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FINITE GROUPS OF OTP PROJECTIVE REPRESENTATION TYPE OVER A COMPLETE DISCRETE VALUATION DOMAIN OF POSITIVE CHARACTERISTIC

BY

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*Dedicated to the memory of Petro Mykhailovich Gudyvok**

Abstract. Let S be a commutative complete discrete valuation domain of positive characteristic p, S^* the unit group of S, Ω a subgroup of S^* and $G = G_p \times B$ a finite group, where G_p is a p-group and B is a p'-group. Denote by $S^\lambda G$ the twisted group algebra of G over S with a 2-cocycle $\lambda \in Z^2(G, S^*)$. For Ω satisfying a specific condition, we give necessary and sufficient conditions for G to be of OTP projective (S,Ω)-representation type, in the sense that there exists a cocycle $\lambda \in Z^2(G,\Omega)$ such that every indecomposable $S^\lambda G$-module is isomorphic to the outer tensor product $V \# W$ of an indecomposable $S^\lambda G_p$-module V and an irreducible $S^\lambda B$-module W.

1. Introduction. Let $p \geq 2$ be a prime, S either a field of characteristic p, or a commutative complete discrete valuation domain of characteristic p, and G a finite group. Denote by $Z^2(G, S^*)$ the group of all S^*-valued normalized 2-cocycles of the group G that acts trivially on S^*. The *twisted group algebra* of G over S with a 2-cocycle $\lambda \in Z^2(G, S^*)$ is the free S-algebra $S^\lambda G$ with an S-basis $\{u_g : g \in G\}$ satisfying $u_a u_b = \lambda_{a,b} u_{ab}$ for all $a, b \in G$. The S-basis $\{u_g : g \in G\}$ of $S^\lambda G$ is called *canonical* (corresponding to λ). Assume now that $G = G_p \times B$, where G_p is a p-group, B is a p'-group and $|G_p| > 1$, $|B| > 1$. Given $\mu \in Z^2(G_p, S^*)$ and $\nu \in Z^2(B, S^*)$, the map $\mu \times \nu : G \times G \to S^*$ defined by the formula

$$(\mu \times \nu)_{x_1 b_1, x_2 b_2} = \mu_{x_1,x_2} \cdot \nu_{b_1,b_2}, \tag{1.1}$$

for all $x_1, x_2 \in G_p$, $b_1, b_2 \in B$, is a 2-cocycle in $Z^2(G, S^*)$. Every cocycle $\lambda \in Z^2(G, S^*)$ is cohomologous to $\mu \times \nu$, where μ is the restriction of λ to $G_p \times G_p$ and ν is the restriction of λ to $B \times B$.

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From now on, we assume that every cocycle $\lambda \in Z^2(G, S^*)$ under consideration satisfies the condition $\lambda = \mu \times \nu$, and all $S^\lambda G$-modules are assumed to be finitely generated left $S^\lambda G$-modules which are S-free. We recall that the study of $S^\lambda G$-modules is essentially equivalent to the study of projective S-representations of G with the 2-cocycle λ.

Let $\lambda = \mu \times \nu \in Z^2(G, S^*)$ and $\{u_g \colon g \in G\}$ be a canonical S-basis of $S^\lambda G$. Then $\{u_h \colon h \in G_p\}$ is a canonical S-basis of $S^\mu G_p$ and $\{u_b \colon b \in B\}$ is a canonical S-basis of $S^\nu B$. Moreover, if $g = hb$, where $g \in G$, $h \in G_p$, $b \in B$, then $u_g = u_h u_b = u_b u_h$. It follows that $S^\lambda G \cong S^\mu G_p \otimes_S S^\nu B$.

Given an $S^\mu G_p$-module V and an $S^\nu B$-module W, we denote by $V \# W$ the $S^\lambda G$-module whose underlying S-module is $V \otimes_S W$, the $S^\lambda G$-module structure is given by

$$u_{hb}(v \otimes w) = u_h v \otimes u_b w$$

for all $h \in G_p$, $b \in B$, $v \in V$, $w \in W$, and it is extended to $S^\lambda G$ and $V \otimes_S W$ by S-linearity. Following [19, p. 122], we call the module $V \# W$ the *outer tensor product* of V and W.

Throughout, Ω is a fixed subgroup of S^*. We recall from [5, p. 10] the following definitions.

DEFINITION 1.1. Assume that S, G, Ω are as fixed above and $\lambda = \mu \times \nu \in Z^2(G, S^*)$ is a 2-cocycle as in (1.1).

(a) We set

$$Z^2(G, \Omega) = \{\lambda \in Z^2(G, S^*) \colon \operatorname{Im} \lambda \subset \Omega\}. \tag{1.2}$$

(b) The algebra $S^\lambda G$ is defined to be of *OTP representation type* if every indecomposable $S^\lambda G$-module is isomorphic to the outer tensor product $V \# W$, where V is an indecomposable $S^\mu G_p$-module and W is an irreducible $S^\nu B$-module.

(c) The group $G = G_p \times B$ is defined to be of *OTP projective* (S, Ω)-*representation type* if there exists a cocycle $\lambda \in Z^2(G, \Omega)$ such that the algebra $S^\lambda G$ is of OTP representation type.

(d) The group $G = G_p \times B$ is said to be of *purely OTP projective* (S, Ω)-*representation type* if $S^\lambda G$ is of OTP representation type for any $\lambda \in Z^2(G, \Omega)$.

If $\Omega = S^*$, we write "S-representation type" instead of "(S, Ω)-representation type".

In [8], Brauer and Feit proved that if S is an algebraically closed field of characteristic p, then the group algebra SG is of OTP representation type.

Blau [7] and Gudyvok [15, 16] independently showed that if S is an arbitrary field of characteristic p, then SG is of OTP representation type if and only if G_p is cyclic or S is a splitting field for B. In [17, 18], Gudyvok also investigated a similar problem for the group algebra SG, where S is a commutative complete discrete valuation domain. In particular, he proved that if S is of characteristic p and T is the quotient field of S, then SG is of OTP representation type if and only if $|G_p| = 2$ or T is a splitting field for B.

In [2]–[6], the results of Blau and Gudyvok were generalized to twisted group algebras $S^\lambda G$, where $G = G_p \times B$, S is either a field of characteristic p, or a commutative complete discrete valuation domain of characteristic p, and $\lambda \in Z^2(G, S^*)$ satisfies a specific condition. The main theorem in [3] asserts that if S is a field of characteristic p, then, under suitable assumptions, an algebra $S^\lambda G$ is of OTP representation type if and only if $S^\lambda G_p$ is a uniserial algebra or S is a splitting field for $S^\lambda B$.

In [4], necessary and sufficient conditions on G and a field S were given for G to be of OTP projective S-representation type and of purely OTP projective S-representation type. Let K be a field of characteristic p and $S := K[[X]]$ the ring of formal power series in the indeterminate X with coefficients in K.

The groups $G = G_p \times B$ of OTP projective (S, K^*)-representation type and of purely OTP projective S-representation type were described in [5].

Denote by T the quotient field of S and by Ω the subgroup of S^* generated by K^* and $f(X)$, where $f(X) \equiv 1 \pmod X$ and $f(X) \not\equiv 1 \pmod{X^2}$. Let $G = G_p \times B$, $|G'_p| \neq 2$, $\mu \in Z^2(G_p, \Omega)$, $\nu \in Z^2(B, K^*)$ and $\lambda = \mu \times \nu$. We recall from [6] that $S^\lambda G$ is of OTP representation type if and only if one of the following three conditions is satisfied:

(i) G_p is abelian and $T^\mu G_p$ is a field;
(ii) $p = 2$, G_2 is abelian and $\dim_T(T^\mu G_2/\operatorname{rad} T^\mu G_2) = |G_2|/2$;
(iii) K is a splitting field for $K^\nu B$.

In the present article we describe the groups $G = G_p \times B$ of OTP projective (S, Ω)-representation type, where S is a commutative complete discrete valuation domain of positive characteristic p and $\Omega \subset S^*$ satisfies specific conditions (see Theorem 3.1, (1.4) and (1.5)).

In view of the Cohen Theorem [25, p. 304], S is isomorphic to the algebra $K[[X]]$, where K is a field of characteristic p.

Throughout this paper, $S = K[[X]]$ denotes the power series algebra and $T = K((X))$ the quotient field of S. For simplicity of presentation, we set

$$i(K) = \begin{cases} t & \text{if } [K\colon K^p] = p^t, \\ \infty & \text{if } [K\colon K^p] = \infty. \end{cases} \tag{1.3}$$

Assume that G_p is an abelian p-group, m is the number of invariants of G_p and $G = G_p \times B$. Let Ω be the subgroup of S^* generated by K^* and $(S^*)^p$. We prove in Theorem 3.1 that G is of OTP projective (S, Ω)-representation type if and only if one of the following conditions is satisfied:

(i) $m \leq i(K)$;
(ii) $p = 2$ and $m = i(K) + 1$;
(iii) K is a splitting field for some K-algebra $K^\nu B$.

Let $p \geq 3$ be a prime and let

$$\Omega = \langle K^*, (S^*)^p, f(X)\rangle \subset S^* \tag{1.4}$$

be the subgroup of S^* generated by K^*, $(S^*)^p$ and $f(X)$, where $f(X) \equiv 1 \pmod X$ and $f(X) \not\equiv 1 \pmod{X^2}$. We prove in Theorem 3.2 that G is of OTP projective (S, Ω)-representation type if and only if $m \leq i(K)+1$ or K is a splitting field for some K-algebra $K^\nu B$.

Suppose now that $p = 2$ and

$$\Omega = \langle K^*, (S^*)^4, f(X)\rangle \subset S^* \tag{1.5}$$

is a subgroup of S^* generated by K^*, $(S^*)^4$ and $f(X)$, where $f(X) \equiv 1 \pmod X$ and $f(X) \not\equiv 1 \pmod{X^2}$. We show in Theorem 3.4 that G is of OTP projective (S, Ω)-representation type if and only if one of the following conditions is satisfied:

(i) $m \leq i(K) + 1$;
(ii) $m = i(K) + 2$ and G_2 has at least one invariant equal to 2;
(iii) K is a splitting field for some K-algebra $K^\nu B$.

Moreover we establish in Theorem 4.2 that the finite group $G = G_p \times B$, where G_p is an arbitrary p-group and B is a p'-group, is of purely OTP projective S-representation type if and only if one of the following conditions is satisfied:

(i) $p = 2$ and $|G_2| = 2$.
(ii) There exists a finite central group extension $1 \to A \to \hat{B} \to B \to 1$ such that any projective K-representation of B lifts projectively to an ordinary K-representation of $\hat{B}$ and K is a splitting field for $\hat{B}$.

Throughout the paper, we use the standard group representation theory notation and terminology introduced in the monographs by Curtis and Reiner [9, 10, 11], and Karpilovsky [19]. The monograph by Karpilovsky gives a systematic account of the projective representation theory. For problems of the representation theory of orders in finite-dimensional algebras, we refer to the books by Curtis and Reiner.

A background of the representation theory of finite-dimensional algebras can be found in the monographs by Assem, Simson and Skowroński [1], Drozd and Kirichenko [14], Simson [21], and Simson and Skowroński [24], where among other things the representation types (finite, tame, wild) of finite groups and algebras are discussed. Various aspects of the representation types are considered also by Dowbor and Simson [12, 13], Simson [22], and Simson and Skowroński [23].

2. On twisted group algebras of OTP representation type. Throughout this paper, we use the following notations: $p \geq 2$ is a prime; K is a field of characteristic p; K^* is the multiplicative group of K; $S = K[[X]]$ is the ring of formal power series in the indeterminate X with coefficients in K, $S^l = \{a^l : a \in S\}$; S^* is the unit group of S, $(S^*)^l = \{a^l : a \in S^*\}$; T is the quotient field of S; $G = G_p \times B$ is a finite group, where G_p is a Sylow p-subgroup; H' is the commutator subgroup of a group H, e is the identity element of H, $|h|$ is the order of $h \in H$. We assume that $|G_p| > 1$ and $|B| > 1$.

Unless stated otherwise, we suppose that if G_p is non-abelian; then $[K(\xi) : K]$ is not divisible by p, where ξ is a primitive $(\exp B)$th root of 1. Given a subgroup Ω of S^*, we denote by $Z^2(H, \Omega)$ the group of all Ω-valued normalized 2-cocycles of the group H, where we assume that H acts trivially on Ω (see (1.2)).

A basis $\{u_h : h \in H\}$ of $S^\lambda H$ satisfying $u_a u_b = \lambda_{a,b} u_{ab}$ for all $a, b \in H$ is called *canonical* (corresponding to $\lambda \in Z^2(H, S^*)$). We often identify γu_e with $\gamma \in S$. If D is a subgroup of H, then the restriction of $\lambda \in Z^2(H, S^*)$ to $D \times D$ will also be denoted by λ. We assume that in this case $S^\lambda D$ is the S-subalgebra of $S^\lambda H$ consisting of all S-linear combinations of elements $\{u_d : d \in D\}$, where $\{u_h : h \in H\}$ is a canonical S-basis of $S^\lambda H$ corresponding to λ. Given an $S^\lambda H$-module V, we write $\mathrm{End}_{S^\lambda H}(V)$ for the ring of all $S^\lambda H$-endomorphisms of V, $\mathrm{rad}\,\mathrm{End}_{S^\lambda H}(V)$ for the Jacobson radical of $\mathrm{End}_{S^\lambda H}(V)$, and we set

$$\overline{\mathrm{End}_{S^\lambda H}(V)} = \mathrm{End}_{S^\lambda H}(V)/\mathrm{rad}\,\mathrm{End}_{S^\lambda H}(V).$$

Given $\lambda \in Z^2(H, K^*)$, $K^\lambda H$ denotes the twisted group algebra of H over K and $\overline{K^\lambda H}$ the quotient algebra of $K^\lambda H$ by the radical $\mathrm{rad}\, K^\lambda H$.

By a *principal unit* in S we understand an element $f(X) \in S$ such that $f(X) \equiv 1 \pmod{X}$. Denote by S_0^* the group of principal units of S. Then $S^* = K^* \times S_0^*$. Let q be a prime and $q \neq p$. Then $(S_0^*)^q = S_0^*$. Moreover S_0^* contains no primitive qth root of 1. By Theorem 1.7 in [19, p. 11], every 2-cocycle $\sigma \in Z^2(B, S_0^*)$ is a coboundary. Hence each 2-cocycle $\tau \in Z^2(B, S^*)$ is cohomologous to a 2-cocycle $\nu \in Z^2(B, K^*)$.

Let $G_p = \langle a_1 \rangle \times \cdots \times \langle a_m \rangle$ be an abelian p-group of type $(p^{n_1}, \ldots, p^{n_m})$. For any cocycle $\mu \in Z^2(G_p, S^*)$, the algebra $S^\mu G_p$ is commutative. The algebra $S^\mu G_p$ has a canonical S-basis $\{v_g : g \in G_p\}$ satisfying the following conditions:

1) if $g = a_1^{j_1} \ldots a_m^{j_m}$ and $0 \leq j_i < p^{n_i}$ for each $i \in \{1, \ldots, m\}$, then

$$v_g = v_{a_1}^{j_1} \ldots v_{a_m}^{j_m};$$

2) $v_{a_i}^{p^{n_i}} = \gamma_i v_e$, where $\gamma_i = \mu_{a_i,a_i} \mu_{a_i,a_i^2} \ldots \mu_{a_i,a_i^{r_i}}$, $r_i = p^{n_i} - 1$.

We denote the algebra $S^\mu G_p$ also by $[G_p, S, \gamma_1, \dots, \gamma_m]$. Similarly if $\mu \in Z^2(G_p, K^*)$, then we denote the algebra $K^\mu G_p$ by $[G_p, K, \gamma_1, \dots, \gamma_m]$ as well.

Now we collect several facts we apply later.

Lemma 2.1. *Let R be either a field of characteristic p, or a commutative complete discrete valuation domain of characteristic p, $G = G_p \times B$, $\mu \in Z^2(G_p, R^*)$, $\nu \in Z^2(B, R^*)$ and $\lambda = \mu \times \nu$ be as in* (1.1). *The algebra $R^\lambda G$ is of OTP representation type if and only if the outer tensor product of any indecomposable $R^\mu G_p$-module and any irreducible $R^\nu B$-module is an indecomposable $R^\lambda G$-module.*

The proof is similar to that of the corresponding fact for a group algebra (see [7, p. 41], [18, p. 68]).

Lemma 2.2. *Let R be either a field of characteristic p, or a commutative complete discrete valuation domain of characteristic p, $G = G_p \times B$, $\mu \in Z^2(G_p, R^*)$, $\nu \in Z^2(B, R^*)$ and $\lambda = \mu \times \nu$ be as in* (1.1). *If V is an indecomposable $R^\mu G_p$-module and W is an irreducible $R^\nu B$-module, then*

$$\overline{\mathrm{End}_{R^\lambda G}(V \# W)} \cong \overline{\mathrm{End}_{R^\mu G_p}(V)} \otimes_{\overline{R}} \overline{\mathrm{End}_{R^\nu B}(W)},$$

where $\overline{R}$ is the residue class field of R.

Proof. See [5, p. 15]. ■

Lemma 2.3. *Let K be an arbitrary field of characteristic p, $S = K[[X]]$, $G = G_p \times B$, $\mu \in Z^2(G_p, S^*)$, $\nu \in Z^2(B, K^*)$ and $\lambda = \mu \times \nu$ be as in* (1.1). *If K is a splitting field for the K-algebra $K^\nu B$, then $S^\lambda B$ is of OTP representation type.*

Proof. See [5, p. 15]. ■

Lemma 2.4. *Let K be an arbitrary field of characteristic p, $S = K[[X]]$, $G = G_p \times B$, $\mu \in Z^2(G_p, S^*)$, $\nu \in Z^2(B, K^*)$ and $\lambda = \mu \times \nu$ be as in* (1.1). *Assume that V is an indecomposable $S^\mu G_p$-module and $\overline{\mathrm{End}_{S^\mu G_p}(V)}$ is isomorphic to a field that is a finite purely inseparable field extension of K. Then the $S^\lambda G$-module $V \# W$ is indecomposable for any irreducible $S^\nu B$-module W.*

Proof. Suppose that L is a finite purely inseparable field extension of K and L is K-isomorphic to $\overline{\mathrm{End}_{S^\mu G_p}(V)}$. Denote by Δ the K-algebra $\overline{\mathrm{End}_{S^\nu B}(W)}$. Then $\Delta \cong \mathrm{End}_{K^\nu B}(\widetilde{W})$, where $\widetilde{W}$ is the quotient module W/XW. Since $K^\nu B$ is a separable algebra, the center of the division K-algebra Δ is a finite separable field extension of K (see [9, p. 485]). The index of Δ is not divisible by p [20]. It follows that $L \otimes_K \Delta$ is a skew field. By Proposition 6.10 in [10, p. 125] and Lemma 2.2, $V \# W$ is an indecomposable $S^\lambda G$-module. ■

Proposition 2.5. *Assume that G_p is an abelian group, $G = G_p \times B$, $\mu \in Z^2(G_p, K^*)$, $\nu \in Z^2(B, K^*)$ and $\lambda = \mu \times \nu$ be as in* (1.1). *If the K-algebra $K^\mu G_p$ is a field then the algebra $S^\lambda G$ is of OTP representation type.*

Proof. Let $L := K^\mu G_p$. Then $S^\mu G_p = L[[X]]$ is a principal ideal ring. Every indecomposable $S^\mu G_p$-module is isomorphic to $S^\mu G_p$. We have

$$\overline{\mathrm{End}_{S^\mu G_p}(S^\mu G_p)} \cong S^\mu G_p / X S^\mu G_p \cong L.$$

The field L is a finite purely inseparable field extension of K (see [19, p. 74]). Applying Lemmas 2.1 and 2.4, we conclude that $S^\lambda G$ is of OTP representation type. ∎

Proposition 2.6. *Let $G_p = \langle a_1 \rangle \times \cdots \times \langle a_m \rangle$, $m \geq 2$, $G = G_p \times B$, $\mu \in Z^2(G_p, S^*)$, $\nu \in Z^2(B, K^*)$ and $\lambda = \mu \times \nu$ be as in* (1.1). *Assume that $S^\mu G_p = [G_p, S, \gamma_1, \dots, \gamma_{m-1}, 1 + X]$, where $\gamma_1, \dots, \gamma_{m-1} \in K^*$. If $\left[K\left(\sqrt[p]{\gamma_1}, \dots, \sqrt[p]{\gamma_{m-1}}\right) : K\right] = p^{m-1}$, then $S^\lambda G$ is of OTP representation type.*

Proof. The T-algebra $T^\mu G_p$ is a field and $S^\mu G_p$ is the valuation domain in $T^\mu G_p$. Any indecomposable $S^\mu G_p$-module is isomorphic to the regular $S^\mu G_p$-module. Let $\sigma \in Z^2(G_p, K^*)$ and $\sigma_{a,b} \equiv \mu_{a,b} \pmod X$ for all $a, b \in G_p$. Then $S^\mu G_p / X S^\mu G_p \cong K^\sigma G_p$. Since $\mathrm{End}_{S^\mu G_p}(S^\mu G_p) \cong S^\mu G_p$, we conclude, by Proposition 5.22 in [10, p. 112], that

$$\overline{\mathrm{End}_{S^\mu G_p}(S^\mu G_p)} \cong (S^\mu G_p / X S^\mu G_p)/\mathrm{rad}(S^\mu G_p / X S^\mu G_p) \cong \overline{K^\sigma G_p}.$$

The K-algebra $\overline{K^\sigma G_p}$ is isomorphic to a field that is a finite purely inseparable field extension of K. By Lemmas 2.1 and 2.4, $S^\lambda G$ is of OTP representation type. ∎

Assume that $S = K[[X]]$, H is a subgroup of G_p, $\mu \in Z^2(G_p, S^*)$ and $\tau \in Z^2(H, S^*)$. Suppose also that $S^\tau H$ is an S-subalgebra of the algebra $S^\mu G_p$. We say that $S^\tau H$ is a *μ-extended algebra* if there exists a subgroup D of G_p and a cocycle $\sigma \in Z^2(D, S^*)$ such that $H \subset D$, $S^\mu D = S^\sigma D$ as S-algebras and the restriction of σ to $H \times H$ is equal to τ.

Lemma 2.7 (see [6]). *Let G_p be an abelian p-group, $G = G_p \times B$, $\mu \in Z^2(G_p, S^*)$, $\nu \in Z^2(B, K^*)$ and $\lambda = \mu \times \nu$ be as in* (1.1). *Assume that $S^\mu G_p$ contains a μ-extended group algebra of a group of order greater than two over S. Then $S^\lambda G$ is of OTP representation type if and only if K is a splitting field for $K^\nu B$.*

Assume now that F is a field of characteristic 2 complete with respect to a discrete valuation, R is the valuation domain in F, $G_2 = \langle a \rangle$ is a cyclic group of order 2^n ($n \geq 1$) and $R^\mu G_2 = [G_2, R, \gamma^{2^l}]$, where $l \in \{0, 1\}$, $\gamma \in R^*$ and $\gamma \notin R^2$ if $n \geq 2$. Denote by ξ a root of the polynomial

$$Y^{2^n} - \gamma^{2^l}.$$

Let $G = G_2 \times B$, $\nu \in Z^2(B, R^*)$ and $\lambda = \mu \times \nu$. The following fact is also proved in [6].

PROPOSITION 2.8. *If $R[\xi]$ is the valuation domain in $F(\xi)$, then $R^\lambda G$ is of OTP representation type.*

3. On groups of OTP projective representation type. We recall that $G = G_p \times B$, $S = K[[X]]$, T is the quotient field of S, and $i(K)$ is as in (1.3). Let $|G'_p| > 2$, $\mu \in Z^2(G_p, S^*)$, $\nu \in Z^2(B, K^*)$ and $\lambda = \mu \times \nu$. By the corollary to Theorem 1 in [5, p. 16], the algebra $S^\lambda G$ is of OTP representation type if and only if K is a splitting field for $K^\nu B$. Therefore, unless stated otherwise, we assume that G_p is an abelian p-group. Denote by m the number of invariants of G_p. In view of Theorem 2 in [5, p. 19], the group G is of OTP projective (S, K^*)-representation type if and only if one of the following conditions is satisfied:

1) $m \leq i(K)$;
2) $p = 2$, $m = i(K) + 1$ and G_2 has at least one invariant equal to 2;
3) K is a splitting field for $K^\sigma B$ for some $\sigma \in Z^2(B, K^*)$.

In this section, we describe the groups $G = G_p \times B$ of OTP projective (S, Ω)-representation type, where G_p is abelian and $\Omega \neq K^*$.

THEOREM 3.1. *Let Ω be the subgroup of S^* generated by K^* and $(S^*)^p$. The group $G = G_p \times B$ is of OTP projective (S, Ω)-representation type if and only if one of the following conditions is satisfied:*

(i) $m \leq i(K)$;
(ii) $p = 2$ *and* $m = i(K) + 1$;
(iii) K *is a splitting field for some K-algebra $K^\nu B$.*

Proof. Let $\mu \in Z^2(G_p, \Omega)$, $\nu \in Z^2(B, K^*)$ and $\lambda = \mu \times \nu$. Suppose that

$$S^\mu G_p = [G_p, S, \gamma_1 f_1(X)^p, \ldots, \gamma_m f_m(X)^p],$$

where $\gamma_1, \ldots, \gamma_m \in K^*$ and $f_1(X), \ldots, f_m(X)$ are principal units in S. If $p \neq 2$ and $m > i(K)$ then $S^\mu G_p$ contains a μ-extended group algebra of a group of order $p \geq 3$ over S. If $p = 2$ and $m > i(K)+1$ then $S^\mu G_2$ contains a μ-extended group algebra of an abelian group of type $(2, 2)$ over S. In these cases, by Lemma 2.7, $S^\lambda G$ is of OTP representation type if and only if K is a splitting field for $K^\nu B$. The necessity is proved.

To prove the sufficiency, assume that $m \leq i(K)$. Then there exists $\sigma \in Z^2(G_p, K^*)$ such that $K^\sigma G_p$ is a field. By Proposition 2.5, the algebra $S^\lambda G$ with $\lambda = \sigma \times \nu$ is of OTP representation type for each $\nu \in Z^2(B, K^*)$. Assume now that $p = 2$, $i(K) \neq 0$ and $m = i(K) + 1$. There exist $\gamma_1, \ldots, \gamma_{m-1} \in K^*$ such that $\left[K\left(\sqrt{\gamma_1}, \ldots, \sqrt{\gamma_{m-1}}\right) \colon K\right] = 2^{m-1}$. Let $G_2 = \langle a_1 \rangle \times \cdots \times \langle a_m \rangle$,

$A = \langle a_1\rangle \times \cdots \times \langle a_{m-1}\rangle$ and $H = \langle a_m\rangle$. We put

$$S^\mu G_2 = [G_2, S, \gamma_1, \ldots, \gamma_{m-1}, (1+X)^2] \quad \text{and} \quad S^\lambda G = S^\mu G_2 \otimes_S S^\nu B,$$

where $\nu \in Z^2(B, K^*)$ is an arbitrary cocycle. Denote by τ the restriction of μ to $A \times A$. Then $\tau \in Z^2(A, K^*)$ and $L := K^\tau A$ is a field. It follows that $F := T^\tau A$ is also a field and $R := S^\tau A$ is the valuation domain in F. Moreover $R \cong L[[X]]$. Let $D = H \times B$. The algebra $S^\lambda G$ is a twisted group algebra of D over R. If we denote it by $R^\sigma D$, we have an algebra isomorphism $R^\sigma D \cong R^\mu H \otimes_R R^\nu B$.

Let M be an $S^\lambda G$-module. Then M is a finitely generated R-module. Denote by 2^n the exponent of A. We have $r^{2^n} \in S$ for any $r \in R$. Suppose that $r \in R$, $v \in M$, $v \neq 0$ and $rv = 0$. Then $r^{2^n} \cdot v = 0$. Since M is a free S-module, $r^{2^n} = 0$, and consequently $r = 0$. This means that M is a torsion-free R-module. Since R is a principal ideal ring, M is a free R-module, i.e. M is an $R^\sigma D$-module. Conversely, if M is an $R^\sigma D$-module then M is an $S^\lambda G$-module. Note also that M is an indecomposable $S^\lambda G$-module if and only if M is an indecomposable $R^\sigma D$-module.

By Proposition 2.8, $R^\sigma D$ is of OTP R-representation type. Assume that V is an indecomposable $S^\mu G_p$-module and W is an irreducible $S^\nu B$-module. In view of Proposition 2.5, $U := R \otimes_S W$ is an irreducible $R^\nu B$-module. Because V is an indecomposable $R^\mu H$-module then, by Lemma 2.1, the $R^\sigma D$-module $V \otimes_R U$ is indecomposable. Since $V \otimes_R U$ is also an indecomposable $S^\lambda G$-module and

$$V \otimes_R U \cong (V \otimes_R R) \otimes_S W \cong V \otimes_S W,$$

we conclude that $V \otimes_S W$ is an indecomposable $S^\lambda G$-module. Consequently, in view of Lemma 2.1, $S^\lambda G$ is of OTP S-representation type and therefore the group G is of OTP projective (S, Ω)-representation type.

In the case when $p = 2$, $i(K) = 0$ and $m = 1$, we set $S^\mu G_2 = [G_2, S, (1+X)^2]$. By Proposition 2.8, the algebra $S^\lambda G := S^\mu G_2 \otimes_S S^\nu B$ is of OTP representation type for any $\nu \in Z^2(B, K^*)$. Hence G is of OTP projective (S, Ω)-representation type. ■

THEOREM 3.2. *Let $p \neq 2$ and Ω be the subgroup of S^* generated by K^*, $(S^*)^p$ and $f(X)$, where $f(X) \equiv 1 \pmod X$ and $f(X) \not\equiv 1 \pmod{X^2}$. The group $G = G_p \times B$ is of OTP projective (S, Ω)-representation type if and only if $m \leq i(K) + 1$ or K is a splitting field for some K-algebra $K^\nu B$.*

Proof. Since $(f(X)-1)S = XS$, we may assume that $f(X) = 1+X$. Let $\mu \in Z^2(G_p, \Omega)$, $\nu \in Z^2(B, K^*)$ and $\lambda = \mu \times \nu$. Choose a canonical S-basis of $S^\mu G_p$ such that

$$S^\mu G_p = [G_p, S, \gamma_1(1+X)^i f_1(X)^p, \gamma_2 f_2(X)^p, \ldots, \gamma_m f_m(X)^p],$$

where $\gamma_1, \ldots, \gamma_m \in K^*$ and $f_1(X), \ldots, f_m(X)$ are principal units of S. If

$m-1 > i(K)$ then $S^{\mu}G_p$ contains a μ-extended group algebra of a group of order p over S. By Lemma 2.7, $S^{\lambda}G$ is of OTP representation type if and only if K is a splitting field for $K^{\nu}B$. The necessity of the theorem is proved.

To prove the sufficiency, assume that $m \le i(K)$. Then there exists $\sigma \in Z^2(G_p, K^*)$ such that $K^{\sigma}G_p$ is a field. By Proposition 2.5, $S^{\lambda}G := S^{\sigma}G_p \otimes_S S^{\nu}B$ is of OTP representation type for each $\nu \in Z^2(B, K^*)$. If $m = i(K)+1$, $i(K) \neq 0$, then there exist elements $\gamma_1, \dots, \gamma_{m-1} \in K^*$ such that $S^{\mu}G_p := [G_p, S, \gamma_1, \dots, \gamma_{m-1}, 1+X]$ is the valuation domain in the field $T^{\mu}G_p$. By Proposition 2.6, the algebra $S^{\lambda}G := S^{\mu}G_p \otimes_S S^{\nu}B$ is of OTP representation type, for any $\nu \in Z^2(B, K^*)$. If K is a splitting field for some K-algebra $K^{\nu}B$ then, by Lemma 2.3, the algebra $S^{\lambda}G := S^{\mu}G_p \otimes_S S^{\nu}B$ is of OTP representation type for every $\mu \in Z^2(G_p, \Omega)$. ∎

PROPOSITION 3.3. *Let $p = 2$ and Ω be a subgroup of S^* generated by K^*, $(S^*)^2$ and $f(X)$, where $f(X) \equiv 1 \pmod X$ and $f(X) \not\equiv 1 \pmod{X^2}$. If $G = G_2 \times B$ is of OTP projective (S, Ω)-representation type then $m \le i(K)+2$ or K is a splitting field for some K-algebra $K^{\nu}B$.*

Proof. Apply the arguments used in the proof of Theorem 3.2. ∎

THEOREM 3.4. *Let $p = 2$, $G = G_2 \times B$ and Ω be the subgroup of S^* generated by K^*, $(S^*)^4$ and $f(X)$, where $f(X) \equiv 1 \pmod X$ and $f(X) \not\equiv 1 \pmod{X^2}$. The group G is of OTP projective (S, Ω)-representation type if and only if one of the following conditions is satisfied:*

(i) $m \le i(K)+1$;
(ii) $m = i(K)+2$ *and G_2 has at least one invariant equal to* 2;
(iii) *K is a splitting field for some K-algebra $K^{\nu}B$.*

Proof. We may assume that $f(X) = 1+X$. Let $G_2 = \langle a_1 \rangle \times \dots \times \langle a_m \rangle$, $H = \{g \in G \colon g^4 = e\}$, $H = \langle h_1 \rangle \times \dots \times \langle h_m \rangle$, where $h_i \in \langle a_i \rangle$ for every $i \in \{1, \dots, m\}$; $\mu \in Z^2(G_2, \Omega)$, $\nu \in Z^2(B, K^*)$ and $\lambda = \mu \times \nu$. Let $S^{\lambda}G$ be of OTP representation type and assume that K is not a splitting field for the K-algebra $K^{\nu}B$. By Theorem 3.1, we may suppose that

$$S^{\mu}G_2 = [G_2, S, \gamma_1(1+X)f_1(X)^4, \gamma_2(1+X)^i f_2(X)^4, \dots, \gamma_m f_m(X)^4],$$

where $\gamma_1, \dots, \gamma_m \in K^*$, $i \in \{0, 2\}$ and $f_1(X), \dots, f_m(X)$ are principal units in S. Therefore

$$S^{\mu}H = [H, S, \gamma_1(1+X), \gamma_2(1+X)^i, \gamma_3, \dots, \gamma_m],$$

where $i = 0$ if $|h_1| \ge |h_2|$, and $i \in \{0, 2\}$ if $|h_1| = 2$, $|h_2| = 4$. Denote by $\{v_h \colon h \in H\}$ a canonical S-basis of $S^{\mu}H$. If

$$v_{h_1}^2 = \gamma_1(1+X)v_e, \qquad v_{h_2}^4 = \gamma_2(1+X)^2 v_e,$$

then $(v_{h_1}^{-1}v_{h_2})^4 = (\gamma_1^{-2}\gamma_2)v_e$. Since $\langle h_1 \rangle \times \langle h_2 \rangle = \langle h_1 \rangle \times \langle h_1 h_2 \rangle$, we shall assume that $i = 0$. By Lemma 2.7, $m-1 \le i(K)+1$, hence $m \le i(K)+2$.

Let $m = i(K) + 2$, $i(K) \neq 0$ and H be a direct product of m cyclic subgroups of order 4 each. Suppose that $L := K[v_{h_2}, \dots, v_{h_{m-1}}]$ is a field. Let $F := K[v_{h_2}^2, \dots, v_{h_{m-1}}^2]$. For each $\alpha \in K$ there exists $\beta \in F$ such that $\alpha = \beta^2$. The element β is uniquely expressible as

$$\beta = \sum_{i_2,\dots,i_{m-1}} \delta_{i_2,\dots,i_{m-1}} v_{h_2}^{2i_2} \dots v_{h_{m-1}}^{2i_{m-1}},$$

where $i_j = 0, 1$ and $\delta_{i_2,\dots,i_{m-1}} \in K$. However, $\delta_{i_2,\dots,i_{m-1}} = \eta_{i_2,\dots,i_{m-1}}^2$ for some $\eta_{i_2,\dots,i_{m-1}} \in F$. This implies $\beta = \rho^2$ for $\rho \in L$, and hence $\alpha = \rho^4$. It follows that $S^\mu H$ contains the μ-extended group algebra of a group of order 4 over S. By Lemma 2.7, K is a splitting field for $K^\nu B$, a contradiction. Consequently, G_2 has at least one invariant equal to 2. The necessity is proved.

To prove the sufficiency, we assume that $m \leq i(K) + 1$ and we set

$$S^\mu G_2 = [G_2, S, \gamma_1, \dots, \gamma_{m-1}, 1 + X],$$

where $\gamma_1, \dots, \gamma_{m-1} \in K^*$ and $\left[K\left(\sqrt{\gamma_1}, \dots, \sqrt{\gamma_{m-1}}\right) \colon K\right] = 2^{m-1}$. If $m = i(K) + 2$ and $|a_m| = 2$, we put $S^\mu G_2 = [G_2, S, \gamma_1, \dots, \gamma_{m-2}, 1 + X, 1]$, where $\gamma_1, \dots, \gamma_{m-2} \in K^*$ and $\left[K\left(\sqrt{\gamma_1}, \dots, \sqrt{\gamma_{m-2}}\right) \colon K\right] = 2^{m-2}$. Arguing as in the proof of Theorem 3.1, we conclude that the algebra

$$S^\lambda G := S^\mu G_2 \otimes_S S^\nu B$$

is of OTP representation type for any $\nu \in Z^2(B, K^*)$. ∎

Proposition 3.5. *Let K be an arbitrary field of characteristic p, $S = K[[X]]$, G_p a finite p-group and $G = G_p \times B$. The group G is of OTP projective $(S, (S^*)^p)$-representation type if and only if one of the following conditions is satisfied:*

- (i) *$p = 2$ and G_2 is cyclic;*
- (ii) *K is a splitting field for some K-algebra $K^\nu B$, where $\nu \in Z^2(B, (K^*)^p)$.*

Proof. Let $\mu \in Z^2(G_p, (S^*)^p)$, $\nu \in Z^2(B, (K^*)^p)$ and $\lambda = \mu \times \nu$. Assume that $p = 2$ and G_2 is non-cyclic. Then $\hat{G}_2 := G_2/G_2'$ is non-cyclic. The restriction of μ to $G_2' \times G_2'$ is a coboundary [19, p. 42]. We may assume that $\mu_{h_1,h_2} = 1$ for all $h_1, h_2 \in G_2'$. Denote $\hat{G} = \hat{G}_2 \times B$, let $\{u_h \colon h \in G_2\}$ be a canonical S-basis of $S^\mu G_2$ corresponding to μ, and set

$$U = \bigoplus_{h \in G_2' \setminus \{e\}} S^\mu G_2(u_h - u_e)$$

and $S^{\hat{\mu}} \hat{G}_2 = S^\mu G_2 / U$. By Lemma 2.7, the algebra $S^{\hat{\lambda}} \hat{G} := S^{\hat{\mu}} \hat{G}_2 \otimes_S S^\nu B$ is of OTP representation type if and only if K is a splitting field for $K^\nu B$. If $p \neq 2$, we argue as in the case $p = 2$. This completes the proof of the necessity.

To prove the sufficiency, assume that $p = 2$, G_2 is cyclic, and put $S^\mu G_2 = [G_2, S, (1+X)^2]$, $S^\lambda G = S^\mu G_2 \otimes_S S^\nu B$, where $\nu \in Z^2(B, (K^*)^2)$ is an arbitrary cocycle. By Proposition 2.8, $S^\lambda G$ is of OTP representation type. If the condition (ii) holds, apply Lemma 2.3. ∎

PROPOSITION 3.6. *Let $p = 2$, K be an arbitrary field of characteristic 2, $S = K[[X]]$, G_2 a finite 2-group, and $G = G_2 \times B$. The group G is of OTP projective $(S, (S^*)^4)$-representation type if and only if $|G_2| = 2$ or K is a splitting field for some K-algebra $K^\nu B$, where $\nu \in Z^2(B, (K^*)^4)$.*

Proof. Apply Proposition 3.5 and Lemma 2.7. ∎

PROPOSITION 3.7. *Let K be an arbitrary field of characteristic p, $S = K[[X]]$, G_p a finite p-group, and $G = G_p \times B$. The group G is of OTP projective $(S, (K^*)^p)$-representation type if and only if one of the following conditions is satisfied:*

(i) *$p = 2$, K is a perfect field and $|G_2| = 2$;*
(ii) *$p = 2$, K is a non-perfect field and G_2 is a cyclic group;*
(iii) *K is a splitting field for some K-algebra $K^\nu B$, where $\nu \in Z^2(B, (K^*)^p)$.*

Proof. Apply Propositions 3.5, 2.8 and Lemma 2.7. ∎

4. On groups of purely OTP projective representation type. In this section, K is an arbitrary field of characteristic p, $t(K^*)$ is the torsion subgroup of K^*, $S = K[[X]]$ and $G = G_p \times B$ is a finite group, where G_p is a p-group, B is a p'-group and $|G_p| > 1$, $|B| > 1$.

A short exact sequence of groups

$$E\colon 1 \to D \xrightarrow{\varphi} \hat{B} \to B \to 1$$

is called an *extension* of D by B. If $\varphi(D)$ is contained in the center of $\hat{B}$, then E is called a *central extension.* If $\hat{B}$ is a finite group, then E is a *finite extension.*

Let V be a finite-dimensional vector space over K, $\mathrm{GL}(V)$ the group of all automorphisms of V, 1_V the identity automorphism of V, and let

$$1 \to D \to \hat{B} \xrightarrow{\psi} B \to 1$$

be a finite central group extension. Denote by $\pi\colon \mathrm{GL}(V) \to \mathrm{GL}(V)/K^*1_V$ the canonical group epimorphism. Let $\hat{\Gamma}$ be an ordinary K-representation of $\hat{B}$ in V such that $\hat{\Gamma}(d) \in K^*1_V$ for any $d \in D$. There is a projective K-representation Γ of B in V such that the diagram

$$\begin{tikzcd}
\hat{B} \arrow[r, "\hat{\Gamma}"] \arrow[d, "\psi"'] & \mathrm{GL}(V) \arrow[r, "\pi"] & \mathrm{GL}(V)/K^*1_V \arrow[d, "\mathrm{id}"] \\
B \arrow[r, "\Gamma"] & \mathrm{GL}(V) \arrow[r, "\pi"] & \mathrm{GL}(V)/K^*1_V
\end{tikzcd}$$

is commutative. We say that Γ *lifts projectively* to the ordinary K-representation $\hat{\Gamma}$ of $\hat{B}$. If $|D| = |H^2(B, K^*)|$ and any projective K-representation of B lifts projectively to an ordinary K-representation of $\hat{B}$, then $\hat{B}$ is called a *covering group* of B over K (see [19, p. 138]).

Here $H^2(B, K^*) = Z^2(B, K^*)/B^2(B, K^*)$ is the second cohomology group of B over K^* (see [19, p. 6]).

Lemma 4.1. *The group $G = G_p \times B$ is of purely OTP projective S-representation type if and only if $|G_p| = 2$ or K is a splitting field for $K^\nu B$ for any $\nu \in Z^2(B, K^*)$.*

Proof. See [5, p. 22]. ■

Now we prove the main results of this section.

Theorem 4.2. *The group $G = G_p \times B$ is of purely OTP projective S-representation type if and only if one of the following two conditions is satisfied:*

(i) $p = 2$ *and* $|G_2| = 2$.
(ii) *There exists a finite central group extension $1 \to A \to \hat{B} \to B \to 1$ such that any projective K-representation of B lifts projectively to an ordinary K-representation of $\hat{B}$ and K is a splitting field for $\hat{B}$.*

Proof. By Proposition 2.9 in [4, p. 45], K is a splitting field for all twisted group algebras of B over K if and only if the condition (ii) holds. Hence the theorem follows by applying Lemma 4.1. ■

Proposition 4.3. *Let S_0^* be the group of principal units in S. A group $G = G_p \times B$ is of purely OTP projective (S, S_0^*)-representation type if and only if $|G_p| = 2$ or K is a splitting field for B.*

Proof. By Theorem 3 in [18], the group algebra SG is of OTP representation type if and only if $|G_p| = 2$ or K is a splitting field for B. If $|G_p| = 2$ then, by Lemma 4.1, $S^\lambda G$ is of OTP representation type for any $\lambda \in Z^2(G, S_0^*)$. Every cocycle $\nu \in Z^2(B, S_0^*)$ is a coboundary, hence $S^\nu B$ is isomorphic to SB. If K is a splitting field for B, then, by Lemma 2.3, an algebra $S^\lambda B := S^\mu G_p \otimes_S SB$ is of OTP representation type for any $\mu \in Z^2(G_p, S_0^*)$. ■

Theorem 4.4. *Let $S = K[[X]]$ and $G = G_p \times B$. Assume that either $t(K^*) = t(K^*)^q$ for any prime q that divides $|B'|$, or every prime divisor of $|B'|$ is also a divisor of $|B\colon B'|$. Then G is of purely OTP projective*

S-representation type if and only if $|G_p| = 2$ or there exists a covering group $\hat{B}$ of B over K such that K is a splitting field for $\hat{B}$.

Proof. By Proposition 2.10 in [4, p. 45], K is a splitting field for any twisted group algebra of B over K if and only if there exists a covering group $\hat{B}$ of B over K such that K is a splitting field for $\hat{B}$. Hence the theorem follows by applying Lemma 4.1. ∎

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