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HERMITIAN OPERATORS ON H_E^∞ AND $S_{\mathcal{K}}^\infty$

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Abstract. A complete characterization of bounded and unbounded norm hermitian operators on H_E^∞ is given for the case when E is a complex Banach space with trivial multiplier algebra. As a consequence, the bi-circular projections on H_E^∞ are determined. We also characterize a subclass of hermitian operators on $S_{\mathcal{K}}^\infty$ for $\mathcal{K}$ a complex Hilbert space.

1. Introduction. The notion of a hermitian operator has been extended to the Banach space setting in several ways (see for example Lumer [19], Vidav [24] and Bonsall–Duncan [8]). In this paper we follow the approach introduced by Berkson and Porta, Kaufman, and Palmer in a series of papers [5], [6], [7], [4] and [21]. For other works in the same genre the reader should consult [3], [2] and [23]. We say that A is *hermitian* if iA is the generator of a one-parameter C_0-group of isometries. In the case that A is bounded, the definition of hermitian operator is equivalent to that given by Vidav [24] which is equivalent to requiring that $\|e^{itA}\| = 1$ for all $t \in \mathbb{R}$. Recently the bounded hermitian operators on some Banach algebras of Lipschitz functions were characterized in [10].

In this paper we investigate the hermitian operators on spaces of analytic vector-valued functions. Specifically we consider a Banach space E with trivial multiplier algebra, i.e. $\mathrm{Mult}(E) = \mathbb{C}$, and H_E^∞ represents the space of all bounded analytic functions from the open unit disc Δ into E. We take the norm of $F \in H_E^\infty$ to be $\|F\| = \sup_{z\in\Delta} \|F(z)\|_E$. We also consider the space $S_{\mathcal{K}}^\infty$ of analytic functions F on the disk with values in a complex Hilbert space $\mathcal{K}$ such that F' belongs to $H_{\mathcal{K}}^\infty$. The norm on this space is given by $\|F\| = \|F(0)\|_{\mathcal{K}} + \|F'\|_{H_{\mathcal{K}}^\infty}$. We will characterize the hermitian operators on these spaces in terms of one-parameter groups of isometries.

In an earlier work, Botelho and Jamison [9] characterized the generalized bi-circular projections in this setting. A *generalized bi-circular projection* on a Banach space X is a projection P such that for some modulus one complex number λ ($\lambda \neq 1$) we know that $P + (1 - \lambda)P$ is an isometry. It is easy to

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see that this isometry must be surjective. A projection P is bi-circular if for every $\lambda \neq 1$ and of modulus one, $P + (1-\lambda)P$ is a surjective isometry. The author [17] showed that every bi-circular projection is a bounded hermitian projection. As a consequence of the main result in the present paper, we also characterize the bi-circular projections on H^∞_E. As has been noticed in [18], generalized bi-circular projections are bi-contractive but the form of a general bi-contractive projection on H^∞_E is yet to be derived.

2. Bounded hermitian operators on H^∞_E. In what follows, E is a complex Banach space with $\mathrm{Mult}(E) = \mathbb{C}$. We refer the reader to Behrends [1] for the notion of multiplier algebra. We note that the condition $\mathrm{Mult}(E) = \mathbb{C}$ is satisfied by any strictly convex Banach space E. We will not need any further technical properties of $\mathrm{Mult}(E)$ but we require this constraint in order to apply the following result due to Cambern and Jarosz.

THEOREM 2.1 (cf. [11]). *Let E be a complex Banach space with $\mathrm{Mult}(E) = \mathbb{C}$, and let T be a surjective isometry on H^∞_E. Then T is of the form*

$$TF(z) = U \cdot F(\tau(z)), \qquad \forall F \in H^\infty_E,\ z \in D,$$

with U a constant surjective isometry of E and τ a conformal map of the open disc Δ onto itself.

We now suppose that $\{T_t\}$ is a uniformly continuous C_0-group of isometries acting on H^∞_E. Then there exists a one-parameter family $\{U_t\}$ of surjective isometries of E and a one-parameter family of disc automorphisms $\{\tau_t(z)\}$ such that, for every $F \in H^\infty_E$,

$$T_tF(z) = U_tF(\tau_t(z)), \qquad \forall z \in \Delta.$$

It is important to emphasize that each U_t does not depend on z. We now observe that $\{U_t\}$ is a uniformly continuous C_0-group of isometries acting on E. To see this, let $e \in E$ and choose F to be the constant function $F(z) = e$. Note that

$$\|T_t - I\| \geq \|T_tF - F\|_\infty = \|U_te - e\|_E.$$

Since this inequality holds for arbitrary $e \in E$, clearly $\|U_t - I\| \to 0$ as $t \to 0$. Since $T_sT_tF = T_{s+t}F$ for every s and t in $\mathbb{R}$, it follows that $U_sU_t = U_{s+t}$. Consequently, $\{U_t\}$ is a uniformly continuous C_0-group of isometries. Hence there exists a bounded hermitian operator $\mathcal{A}$ on E such that $U_t = e^{it\mathcal{A}}$ for every $t \in \mathbb{R}$. Now, consider $\widetilde{T}_t = e^{-it\mathcal{A}}T_t$. It is also clear that $\{\widetilde{T}_t\}$ is a uniformly continuous C_0-group of isometries on H^∞_E. For each $F \in H^\infty_E$, we have

$$(\widetilde{T}_tF)(z) = F(\tau_t(z)).$$

Choosing an arbitrary $e \in E$ and letting $F_1(z) = z \cdot e$, we have

$$(\widetilde{T}_t F_1)(z) = \tau_t(z) \cdot e. \tag{2.1}$$

The continuity of $t \mapsto \tau_t(z)$ follows immediately from (2.1) by applying linear functionals from E^* and using the uniform continuity of $\{\widetilde{T}_t\}$. Furthermore, the group property of $\{\widetilde{T}_t\}$ applied to F_1 implies that

$$\tau_{t+s}(z) = \tau_t(\tau_s(z)) = \tau_s(\tau_t(z)), \qquad \forall z \in \Delta.$$

We summarize these considerations in the following proposition.

PROPOSITION 2.2. *Suppose that E is a complex Banach space with* $\mathrm{Mult}(E) = \mathbb{C}$ *and that E supports nontrivial bounded hermitian operators. Then $\{T_t\}$ is a uniformly continuous C_0-group of surjective isometries acting on H_E^∞ if and only if there exist a uniformly continuous C_0-group $\{U_t\}$ acting on E and a continuous group $\{\tau_t\}$ of disk automorphisms such that*

$$(T_t F)(z) = U_t F(\tau_t(z)), \qquad \forall z \in \Delta.$$

In Berkson and Porta's paper [6], it was proven that the scalar-valued H^∞ space only supports hermitian operators. We will show that this is no longer the case for the vector-valued H_E^∞ under the hypothesis that $\mathrm{Mult}(E) = \mathbb{C}$ and also assuming that E supports nontrivial hermitian operators. A simple example of this situation is when E is a Hilbert space. The main result of this section is the following.

THEOREM 2.3. *Suppose that E is a complex Banach space with* $\mathrm{Mult}(E) = \mathbb{C}$ *and that E supports nontrivial bounded hermitian operators. Then $\mathfrak{H}$ is a bounded hermitian operator on H_E^∞ if and only if there exists a hermitian operator $\mathcal{A} \in B(E)$ such that*

$$\mathfrak{H}(F)(z) = \mathcal{A}[F(z)], \qquad \forall z \in \Delta.$$

Proof. Let $\mathfrak{H}$ be a bounded hermitian operator on H_E^∞. Then $\mathfrak{H}$ is the generator of a uniformly continuous C_0-group $\{T_t\}$ of surjective isometries acting on H_E^∞. By Proposition 2.2 there exists a bounded hermitian operator $\mathcal{A}$ acting on E and a continuous group $\{\tau_t\}$ of disk automorphisms such that

$$T_t(F)(z) = e^{it\mathcal{A}} F(\tau_t(z))$$

for every $F \in H_E^\infty$ and $z \in \Delta$. The generator of $\{T_t\}$ is given by

$$(\mathfrak{H}F)(z) = \left(-i\frac{d}{dt}T_t\right)_{t=0} F(z) = \mathcal{A} \cdot F(z) + F'(z) \cdot [\partial_t(\tau_t(z))]_{t=0}.$$

From the equation (1.6) in Berkson and Porta's paper [5, p. 335], we deduce that $[\partial_t(\tau_t(z))]_{t=0}$ is a quadratic polynomial and hence belongs to H^∞. Since $\{T_t\}$ is uniformly continuous, $\mathfrak{H} : H_E^\infty \to H_E^\infty$ is bounded. Let $F_n(z) = e_n(z) \cdot e$, where e is an arbitrary unit vector in E and $e_n(z) = z^n$. Clearly

$F_n \in H_E^\infty$ and $\|F_n\|_\infty = 1$. However, if $\tau_t(z) \neq z$ for every $z \in \Delta$, we get

$$\infty > \|\mathfrak{H}(F_n)\| \geq n\|\partial_t(\tau_t(z))|_{t=0}\|_\infty - \|\mathcal{A}\|.$$

This leads to an absurd unless $\partial_t(\tau_t(z))|_{t=0} = 0$. Thus $\tau_t(z) = z$ and the proof of the necessity is complete.

Conversely, if we assume that $\mathfrak{H}(F)(z) = \mathcal{A}[F(z)]$ for all $z \in \Delta$, the equation

$$e^{it\mathfrak{H}}F(z) = e^{it\mathcal{A}}F(z) \tag{2.2}$$

together with Cambern and Jarosz's Theorem 2.1 describing the form of isometries of H_E^∞ implies that $\mathfrak{H}$ is a bounded hermitian operator on H_E^∞. ∎

Remark 2.4. This result is reminiscent of the behavior of bounded hermitian operators on $C(\Omega, E)$ (see for example the results of Fleming and Jamison in [14]).

Using the result of A. R. Sourour on hermitian operators on the Schatten class [23, Theorem 1, p. 71] we have the next corollary.

Corollary 2.5. *Let H be a Hilbert space and $C_p(H)$ be the Schatten class of compact operators on H with $1 \leq p < \infty$, $p \neq 2$. Then $\mathfrak{R} : H_{C_p(H)}^\infty \to H_{C_p(H)}^\infty$ is a bounded hermitian operator if and only if there exist hermitian operators $\mathcal{A}$ and $\mathcal{B}$ in $B(H)$ such that*

$$(\mathfrak{H}F)(z) = \mathcal{A} \cdot F(z) + F(z) \cdot \mathcal{B}.$$

The form of the bounded hermitian operators on H_E^∞, with $\mathrm{Mult}(E) = \mathbb{C}$, described in Theorem 2.3, implies the following result on bi-circular projections.

Corollary 2.6. *$\mathfrak{Q}$ is a bi-circular projection on H_E^∞ where E is a complex Banach space with $\mathrm{Mult}(E) = \mathbb{C}$ if and only if there exists a norm hermitian projection $\mathcal{P}$ on E such that*

$$(\mathfrak{Q}F)(z) = \mathcal{P}[F(z)], \qquad \forall F \in H_E^\infty.$$

The idea of a bounded normal operator has also been extended to the Banach space setting in the following way: Let E be a complex Banach space and $A \in B(E)$. Then $\mathfrak{N}$ is normal if $\mathfrak{N} = \mathfrak{H}_1 + i\mathfrak{H}_2$ where $\mathfrak{H}_1$ and $\mathfrak{H}_2$ are hermitian and $\mathfrak{H}_1\mathfrak{H}_2 - \mathfrak{H}_1\mathfrak{H}_2 = 0$. The next corollary follows easily from Theorem 2.3.

Corollary 2.7. *$\mathfrak{N}$ is a bounded normal operator on H_E^∞ if and only if there is an $\mathcal{N} \in B(E)$ which is normal on E and $\mathfrak{N}F(z) = \mathcal{N} \cdot F(z)$ for every $z \in \Delta$.*

3. Hermitian operators on H_E^∞: The unbounded case. We consider strongly continuous groups of isometries on H_E^∞ with $\mathrm{Mult}(E) = \mathbb{C}$. Using an argument from Berkson and Porta [5] we are able to completely characterize the generators of strongly continuous C_0-groups of isometries.

PROPOSITION 3.1. *Suppose that E is a complex Banach space with* $\mathrm{Mult}(E) = \mathbb{C}$. *Let $\{T_t\}$ be a strongly continuous C_0-group of isometries on H_E^∞. Then $T_tF(z) = U_tF(z)$ for some strongly C_0-group of surjective isometries $\{U_t\}$ acting on E.*

Proof. From Theorem 2.1 and the group property of $\{T_t\}$, there exist a one-parameter group $\{U_t\}$ of surjective isometries on E and a group of disc automorphisms $\{\tau_t\}$ such that $T_tF(z) = U_tF(\tau_t(z))$. Since for every F in H_E^∞ we have $\|T_tF - F\| \to 0$ as $t \to 0$, it follows by judicious choices of functions F that $\{U_t\}$ is a strongly continuous group of surjective isometries on E and that $\{\tau_t\}$ is a continuous group of disk automorphisms. Let $e \in E$ and $w \in \varDelta$, and set

$$S_w(z) = \exp\left(\frac{1 + z\overline{w}}{z\overline{w} - 1}\right) \cdot e.$$

Since U_t is a family of isometries independent of z, the same reasoning applied in Berkson and Porta [5, Theorem 2.7, p. 340] implies that the family $\{\tau_t\}$ must be independent of t, for otherwise the action of T_t on $S_w(z)$ would contradict the strong continuity of T_t. Hence $\tau_t(z) = z$ and the argument is complete. ∎

The previous proposition allows us to state the following characterization for the unbounded hermitian operator on H_E^∞.

THEOREM 3.2. *$\mathfrak{A}$ is an unbounded hermitian operator on H_E^∞ if and only if there exists an unbounded hermitian operator $\mathcal{A}$ on E such that $\mathfrak{A}[F](z) = \mathcal{A} \cdot F(z)$ for all F in the domain of $\mathcal{A}$, $\mathcal{D}(\mathcal{A})$.*

REMARK 3.3. It follows from the previous theorem that the unbounded hermitian operators on H_E^∞ are inherited from the unbounded hermitian operators on E. If this class is empty then H_E^∞ supports only bounded hermitians.

We conclude this section with an example to demonstrate that unbounded hermitian operators do occur in nontrivial settings.

EXAMPLE. If we set $E = \ell_2$ and consider a family of operators $U_t : \ell_2 \to \ell_2$ given by $U_t[(x_k)] = (e^{it}x_1, e^{2it}x_2, e^{3it}x_3, \ldots)$ then $\{U_t\}$ defines a C_0-group of isometries of ℓ_2. The generator of this group is $\mathfrak{H}[(x_k)] = (x_1, 2x_2, 3x_3, \ldots)$, which is a closed densely defined linear transformation with domain

$$\mathcal{D}(\mathfrak{H}) = \Big\{x \in \ell_2 : \sum k^2|x_k|^2 < \infty\Big\}.$$

The maps $T_t : H^\infty_{\ell_2} \to H^\infty_{\ell_2}$ given by

$$(T_tF)(z) = U_t[f_1(z), f_2(z), \ldots] = (e^{it} f_1(z), e^{2it} f_2(z), \ldots)$$

clearly form a strongly continuous group of isometries with generator

$$(\mathfrak{H}F)(z) = (f_1(z), 2f_2(z), 3f_3(z), \ldots), \qquad \forall z \in \Delta.$$

It is easy to see that $\mathfrak{H}$ is an unbounded hermitian operator on $H^\infty_{\ell_2}$.

4. A class of hermitian operators on $S^\infty_{\mathcal{K}}$. In this section we consider a restricted class of hermitian operators on the Banach space $S^\infty_{\mathcal{K}}$ of all analytic functions F on the disk with values in a Hilbert space $\mathcal{K}$ such that F' belongs to $H^\infty_{\mathcal{K}}$. The norm in this space is given by $\|F\| = \|F(0)\|_{\mathcal{K}} + \|F'\|_{H^p_{\mathcal{K}}}$. A complete classification of the surjective isometries of $S^\infty_{\mathcal{K}}$ is not known at this juncture but we consider a class of isometries which may indeed be the most general form of isometry on this space. We examine the strongly continuous groups of such isometries and their generators. In particular we consider isometries of the form

$$TF(z) = Vf(0) + \int_0^z W(F')(\xi)\,d\xi, \tag{4.1}$$

where W is a surjective isometry of $H^\infty_{\mathcal{K}}$ and V is a unitary operator on $\mathcal{K}$. In an earlier work (cf. [15]) it was shown that in the case of $S^p_{\mathcal{K}}$ (with $1 \le p < \infty$) all surjective isometries are of this form.

We now suppose that $\{T_t\}$ is a strongly continuous group of surjective isometries given by (4.1). Then there exists a one-parameter family $\{V_t\}$ of unitaries on $\mathcal{K}$ and a one-parameter family of surjective isometries $\{W_t\}$ on $H^\infty_{\mathcal{K}}$ such that

$$T_tF(z) = V_tf(0) + \int_0^z W_t(F')(\xi)\,d\xi. \tag{4.2}$$

If we let $v \in \mathcal{K}$ be an arbitrary vector and set $F_0(z) = v$ for every z in the disk, then the group condition $T_{s+t} = T_sT_t$ applied to the function F_0 yields $V_{s+t}v = V_sV_tv$. Also $T_0F_0 = F_0$ implies that $V_0v = v$. Finally

$$\|T_tF_0 - F_0\| = \|V_tv - v\|_{\mathcal{K}} \tag{4.3}$$

shows that $\|V_tv - v\|_{\mathcal{K}} \to 0$ as $t \to 0$. Then we conclude that $\{V_t\}$ is a strongly continuous group of unitaries on $\mathcal{K}$. Clearly if $\{T_t\}$ is a uniformly continuous group of isometries on $S^\infty_{\mathcal{K}}$, the group $\{V_t\}$ will be a uniformly continuous group. Given the algebraic properties of the family $\{V_t\}$, the group equation $T_{s+t} = T_sT_t$ yields

$$V_sV_tF(0) + \int_0^z W_sW_t(F'(\xi))\,d\xi = V_{s+t} + \int_0^z W_{s+t}(F'(\xi))\,d\xi \tag{4.4}$$

for all $F \in S_{\mathcal{K}}^\infty$. Differentiation of equation (4.4) with respect to z yields $W_s W_t(F'(z)) = W_{s+t}(F'(z))$ for all $F \in H_{\mathcal{K}}^\infty$. It is clear that $W_0 = I_{H_{\mathcal{K}}^\infty}$ and hence $\{W_t\}$ is a one-parameter group of surjective isometries on $S_{\mathcal{K}}^\infty$. From the equation

$$\|T_t F(z) - F(z)\| = \|V_t F(0) - F(0)\|_{\mathcal{K}} + \|W_t F'(z) - F'(z)\|_{H_{\mathcal{K}}^\infty} \tag{4.5}$$

it is clear that strong (uniform) continuity of $\{T_t\}$ implies strong (uniform) continuity of $\{W_t\}$ on $H_{\mathcal{K}}^\infty$. We summarize these results in the following proposition.

Proposition 4.1. *If $\{T_t\}$ is a one-parameter C_0-group of surjective isometries on $S_{\mathcal{K}}^\infty$ given by* (4.2) *then the families $\{V_t\}$ and $\{W_t\}$ are strongly continuous one-parameter groups on $\mathcal{K}$ and $H_{\mathcal{K}}^\infty$, respectively. Moreover, the generator $\mathcal{G}$ of $\{T_t\}$ is given by*

$$\mathcal{G}[F(z)] = \mathfrak{A}F(0) + \int_0^z \mathfrak{R}[F'](\xi)\, d\xi, \tag{4.6}$$

where $\mathfrak{A}$ is the generator of $\{V_t\}$ on $\mathcal{K}$ and $\mathfrak{R}$ is the generator of $\{W_t\}$ on $H_{\mathcal{K}}^\infty$. The domain of the generator is given by

$$\mathcal{D}(\mathcal{G}) = \{F \in S_{\mathcal{K}}^\infty : \mathcal{G}[F] \textit{ is defined}\}.$$

Proof. The generator $\mathcal{G}$ of $\{T_t\}$ is given by

$$\begin{aligned} \mathcal{G}[F(z)] &= \left[\left(-i\frac{d}{dt}T_t\right)_{t=0}\right] F(z) \\ &= \left[\left(-i\frac{d}{dt}V_t\right)_{t=0} F(0)\right] + \left[\left(-i\frac{d}{dt}\right)_{t=0}\right]\left(\int_0^z W_t(F')(\xi)\, d\xi\right). \end{aligned} \tag{4.7}$$

Since

$$\mathfrak{A} = \left(-i\frac{d}{dt}V_t\right)_{t=0} \quad \text{and} \quad \mathfrak{R} = \left(-i\frac{d}{dt}W_t\right)_{t=0},$$

the result follows. ∎

Corollary 4.2. *If $\{T_t\}$ is a uniformly continuous one-parameter group on $S_{\mathcal{K}}^\infty$ then there exist bounded hermitian operators $\mathcal{A}$ and $\mathcal{B}$ on $\mathcal{K}$ such that*

$$\mathcal{G}[F](z) = \mathcal{A}[F(0)] + \mathcal{B}[F(z)] \tag{4.8}$$

for all $F \in \mathcal{D}(\mathcal{G})$, the domain of $\mathcal{G}$.

Remark 4.3. This corollary is a generalization of Theorem 6.1 in [15, p. 422] for scalar-valued functions in certain Banach spaces of analytic functions.

5. Remarks on isometric equivalence of some operators on H_E^∞. Let E be a Banach space and A_k be bounded operators on E. The operators are said to be *isometrically equivalent* if there exist surjective isometries T_1 and T_2 of E with the property that $T_1A_1 = A_2T_2$. If the operators are defined via symbols like for example composition operators, then it is of interest to characterize isometric equivalence in terms of the defining symbols (see for example [16]). We have shown that bounded hermitian operators on H_E^∞ are defined by hermitian operators on E in the sense that $\mathfrak{H}_kF(z) = \mathcal{A}_k(F(z))$. We state a theorem (without proof) which gives isometric equivalence results on hermitian operators and composition operators on H_E^∞. The theorem follows easily from our Theorem 2.3 and techniques in [15], [16].

THEOREM 5.1. *Let E be a Banach space with* $\mathrm{Mult}(E) = \mathbb{C}$ *and suppose that E supports nontrivial bounded hermitian operators. Let $\mathfrak{H}_k$ be bounded hermitian operators on H_E^∞. Let ψ_k be analytic maps of the disk into the disk and let C_{ψ_k} denote the associated composition operators on H_E^∞.*

(i) *$\mathfrak{H}_1$ is isometric equivalent to $\mathfrak{H}_2$ if and only if the symbols $\mathcal{A}_1$ and $\mathcal{A}_2$ are isometrically equivalent on E.*

(ii) *C_{ψ_1} is isometrically equivalent to C_{ψ_2} if and only if there exists a real number θ such that $\psi_1(z) = e^{-i\theta}\psi_2(e^{-i\theta}z)$.*

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