BANACH ALGEBRAS 2009
BANACH CENTER PUBLICATIONS, VOLUME 91
INSTITUTE OF MATHEMATICS
POLISH ACADEMY OF SCIENCES
WARSZAWA 2010

COMPACTNESS OF DERIVATIONS FROM COMMUTATIVE BANACH ALGEBRAS

MATTHEW J. HEATH

Departamento de Matemática, Instituto Superior Técnico
Av. Rovisco Pais, 1049-001 Lisboa, Portugal
E-mail: mheath@math.ist.utl.pt

Abstract. We consider the compactness of derivations from commutative Banach algebras into their dual modules. We show that if there are no compact derivations from a commutative Banach algebra, A, into its dual module, then there are no compact derivations from A into any symmetric A-bimodule; we also prove analogous results for weakly compact derivations and for bounded derivations of finite rank. We then characterise the compact derivations from the convolution algebra $\ell^1(\mathbb{Z}_+)$ to its dual. Finally, we give an example (due to J. F. Feinstein) of a non-compact, bounded derivation from a uniform algebra A into a symmetric A-bimodule.

1. Introduction. The question of the compactness of endomorphisms of Banach algebras has been studied in, for example [10], [7], and [8]. In this paper we consider compactness for another class of maps of interest in Banach algebra theory, derivations from a Banach algebra to its dual. In [3] Yemon Choi and the present author showed that all derivations from the disc algebra to its dual are compact. In [4] the same two authors characterised when derivations from $\ell^1(\mathbb{Z}_+)$ to its dual are weakly compact.

1.1. Definitions and notation. Throughout we shall take all Banach spaces to be over the field of complex numbers.

Let A be a Banach algebra. Recall that a *Banach A-bimodule* is a Banach space together with two bilinear maps $A \times E \to E$ denoted $(a, x) \mapsto a \cdot x$ and $(a, x) \mapsto x \cdot a$ such that:

$$a \cdot (b \cdot x) = (ab) \cdot x,\ (x \cdot a) \cdot b = x \cdot (ab),\ a \cdot (x \cdot b) = (a \cdot x) \cdot b \qquad (a, b \in A, x \in E).$$

Clearly, if we define both actions to be the product, A becomes an A-bimodule. If E and F are Banach A-bimodules we call a linear map $R : E \to F$ an *A-bimodule* homomorphism

2010 *Mathematics Subject Classification*: Primary 46J10; Secondary 46H25, 46J05.
Key words and phrases: commutative Banach algebras, compact derivations, convolution Banach algebras, uniform algebras.
The paper is in final form and no version of it will be published elsewhere.

DOI: 10.4064/bc91-0-11

if

$$R(a \cdot e) = a \cdot R(e),\ R(e \cdot a) = R(e) \cdot a \qquad (a \in A, e \in E).$$

We say E is a *symmetric* A-bimodule if, for all $a \in A$ and all $x \in E$, we have $a \cdot x = x \cdot a$.

Let A be an algebra and E an A-bimodule. We call a linear map $D : A \to E$ a *derivation* if the following identity holds for all $a, b \in A$:

$$D(ab) = a \cdot D(b) + D(a) \cdot b.$$

The derivation D is called *inner* if there is $e \in E$ such that $D(a) = a \cdot e - e \cdot a$ for all $a \in A$. We call this inner derivation δ_e. If E is symmetric, it is clear that the only inner derivation from A into E is the zero derivation.

For a Banach space E we denote the topological dual of E by E^*. If A is a Banach algebra and E a Banach A-bimodule we make E^* into a Banach A-bimodule by defining the actions

$$(a \cdot \psi)(x) = \psi(x \cdot a),\ (\psi \cdot a)(x) = \psi(a \cdot x), \qquad (a \in A, \psi \in E^*, x \in E).$$

We denote the open unit ball of a Banach space E by $B(E)$.

2. Results

2.1. General results. Recall the well-known result, due to Bade *et al.* ([1], also found as [5, 2.8.63(iii)]), that, if a commutative Banach algebra A has no non-zero, bounded derivations into A^* (i.e. if A is weakly amenable), then it has no non-zero, bounded derivations into any symmetric A-bimodule. In this subsection we prove analogues of this result with "bounded" replaced by "compact", by "weakly compact" and by "bounded and of rank less than n" for some $n \in \mathbb{N}$.

We shall need the following lemma, which is a stronger version of [5, 2.8.63(i)].

LEMMA 2.1. *Let A be a Banach algebra with no non-zero, non-inner, bounded derivations of rank* 1, *into A^*. Then $\overline{A^2} = A$.*

Proof. We prove this result in the contrapositive. Let A be a Banach algebra such that $\overline{A^2} \neq A$; we shall construct the required derivation. Take $a_0 \in A \backslash \overline{A^2}$. By the Hahn-Banach theorem we may choose $\lambda_0 \in A^*$ with $\lambda_0|_{A^2} = 0$ and $\lambda_0(a_0) = 1$. We define a function as follows:

$$D : A \to A^*, \qquad a \mapsto \lambda_0(a)\lambda_0.$$

It is clear that D is a bounded linear map. Also, $D(A) = \lambda_0\mathbb{C}$ and so D is of rank 1. Since $\lambda_0(A^2) = 0$, we have, for $a, b, c \in A$,

$$D(ab)(c) = \lambda_0(ab)\lambda_0(c) = 0$$

and

$$(a \cdot D(b) + D(a) \cdot b)(c) = D(b)(ca) + D(a)(bc) = \lambda_0(b)\lambda_0(ca) + \lambda_0(a)\lambda_0(bc) = 0.$$

Hence, $D(ab) = a \cdot D(b) + D(a) \cdot b = 0$ and so D is a derivation. Furthermore,

$$\|D(a_0)(a_0)\| = |\lambda_0(a_0)||\lambda_0(a_0)| = 1$$

while, for each $\lambda \in A^*$,

$$\delta_\lambda(a_0)(a_0) = (a_0 \cdot \lambda)(a_0) - (\lambda \cdot a_0)(a_0) = \lambda(a_0^2) - \lambda(a_0^2) = 0.$$

Thus D is not inner, and so the result follows. ∎

We shall also need the following elementary lemma (see [5, 2.6.6(i)]).

LEMMA 2.2. *Let A be a commutative Banach algebra, let E be a symmetric A-bimodule and let $\lambda \in E^*$. Then there is a bounded A-bimodule homomorphism $R_\lambda : E \to A^*$ such that*

$$R_\lambda(x)(a) = \lambda(a \cdot x) \quad (a \in A, x \in E).$$

We can now prove the main result of this subsection. Each part of the proof follows the pattern of [5, 2.8.63(iii)].

THEOREM 2.3. *Let A be a commutative Banach algebra. Then the following are true:*

1. *if A has no non-zero, bounded, derivations of rank less than $n \in \mathbb{N}$ into A^* then it has no non-zero derivations of rank less than n into any symmetric Banach A-bimodule E;*
2. *if A has no non-zero, compact derivations into A^* then it has no non-zero compact derivations into any symmetric Banach A-bimodule E;*
3. *if A has no non-zero, weakly compact derivations into A^* then it has no non-zero weakly compact derivations into any symmetric Banach A-bimodule E.*

Proof. In each case we shall assume, towards a contradiction, that such a derivation exists. First, let E be a symmetric A-bimodule and let $D : A \to E$ be any non-zero, bounded derivation. By Lemma 2.1, $\overline{A^2} = A$, and so there is $a_0 \in A$ with $D\left(a_0^2\right) \neq 0$. Thus, $a_0 \cdot D(a_0) = 1/2D\left(a_0^2\right) \neq 0$, and so, by the Hahn-Banach theorem, there exists $\lambda_D \in E^*$ such that $\lambda_D(a_0 \cdot D(a_0)) = 1$. By Lemma 2.2 there is a continuous A-bimodule homomorphism $R_{\lambda_D} : E \to A^*$ such that $R_{\lambda_D}(x)(a) = \lambda_D(a \cdot x)$ for each $x \in E$. Now let $D' = R_{\lambda_D} \circ D : A \to A^*$. Clearly D is a bounded linear map, and since R_{λ_D} is an A-bimodule homomorphism we have, for $a, b \in A$,

$$\begin{aligned} D'(ab) &= R_{\lambda_D}(D(ab)) = R_{\lambda_D}(a \cdot D(b) + b \cdot D(a)) \\ &= a \cdot R_{\lambda_D}(D(b)) + b \cdot R_{\lambda_D}(D(a)) = a \cdot D'(b) + b \cdot D'(a). \end{aligned}$$

Thus, D' is a derivation. Also, $D'(a_0)(a_0) = \lambda_D(a_0 \cdot D(a_0)) = 1$ and so $D' \neq 0$.

To show part (1) we now let $n \in \mathbb{N}$ and D be a bounded derivation of rank less than n. Then $D'(A) = R_{\lambda_D}(D(A))$ is a linear image of a space of dimension less than n. Hence, $D'(A)$ has dimension less than n and so D' has rank less than n.

To show part (2) we let D be a compact derivation. Then

$$\overline{R_{\lambda_D}\big(\overline{D(B(A))}\big)} \supseteq \overline{D'(B(A))} = \overline{R_{\lambda_D}(D(B(A)))} \supseteq R_{\lambda_D}\big(\overline{D(B(A))}\big). \tag{1}$$

Now, since D is compact, $\overline{D(B(A))}$ is compact and so $R_{\lambda_D}\big(\overline{D(B(A))}\big)$ is compact. In particular, $R_{\lambda_D}\big(\overline{D(B(A))}\big)$ is closed. Thus, (1) gives

$$\overline{R_{\lambda_D}\big(\overline{D(B(A))}\big)} = \overline{D'(B(A))} = R_{\lambda_D}\big(\overline{D(B(A))}\big), \tag{2}$$

and so $\overline{D'(B(A))}$ is compact. Hence, D' is a compact linear map.

To show part (3) let D be a weakly compact derivation. Since D is weakly compact, $\overline{D(B(A))}$ is weakly compact and so $R_{\lambda_D}(\overline{D(B(A))})$ is weakly compact since bounded linear maps are weak-weak continuous. Thus equation (1) holds with the closures taken in the weak topology, and so the weak closure of $D'(B(A))$ is weakly compact. Hence, D' is a weakly compact linear map.

In each case we have a contradiction and so the result follows. ∎

2.2. Compact derivations from $\ell^1(\mathbb{Z}^+)$. In this section we look at compactness of derivations from the semigroup algebra $\ell^1(\mathbb{Z}^+)$—that is, the Banach space $\ell^1(\mathbb{Z}^+)$ together with the product

$$ab := \Big(\sum_{r=0}^{n} a_r b_{n-r} : n \in \mathbb{Z}^+\Big)_{n\in\mathbb{Z}^+}, \quad (a = (a_n)_{n\in\mathbb{Z}^+}, b = (b_n)_{n\in\mathbb{Z}^+} \in \ell^1)$$

—into its dual. It is standard (see for example [5, 2.1.13(v)]) that this is a Banach algebra, which we shall call A, and that c_{00} is dense in A. We identify c_{00} with the algebra $\mathbb{C}[t]$ of complex valued polynomials in one variable, so that the sequence $(0, 1, 0, \dots) = t$. It is standard that $\phi \mapsto (\phi(t^k))_{k\in\mathbb{Z}^+}$ is an isometric linear isomorphism from A^* to ℓ^∞. The following proposition follows trivially from [2, Lemma 3.3.1]. We provide a direct proof for the convenience of the reader.

Proposition 2.4. *Let $\phi \in A^*$. The following are equivalent:*

1. $(n\phi(t^{n-1}))_{n\in\mathbb{N}} \in \ell^\infty$,
2. $\phi = D(t)$ *for some continuous derivation* $D : A \to A^*$.

Furthermore $\|(n\phi(t^{n-1}))_{n\in\mathbb{Z}^+}\|_\infty = \|D\|$.

Proof. We first show that (1) implies (2) and that $\|(n\phi(t^{n-1}))_{n\in\mathbb{Z}^+}\|_\infty \ge \|D\|$. Simple algebra yields that, for every $\phi \in A^*$, there is a unique derivation, D, from $\mathbb{C}[t]$ into A^* with $\phi = D(t)$. By the derivation identity we have $D(t^k)(t^n) = k(t^{k-1}) \cdot \phi(t^n) = k\phi(t^{k+n-1})$ and so, if we let f be the polynomial $f = \sum_{k=0}^{N} a_k t^k$, we have, by linearity,

$$D(f)(t^n) = \sum_{k=1}^{N} k a_k \phi(t^{k+n-1}).$$

Hence, since $\psi \mapsto (\psi(t^k))_{n\in\mathbb{N}}$ is an isometric isomorphism,

$$\|D(f)\| = \sup_{n\in\mathbb{N}} \Big|\sum_{k=1}^{N} k a_k \phi(t^{k+n-1})\Big|.$$

For each $n \ge 0$,

$$|k\phi(t^{k+n-1})| \le |(k+n)\phi(t^{k+n-1})| \le \|(n\phi(t^{n-1}))_{n\in\mathbb{Z}^+}\|_\infty,$$

and so

$$\|D(f)\| \le \sup_{k,n\in\mathbb{N}} |k\phi(t^{k+n-1})| \sum_{k=0}^{N} |a_k| \le \|(n\phi(t^{n-1}))_{n\in\mathbb{Z}^+}\|_\infty \|f\|_1.$$

Hence D is bounded with norm at most $\|(n\phi(t^{n-1}))\|_\infty$ and so extends continuously to a derivation $D : A \to A^*$ with $\|D\| \le \|(n\phi(t^{n-1}))\|_\infty$.

To prove that (2) implies (1) and that $\|D\| \geq \|(n\phi(t^{n-1}))\|_\infty$, note that

$$D(t^k)(1) = kt^{k-1} \cdot \phi(1) = k\phi(t^{k-1}),$$

and so

$$|k\phi(t^{k-1})| = |D(t^k)(1)| \leq \|D\|.$$

Hence $\|D\| \geq \|(n\phi(t^{n-1})\|_\infty$. The result follows. ∎

We denote the space of bounded derivations from A to A^*, given the operator norm, by $\mathcal{D}(A)$.

COROLLARY 2.5. *The map*

$$T : \mathcal{D}(A) \to A^*, \qquad D \mapsto D(\cdot)(1)$$

is an isometric isomorphism.

Proof. By the derivation identity, $D(t^k)(1) = kD(t)(t^{k-1})$, and so

$$\|D(\cdot)(1)\| = \|(D(t^k)(1))_{k\in\mathbb{Z}^+}\|_\infty = \|(kD(t)(t^{k-1}))_{k\in\mathbb{Z}^+}\|_\infty,$$

which is equal to $\|D\|$ by Proposition 2.4. ∎

THEOREM 2.6. *A bounded derivation $D : A \to A^*$ is compact if and only if it has $(D(t^n)(1))_{n\in\mathbb{N}} \in c_0$.*

Proof. If $(D(t^n)(1))_{n\in\mathbb{N}} \in c_0$, then, by Corollary 2.5, we have that it is in the closure of the set $\{D : (D(t^n)(1))_{n\in\mathbb{N}} \in c_{00}\}$, which consists of finite rank derivations. Hence D is compact. Now let $D : A \to A^*$ be a derivation such that $(D(t^n)(1))_{n\in\mathbb{N}} \in \ell^\infty \setminus c_0$. We shall show that the sequence $(D(t^k))_{k\in\mathbb{N}}$ has a subsequence with no convergent subsubsequence. Without any loss of generality, we assume that D has $\|D\| = 1$. There exists $\varepsilon > 0$ and a sequence, $(n_k)_{k\in\mathbb{N}} \subseteq \mathbb{N}$, such that, for all $k \in \mathbb{N}$, $n_k > n_{k-1}$ and $|D(t^{n_k})(1)| > \varepsilon$. Let $k, l \in \mathbb{N}$. Then

$$|D(t^k)(t^l)| = |kt^{k+l-1} \cdot D(t)(1)| = \frac{k}{k+l}|D(t^{k+l})(1)| \leq \frac{k}{k+l}. \tag{3}$$

Now suppose that $k + l \in \{n_k : k \in \mathbb{N}\}$. Then

$$|D(t^k)(t^l)| = |kt^{k+l-1} \cdot D(t)(1))| = \frac{k}{k+l}|D(t^{k+l})(1)| \geq \frac{\varepsilon k}{k+l}. \tag{4}$$

Suppose that we have already chosen $j_1, \ldots, j_{k-1} \in \mathbb{N}$ such that for all $i, i' \in \mathbb{N}$ with $i < i' < k$, we have $j_i < j_{i'}$ and $\|D(t^{j_i}) - D(t^{j_{i'}})\| > \frac{\varepsilon}{4}$. Choose $N \in \{n_k : k \in \mathbb{N}\}$ with $N > 1000\varepsilon^{-1}j_{k-1}$, and let $l_k = \lfloor N/2 \rfloor$ and $j_k = N - l_k$. Then, by (4),

$$|D(t^{j_k})(t^{l_k})| \geq \frac{\varepsilon j_k}{N} > \frac{\varepsilon}{3}.$$

Also, if $m \leq j_{k-1}$, then, by (3),

$$|D(t^m)(t^{l_k})| \leq \frac{m}{m+l_k} \leq \frac{j_{k-1}}{250\varepsilon^{-1}j_{k-1}} = \frac{\varepsilon}{250}.$$

Thus

$$|D(t^m)(t^{l_k}) - D(t^{j_k})(t^{l_k})| > \frac{\varepsilon}{4}.$$

In particular, if $i < k$, then $\|D(t^{j_i}) - D(t^{j_k})\| > \frac{\varepsilon}{4}$. Hence, by induction, we obtain a sequence, $(j_i)_{i\in\mathbb{N}}$, such that, if $i, k \in \mathbb{N}$ and $i \neq k$ then $\|D(t^{j_i}) - D(t^{j_k})\| > \frac{\varepsilon}{4}$. Thus $(D(t^{j_i}))_{i\in\mathbb{N}}$ has no convergent subsequence, and so, D is not compact. ∎

We conclude that the space of compact derivations on A is linearly isomorphic to c_0.

We finish with a relevant example due to J. F. Feinstein that appears in the present author's PhD thesis [9].

2.3. A non-compact, bounded derivation from a uniform algebra. For a compact Hausdorff space X let $C(X)$ be the algebra of continuous functions from X to $\mathbb{C}$ equipped with the uniform norm, which we denote by $|\cdot|_X$. For a compact subset, X, of the complex plane we let $R_0(X)$ be the algebra of rational functions with no poles contained in X. We let $R(X)$ be the closure of $R_0(X)$ in $C(X)$. Let Δ be the closed unit disc. We shall construct a plane set X by removing a sequence, $(D_n)_{n\in\mathbb{N}}$, of open discs from Δ such that there is a non-compact, bounded derivation from $R(X)$ into a symmetric Banach $R(X)$-bimodule. We shall need the following result, which is [6, Lemma 3].

PROPOSITION 2.7. *Let Δ be the closed unit disc and $(D_n)_{n\in\mathbb{N}}$ a sequence of open discs each contained in Δ. Set*

$$X := \Delta \setminus \bigcup_{i=1}^{\infty} D_n.$$

We set $r_n = r(D_n)$ and for each $z \in X$ we set $s_n(z) = \operatorname{dist}(z, D_n)$. We also set $r_0 = 1$ and $s_0(z) = 1 - |z|$. If $s_n(z) > 0$ for all $n \in \mathbb{N}$ then for $f \in R_0(X)$ we have

$$|f'(z)| \leq \sum_{j=0}^{\infty} \frac{r_j}{s_j(z)^2} |f|_X.$$

EXAMPLE 2.8. Let $I = \left[0, \frac{1}{2}\right]$. For any compact plane set X with $I \subseteq X$, we make $C(I)$ a symmetric Banach $R(X)$-bimodule by defining the action

$$(f \cdot g)(x) = (g \cdot f)(x) = f(x)g(x), \quad f \in R(X),\ g \in C(I).$$

It is clear that the map $D : R_0(X) \to C(I)$ given by $D(f) = f'|_I$ is a derivation. We shall construct a collection $\{D_n : n \in \mathbb{N}\}$ of disjoint open discs contained in Δ such that, setting $X = \Delta \setminus \bigcup_{n=1}^{\infty} D_n$, we have $I \subseteq X$ and such that the derivation D is bounded and so extends by continuity to a bounded derivation $R(X) \to C(I)$ which is not compact. For $n \in \mathbb{N}$, let $I_n = \left[\frac{1}{2} - 2^{-n}, \frac{1}{2} - 2^{-(n+2)}\right[$ and $x_n = \frac{1}{2} - 3 \cdot 2^{-(n+1)}$; that is, x_n is the midpoint of I_n. Choose $y_n \in \left]0, 1\right[$ small enough that

$$x_n + iy_n \in \Delta, \tag{5}$$

$$\frac{1}{(1 - y_n)^2} < 2, \tag{6}$$

$$\frac{y_n^2}{\left(((2^{-2(n+2)}) - y_n^2)^{\frac{1}{2}} + y_n^2\right)^2} < 2^{-(n+1)}. \tag{7}$$

Set $a_n = x_n + iy_n$, $r_n = y_n^2$, $D_n = B(a_n, r_n)$ and $X = \Delta \setminus \bigcup_{n=1}^{\infty} D_n$. We also set $r_0 = 1$. Let $z \in X$. We let $s_0(z) = 1 - |z|$ and for $n \in \mathbb{N}$, let $s_n(z) = \operatorname{dist}(z, D_n)$. Now let $x \in I$. Then $s_0(x) \geq \frac{1}{2}$ so $\frac{r_0}{s_0(x)^2} = s_0(x)^{-2} \leq 4$. Also, either $x = \frac{1}{2}$, in which case, for each

$j \in \mathbb{N}$, $s_j(x) \geq \operatorname{dist}(D_j, \mathbb{R} \setminus I_j) = \left(2^{-2(j+2)} + y_j^2\right)^{\frac{1}{2}} - y_j^2$; or there exists a unique $n \in \mathbb{N}$ such that $x \in I_n$. In this second case, for $j \in \mathbb{N}$,

$$s_j(x) \geq \begin{cases} \operatorname{dist}(D_n, \mathbb{R}) = y_n - r_n = y_n - y_n^2 & \text{if } j = n, \\ \operatorname{dist}(D_j, \mathbb{R} \setminus I_j) \geq (2^{-2(n+1)} + y_j^2)^{\frac{1}{2}} - y_j^2 & \text{if } j \neq n. \end{cases} \tag{8}$$

Thus, by (6), (7) and (8),

$$\sum_{j=0}^{\infty} \frac{r_j}{s_j(x)^2} \leq 4 + \frac{y_n^2}{(y_n - y_n^2)^2} + \sum_{j=1}^{\infty} \frac{y_j^2}{((2^{-2(j+2)} + y_j^2)^{\frac{1}{2}} - y_j^2)^2}$$
$$< 4 + 2 + \sum_{j=1}^{\infty} 2^{-(j+1)} = \frac{13}{2}.$$

By Proposition 2.7, this implies that $|f'|_I < \frac{13}{2}|f|_X$ for $f \in R_0(X)$. Hence D is a bounded derivation from $R_0(X)$ to $C(I)$. We extend D by continuity to a derivation from $R(X)$ to $C(I)$, which we shall also call D. It remains to show that D is not compact. Let $n \in \mathbb{N}$, and let

$$f_n(z) = \frac{r_n}{z - a_n} \quad (z \in X).$$

Then $|f_n|_X = 1$. Also

$$f_n'(z) = \frac{-r_n}{(z - a_n)^2} \quad (z \in X).$$

Clearly, for each $x \in [0, 1/2[$, $f_n'(x) \to 0$ as $n \to \infty$. Thus, if $(f_n'|_I)_{n \in \mathbb{N}}$ were to have a convergent subsequence the limit would have to be the zero function. However, $|f_n'(x_n)| = 1$ for each $n \in \mathbb{N}$. Hence $(D(f_n))_{n \in \mathbb{N}} = (f_n'|_I)_{n \in \mathbb{N}}$ has no convergent subsequence, and thus D is not a compact linear map.

Acknowledgements. This work is adapted from the author's PhD thesis ([9]), which was produced under the supervision of J. F. Feinstein and with the support of a grant from the EPSRC (UK).

This paper is based on a lecture delivered at the 19th International Conference on Banach Algebras held at Będlewo, July 14–24, 2009. The support for the meeting by the Polish Academy of Sciences, the European Science Foundation under the ESF-EMS-ERCOM partnership, and the Faculty of Mathematics and Computer Science of the Adam Mickiewicz University at Poznań is gratefully acknowledged. The author is grateful to FCT (Portugal) for paying his travel costs to this conference and funding his continuing research under the grant SFRH/BPD/40762/2007.

References

[1] W. G. Bade, Jr., P. C. Curtis, and H. G. Dales, *Amenability and weak amenability for Beurling and Lipschitz algebras*, Proc. London Mathematical Society 55 (1987), 359–377.

[2] Y. Choi, *Cohomology of commutative Banach algebras and ℓ^1-semigroup algebras*, Ph.D. thesis, University of Newcastle, 2006.

[3] Y. Choi and M. J. Heath, *Derivations from the disc algebra to its dual*, in preparation.

[4] Y. Choi and M. J. Heath, *Translation-finite sets and weakly compact derivations from $\ell^1(\mathbb{Z}_+)$ to its dual*, Bull. London Math. Soc. 42 (2010), 429–440.

[5] H. G. Dales, *Banach Algebras and Automatic Continuity*, London Mathematical Society Monographs 24, Oxford University Press, 2000.

[6] J. F. Feinstein, *Trivial Jensen measures without regularity*, Studia Math. 148 (2001), 67–74.

[7] J. F. Feinstein and H. Kamowitz, *Compact endomorphisms of H^∞*, Studia Math. 136 (1999), 87–90.

[8] J. F. Feinstein and H. Kamowitz, *Compact endomorphisms of Banach algebras of infinitely differentiable functions*, J. London Math. Soc. 132 (2004), 489–502.

[9] M. J. Heath, *Bounded derivations from Banach algebras*, Ph.D. thesis, University of Nottingham, 2008.

[10] H. Kamowitz, *Compact endomorphisms of Banach algebras*, Pacific J. Math. 89 (1980), 313–325.