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QUASI-NILPOTENT AND COMPACT

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Let $A(z)$ be an analytic family of compact operators in the open unit disc $\mathbb{D}$. Suppose that $A(z_n)$ are quasi-nilpotent for some infinite sequence $z_n \to 0$. Does it follow that the operators $A(z)$ are quasi-nilpotent for all $z \in \mathbb{D}$?

Remark 1. For a family consisting of finite rank operators, the positive answer can be obtained by observing that the traces of the powers of $A(z)$ are identically zero on $\mathbb{D}$; see [1, Theorem I.2.2] and [2, Theorem 2(j)].

Remark 2. Let

$$F(\lambda, z) = I - \lambda A(z),$$

where $\lambda \in \mathbb{C}$ and $z \in \mathbb{D}$. The above assumption says that $F(\lambda, z_n)$ are invertible for all $\lambda \in \mathbb{C}$ and some $z_n \to 0$. The question is, in fact, whether the Fredholm operators $F(\lambda, z)$ of index zero are actually invertible for all $\lambda \in \mathbb{C}$ and all $z \in \mathbb{D}$.

References

[1] H. Bart, *Meromorphic Operator Valued Functions*, Mathematisch Centrum, Amsterdam, 1973.

[2] D. Petz and J. Zemánek, *Characterizations of the trace*, Linear Algebra Appl. 111 (1988), 43–52.

