)$ and an equivariant algebra homomorphism $t\colon \mathbb{D}^r_m \to A$, $\mathbb{D}^r_m = J^r_0(\mathbb{R}^m, \mathbb{R})$. Proposition 2 states that for a fibered manifold $Y \to M$, $\dim M = m$, and two such functors $F_i = (A_i, H_i, t_i)$, $i = 1, 2$, every K-equivariant algebra homomorphism $\mu\colon A_1 \to A_2$ satisfying $t_2 = \mu \circ t_1$ and every K-reduction $Q \subset P^rM$ determine a map $\tilde{\mu}_{Q,Y}\colon F_1Y \to F_2Y$ that is natural in Q. In the last section, we deduce from Proposition 2 that there exists an exchange map of the iteration of two general nonholonomic jet functors depending naturally on a classical torsion-free connection on the base.

All manifolds and maps are assumed to be infinitely differentiable. Unless otherwise specified, we use the terminology and notation from the book [7].

1. The case of $\mathcal{M}f_m$. The classical natural bundles over m-manifolds, i.e. the bundle functors on the category $\mathcal{M}f_m$ of m-dimensional manifolds and local diffeomorphisms, are in bijection with the left actions $l\colon G^r_m \times S \to S$, where G^r_m means the rth jet group in dimension m, [7]. For every m-manifold M, FM is the bundle $P^rM[S, l]$ associated to the rth order frame bundle P^rM and, for a local diffeomorphism $f\colon M \to M'$, we have $Ff = P^rf[\mathrm{id}_S]$, i.e. $Ff(\{u, z\}) = \{P^rf(u), z\}$, $u \in P^rM$, $z \in S$. If $F_iM = P^rM[S_i, l_i]$, $i = 1, 2$, then natural transformations $F_1 \to F_2$ are in bijection with G^r_m-maps $\varphi\colon S_1 \to S_2$. The induced map $\varphi_M\colon F_1M \to F_2M$ is of the form

$$\varphi_M(\{u, z\}) = \{u, \varphi(z)\}, \qquad u \in P^rM,\ z \in S_1.$$

We are interested in the case where we have a subgroup $K \subset G^r_m$ and $\varphi\colon S_1 \to S_2$ is K-equivariant only. Then we can use a reduction $Q \subset P^rM$ to K. Both F_iM can be interpreted as associated bundles to Q, i.e. $F_iM =$

$Q[S_i]$, $i = 1, 2$, and we define

$$\varphi_Q \colon Q[S_1] \to Q[S_2],\ \varphi_Q(\{u, z\}) = \{u, \varphi(z)\}, \qquad u \in Q,\ z \in S_1.$$

This definition is correct, for

$$\begin{aligned}(1) \qquad \varphi_Q(\{u \circ k, l_1(k^{-1})(z)\}) &= \{u \circ k, \varphi(l_1(k^{-1}))(z)\} \\ &= \{u \circ k, l_2(k^{-1})(\varphi(z))\} = \varphi_Q(\{u, z\}), \qquad k \in K,\end{aligned}$$

by K-equivariance of φ.

If we consider a K-reduction Q' of $P^r M'$ that is f-related to Q, i.e. $P^r f(u) \in Q'$ for all $u \in Q$, then the following diagram commutes:

$$(2) \qquad \begin{array}{ccc} F_1 M & \xrightarrow{\ \varphi_Q\ } & F_2 M \\ {\scriptstyle F_1 f}\downarrow & & \downarrow{\scriptstyle F_2 f} \\ F_1 M' & \xrightarrow{\ \varphi_{Q'}\ } & F_2 M' \end{array}$$

Indeed, $F_2 f(\{u, \varphi(z)\}) = \{P^r f(u), \varphi(z)\}$. So we say that the maps φ_Q are natural with respect to the choice of K-reductions.

The most interesting case is $K = \iota_r(G^1_m)$, where $\iota_r \colon G^1_m \to G^r_m$ is the standard injection $\iota_r(a) = j^r_0 \tilde{a}$, $\tilde{a} \colon \mathbb{R}^m \to \mathbb{R}^m$ being the linear map determined by $a \in G^1_m$. Every classical torsion-free connection Λ on M defines a reduction $\exp^{r-1}_\Lambda \colon P^1 M \to P^r M$ to $\iota_r(G^1_m)$ as follows, [5]. The exponential map $\exp_{\Lambda,x}$ of Λ at x is a local map of $T_x M$ into M, $u \in P^1_x M$ can be interpreted as a linear map $\tilde{u} \colon \mathbb{R}^m \to T_x M$ and we define

$$(3) \qquad \exp^{r-1}_\Lambda(u) = j^r_0(\exp_{\Lambda,x} \circ \tilde{u}) \in P^r_x M.$$

If $Q = \exp^{r-1}_\Lambda(P^1 M)$, we say that $\varphi_Q =: \varphi_\Lambda$ is determined by Λ.

We recall there is a canonical $(\mathbb{R}^m \times \mathfrak{g}^{r-1}_m)$-valued one-form θ_r on $P^r M$ and the torsion of a connection Γ on $P^r M$ is the covariant exterior differential $D_\Gamma \theta_r$. In [3] (see also [5]) we deduced the following assertion.

Lemma 1. *There is a canonical bijection between torsion-free connections on $P^{r-1} M$ and the reductions of $P^r M$ to $\iota_r(G^1_m)$.*

Example. To illustrate this procedure, we consider a very simple case $J^1 TM \to TM \otimes T^* M$. Then S_1 is the G^2_m-space $\mathbb{R}^m \times \mathbb{R}^m \otimes \mathbb{R}^{m*}$ and S_2 is the G^1_m-space $\mathbb{R}^m \otimes \mathbb{R}^{m*}$ interpreted as a G^2_m-space by means of the jet projection $G^2_m \to G^1_m$. We see directly that all G^2_m-maps $S_1 \to S_2$ are the constant maps of S_1 into $k\, \mathrm{id}_{\mathbb{R}^m}$, $k \in \mathbb{R}$. In the case of $\iota_2(G^1_m) \subset G^2_m$, it is reasonable to restrict ourselves to the linear G^1_m-maps $S_2 \to S_1$. We find directly that all of them are of the form

$$(4) \qquad x^i_j = c_1 y^i_j + c_2 \delta^i_j y^k_k, \qquad c_1, c_2 \in \mathbb{R},$$

$(y^i, y^i_j) \in S_1$, $x^i_j \in S_2$. In some local coordinates x^i on M, the geodesics of Λ are determined by

$$\frac{d^2x^i}{dt^2} + \Lambda^i_{jk}(x)\frac{dx^j}{dt}\frac{dx^k}{dt} = 0. \tag{5}$$

Every frame from $\exp^1_\Lambda(P^1_x M)$ is characterized by $\Lambda^i_{jk}(x) = 0$. Hence (4) implies that all vector bundle morphisms $J^1TM \to TM \otimes T^*M$ determined by Λ form the 2-parameter family

$$(j^1_x s) \mapsto c_1(\nabla_\Lambda s)(x) + c_2\, Ctr((\nabla_\Lambda s)(x))\, \mathrm{id}_{T_xM}, \qquad c_1, c_2 \in \mathbb{R}, \tag{6}$$

where $\nabla_\Lambda s$ is the covariant differential of a section s of TM with respect to Λ and $Ctr((\nabla_\Lambda s)(x))$ means the contraction of this $(1,1)$-tensor.

2. The case of $\mathcal{M}f_m \times \mathcal{M}f$. According to [8], if we intend to study a f.p.p.b. functor F on $\mathcal{FM}_m$, we first have to discuss a bundle functor E on the product category $\mathcal{M}f_m \times \mathcal{M}f$ that preserves products in the second factor, i.e.

$$E(M, N_1 \times N_2) = E(M, N_1) \times_M E(M, N_2).$$

These functors are identified with pairs $E = (A, H)$ of a Weil algebra A and a group homomorphism $H\colon G^r_m \to \operatorname{Aut} A$. In general, every algebra homomorphism $\mu\colon A_1 \to A_2$ of two Weil algebras determines a natural transformation $\mu_M\colon T^{A_1}M \to T^{A_2}M$ of the corresponding Weil functors (see [4], [7]). This defines an action $H_N\colon g \mapsto H(g)_N$ of G^r_m on T^AN, and $E(M, N)$ is the corresponding associated bundle

$$(A, H)(M, N) = P^rM[T^AN, H_N].$$

If $f_1\colon M \to M'$ is a local diffeomorphism and $f_2\colon N \to N'$ is a smooth map, we have

$$(A, H)(f_1, f_2)(\{u, Z\}) = \{P^rf_1(u), T^Af_2(Z)\}, \qquad u \in P^rM,\ Z \in T^AN. \tag{7}$$

For two such functors $E_i = (A_i, H_i)$, $i = 1, 2$, the natural transformations $E_1 \to E_2$ are in bijection with G^r_m-equivariant algebra homomorphisms $\mu\colon A_1 \to A_2$, i.e.

$$\mu(H_1(g)(a)) = H_2(g)(\mu(a)), \qquad a \in A_1,\ g \in G^r_m \tag{8}$$

(see [8]). The induced map $\mu_{M,N}\colon (A_1, H_1)(M, N) \to (A_2, H_2)(M, N)$ is of the form

$$\mu_{M,N}(\{u, Z\}) = \{u, \mu_N(Z)\}, \qquad u \in P^rM,\ Z \in T^{A_1}N. \tag{9}$$

Suppose μ is K-equivariant, i.e. (8) holds for $g \in K \subset G^r_m$ only. If we take a K-reduction $Q \subset P^rM$, we may write $E_i(M, N) = Q[T^{A_i}N]$, $i = 1, 2$, and we can define $\mu_{Q,N}\colon E_1(M, N) \to E_2(M, N)$ by

$$\mu_{Q,N}(\{u, Z\}) = \{u, \mu_N(Z)\}, \qquad u \in Q,\ z \in T^{A_1}N. \tag{10}$$

Since μ is K-equivariant, this definition is correct. Further, if Q' is an f_1-related K-reduction of P^rM', then one verifies analogously to Section 1 that the following diagram commutes:

$$\begin{array}{ccc} (A_1,H_1)(M,N) & \xrightarrow{(A_1,H_1)(f_1,f_2)} & (A_1,H_1)(M',N') \\ \Big\downarrow{\scriptstyle \mu_{Q,N}} & & \Big\downarrow{\scriptstyle \mu_{Q',N'}} \\ (A_2,H_2)(M,N) & \xrightarrow{(A_2,H_2)(F_1,f_2)} & (A_2,H_2)(M',N') \end{array} \tag{11}$$

Thus, we have proved

PROPOSITION 1. *For two functors $E_i = (A_i, H_i)$, $i = 1, 2$, with the same r and m, and a subgroup $K \subset G^r_m$, every K-equivariant algebra homomorphism $\mu\colon A_1 \to A_2$ and every K-reduction $Q \subset P^rM$ determine a map $\mu_{Q,N}\colon E_1(M,N) \to E_2(M,N)$ that is natural in the sense of* (11).

3. The case of $\mathcal{FM}_m$. We have an injection of categories $i\colon \mathcal{M}f_m \times \mathcal{M}f \to \mathcal{FM}_m$ transforming (M,N) into the product fibered manifold $M \times N \to M$ and (f_1, f_2) into the product $\mathcal{FM}_m$-morphism $f_1 \times f_2$ with base map f_1. If F is a f.p.p.b. functor on $\mathcal{FM}_m$, then $\bar{F} := F \circ i$ is a bundle functor on $\mathcal{M}f_m \times \mathcal{M}f$ that preserves products in the second factor, so that $\bar{F} = (A, H)$. According to [8], F is identified with a triple $F = (A, H, t)$, where $t\colon \mathbb{D}^r_m \to A$ is an equivariant algebra homomorphism. Hence $t_M\colon T^r_m M \to T^A M$. For a fibered manifold $p\colon Y \to M$, we have $FY \subset \bar{F}(M,Y) = P^rM[T^AY]$ and

$$\{u, Z\} \in FY \text{ means } t_M(u) = T^Ap(Z) \in T^AM,\ u \in P^rM,\ Z \in T^AY, \tag{12}$$

where $T^Ap\colon T^AY \to T^AM$ and we use $P^rM \subset T^r_mM$. Let $p'\colon Y' \to M'$ be another fibered manifold, $\dim M' = m$, and $f\colon Y \to Y'$ be an $\mathcal{FM}_m$-morphism with base map $\underline{f}\colon M \to M'$. Then $Ff\colon FY \to FY'$ is the restriction and corestriction of $\bar{F}(\underline{f}, f)\colon \bar{F}(M,Y) \to \bar{F}(M',Y')$.

Consider $F_i = (A_i, H_i, t_i)$, $i = 1, 2$. According to [8], the natural transformations $F_1 \to F_2$ are in bijection with G^r_m-equivariant algebra homomorphisms $\mu\colon A_1 \to A_2$ satisfying $t_2 = \mu \circ t_1$. The corresponding map $\tilde{\mu}_Y\colon F_1Y \to F_2Y$ is of the form

$$\tilde{\mu}_Y(\{u, Z\}) = \{u, \mu_Y(Z)\}, \qquad u \in P^rM,\ Z \in T^{A_1}Y, \tag{13}$$

where $\mu_Y\colon T^{A_1}Y \to T^{A_2}Y$ is the map determined by μ in the manifold case.

Assume again that μ is K-equivariant only, $K \subset G^r_m$, and we have a K-reduction $Q \subset P^rM$. Then $F_iY \subset Q[T^{A_i}Y]$, $i = 1, 2$, and there is a restricted and corestricted map $\tilde{\mu}_{Q,Y}$,

(14)
$$\begin{array}{ccc} F_1Y & \xrightarrow{\tilde{\mu}_{Q,Y}} & F_2Y \\ \downarrow & & \downarrow \\ Q[T^{A_1}Y] & \xrightarrow{\mu_{Q,Y}} & Q[T^{A_2}Y] \end{array}$$

Analogously to Section 2, one verifies directly that the following diagram commutes:

(15)
$$\begin{array}{ccc} F_1Y & \xrightarrow{\tilde{\mu}_{Q,Y}} & F_2Y \\ {\scriptstyle F_1f}\downarrow & & \downarrow{\scriptstyle F_2f} \\ F_1Y' & \xrightarrow{\tilde{\mu}_{Q',Y'}} & F_2Y' \end{array}$$

Thus, we have proved

PROPOSITION 2. *For two functors $F_i = (A_i, H_i, t_i)$, $i = 1, 2$, with the same r and m, and a subgroup $K \subset G^r_m$, every K-equivariant algebra homomorphism $\mu: A_1 \to A_2$ satisfying $t_2 = \mu \circ t_1$ and every K-reduction $Q \subset P^rM$ determine a map $\tilde{\mu}_{Q,Y}: F_1Y \to F_2Y$ that is natural in the sense of* (15).

4. Iteration of general nonholonomic jet functors. The rth nonholonomic prolongation $\tilde{J}^rY$ of Y is introduced by the iteration $\tilde{J}^rY = J^1(\tilde{J}^{r-1}Y \to M)$, $\tilde{J}^1Y = J^1Y$. The bundle $\tilde{J}^r(M, N)$ of nonholonomic r-jets of M into N is defined as $\tilde{J}^r(M \times N \to M)$. We have $J^rY \subset \tilde{J}^rY$ and $J^r(M, N) \subset \tilde{J}^r(M, N)$. The composition $Z \circ X \in \tilde{J}^r_x(M, Q)_z$ of $X \in \tilde{J}^r_x(M, N)_y$ and $Z \in \tilde{J}^r_y(N, Q)_z$ coincides with the classical one for holonomic r-jets (see [4]). We write $\beta^r_Y: \tilde{J}^rY \to Y$ for the target jet projection. If we consider $\tilde{J}^s\tilde{J}^rY = \tilde{J}^s(\tilde{J}^rY \to M)$, we have the target projection $\beta^s_{\tilde{J}^rY}: \tilde{J}^s\tilde{J}^rY \to \tilde{J}^rY$ and the induced map $\tilde{J}^s\beta^r_Y: \tilde{J}^s\tilde{J}^rY \to \tilde{J}^sY$. A map $\mathrm{e}: \tilde{J}^s\tilde{J}^rY \to \tilde{J}^r\tilde{J}^sY$ is called an *exchange* if $\tilde{J}^r\beta^s_Y \circ \mathrm{e} = \beta^s_{\tilde{J}^rY}$ and $\beta^r_{\tilde{J}^sY} \circ \mathrm{e} = \tilde{J}^s\beta^r_Y$. We use Proposition 2 to prove the following assertion.

PROPOSITION 3. *Every classical torsion-free connection Λ on M determines an exchange map $\mathrm{ex}_\Lambda: \tilde{J}^s\tilde{J}^rY \to \tilde{J}^r\tilde{J}^sY$.*

Proof. According to iteration theory (see [1]), the Weil algebra $\tilde{\mathbb{D}}^r_m$ of $\tilde{J}^r$ is the tensor product $\tilde{\mathbb{D}}^r_m = \otimes^r\mathbb{D}^1_m$, $\mathbb{D}^1_m = J^1_0(\mathbb{R}^m, \mathbb{R}) = \mathbb{R} \times \mathbb{R}^{m*}$. The corresponding action of $\iota_r(G^1_m)$ on $\tilde{\mathbb{D}}^r_m$ is the tensor product of the identities on $\mathbb{R}$ and the classical actions of G^1_m on $\mathbb{R}^{m*}$. By iteration theory, the Weil algebra of $\tilde{J}^s\tilde{J}^r$ is $\tilde{\mathbb{D}}^s_m \otimes \tilde{\mathbb{D}}^r_m$ and the corresponding action of $\iota_{r+s}(G^1_m)$ is of the same type. Hence the exchange map $\mathrm{e}: \tilde{\mathbb{D}}^s_m \otimes \tilde{\mathbb{D}}^r_m \to \tilde{\mathbb{D}}^r_m \otimes \tilde{\mathbb{D}}^s_m$ is $\iota_{r+s}(G^1_m)$-equivariant. Using the exponential map of Λ, we construct ex_Λ. ■

Remark. We underline that for $r > 1$ or $s > 1$ there is no hope for the uniqueness result as in the case $r = s = 1$ mentioned in the introduction. By Lemma 1, instead of $\exp_\Lambda^{r+s-1}$ we can use any natural operator transforming Λ into a torsion-free connection on $P^{r+s}M$. All those operators were characterized by Mikulski [10], with an addendum concerning the torsion-free case in [5].

This approach works even if we replace $\tilde{J}^r$ by a general r-jet category C introduced in [6]. This is a rule transforming every pair (M, N) of manifolds into a fibered submanifold $C(M, N) \subset \tilde{J}^r(M, N)$ such that

(i) $J^r(M, N) \subset C(M, N)$ is a fibered submanifold,
(ii) if $X \in C_x(M, N)_y$ and $Z \in C_y(N, Q)_z$, then $Z \circ X \in C_x(M, Q)_z$,
(iii) if $X \in C_x(M, N)_y$ is regular, i.e. there exists $W \in \tilde{J}_y^r(N, M)_x$ such that $W \circ X = j_x^r \operatorname{id}_M$, then there exists $Z \in C_y(M, N)_x$ with this property,
(iv) $C(M, N \times Q) = C(M, N) \times_M C(M, Q)$.

For every fibered manifold $p\colon Y \to M$, we define its *horizontal C-prolongation*

$$C_hY = \{X \in C(M, Y), (j_{\beta X}^r p) \circ X = j_{\alpha X}^r \operatorname{id}_M\},$$

αX or βX being the source or target of X, and its *vertical C-prolongation*

$$C_vY = \bigcup_{x \in M} C_x(M, Y_x) \subset C(M, Y).$$

Clearly, $\tilde{J}_h^r Y = \tilde{J}^r Y$. If we restrict C_hY or C_vY to fibered manifolds with m-dimensional bases, we obtain a f.p.p.b. functor $C_{h,m}$ or $C_{v,m}$ on $\mathcal{FM}_m$, provided the values of $C_{h,m}$ or $C_{v,m}$ on $\mathcal{FM}_m$-morphisms are defined by means of the jet composition.

If we consider another general s-jet category C', then the proof of Proposition 3 works for every pair from $C_{h,m}$, $C_{v,m}$, $C'_{h,m}$, $C'_{v,m}$.

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