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# SUBNORMALITY FROM BOUNDED VECTORS

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For a densely defined operator $S$ in a Hilbert space $\mathcal{H}$ having invariant domain, that is, $S\mathcal{D}(S) \subset \mathcal{D}(S)$, consider the following *positive definiteness* condition:

$$\sum_{i,j=0}^{p} \langle S^i f_j, S^j f_i \rangle \geq 0, \qquad f_0, \dots, f_p \in \mathcal{D}(S). \tag{1}$$

Moreover, for a densely defined operator $A$ in $\mathcal{H}$, $f \in \bigcap_{n=1}^{\infty} \mathcal{D}(A^n)$ is said to be a *bounded vector* if

$$\|A^n f\| \leq ab^n, \qquad n \in \mathbb{N},$$

with $a$ and $b$ depending on $f$.

QUESTION. *Is $S$ subnormal if it satisfies* (1) *and the set of bounded vectors of $S^*$ is dense?*

## References

[1] F. H. Szafraniec, *Bounded vectors for subnormality via a group of unbounded operators*, Contemp. Math. 341 (2004), 113–118.

 