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THE NATURAL OPERATORS LIFTING 1-FORMS TO SOME VECTOR BUNDLE FUNCTORS

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Abstract. Let $F : \mathcal{M}f \to \mathcal{VB}$ be a vector bundle functor. First we classify all natural operators $T_{|\mathcal{M}f_n} \rightsquigarrow T^{(0,0)}(F_{|\mathcal{M}f_n})^*$ transforming vector fields to functions on the dual bundle functor $(F_{|\mathcal{M}f_n})^*$. Next, we study the natural operators $T^*_{|\mathcal{M}f_n} \rightsquigarrow T^*(F_{|\mathcal{M}f_n})^*$ lifting 1-forms to $(F_{|\mathcal{M}f_n})^*$. As an application we classify the natural operators $T^*_{|\mathcal{M}f_n} \rightsquigarrow T^*(F_{|\mathcal{M}f_n})^*$ for some well known vector bundle functors F.

0. Introduction. In [1], the authors studied the problem of how a 1-form ω on an n-manifold M can naturally induce a 1-form $B(\omega)$ on the cotangent bundle $(TM)^*$. This problem is concerned with natural operators $B : T^*_{|\mathcal{M}f_n} \rightsquigarrow T^*(T_{|\mathcal{M}f_n})^*$ in the sense of Kolář, Michor and Slovák [4], where $\mathcal{M}f_n$ is the category of n-dimensional manifolds and embeddings. The classification result of [1] says that every natural operator $B : T^*_{|\mathcal{M}f_n} \rightsquigarrow T^*(T_{|\mathcal{M}f_n})^*$ is of the form $B(\omega) = a\omega^V + b\lambda$ for some $a, b \in \mathbb{R}$, where ω^V is the vertical lifting of ω to $(TM)^*$ and λ is the canonical Liouville 1-form on $(TM)^*$.

In this paper we study a similar general problem with T replaced by an arbitrary vector bundle functor $F : \mathcal{M}f \to \mathcal{VB}$ from the category $\mathcal{M}f$ of all manifolds and maps into the category $\mathcal{VB}$ of vector bundles and vector bundle maps. First we classify all natural operators $T_{|\mathcal{M}f_n} \rightsquigarrow T^{(0,0)}(F_{|\mathcal{M}f_n})^*$ transforming vector fields to functions on the dual bundle functor $(F_{|\mathcal{M}f_n})^*$. Next we prove that every natural operator $B : T^*_{|\mathcal{M}f_n} \rightsquigarrow T^*(F_{|\mathcal{M}f_n})^*$ transforming a 1-form ω on an n-manifold M into a 1-form $B(\omega)$ on $(FM)^*$ is of the form $B(\omega) = a\omega^V + \lambda$ for some uniquely determined canonical map $a : (FM)^* \to \mathbb{R}$ and some canonical 1-form λ on $(FM)^*$. As an application we describe all natural operators $T^*_{|\mathcal{M}f_n} \rightsquigarrow T^*(F_{|\mathcal{M}f_n})^*$ for some well known vector bundle functors F. For $F = (J^rT^*)^*$ we recover the results of [5].

Natural operators lifting functions, vector fields and 1-forms to some natural bundles were used practically in all papers in which the problem of

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prolongation of geometric structures was studied, e.g. [12]. That is why such natural operators have been classified; see [1], [3]–[11], etc.

From now on the usual coordinates on $\mathbb{R}^n$ will be denoted by $x^1, \ldots, x^n$.

All manifolds are assumed to be finite-dimensional and smooth, i.e. of class $\mathcal{C}^\infty$. Maps between manifolds are assumed to be smooth.

1. Natural operators $T_{|\mathcal{M}f_n} \rightsquigarrow T^{(0,0)}(F_{|\mathcal{M}f_n})^*$. Let $F : \mathcal{M}f \to \mathcal{VB}$ be a vector bundle functor. We have the following example of natural operators $T_{|\mathcal{M}f_n} \rightsquigarrow T^{(0,0)}(F_{|\mathcal{M}f_n})^*$.

EXAMPLE 1. Let $v \in F_0\mathbb{R}$. Consider a vector field X on an n-manifold M. We define $A^v(X) : (FM)^* \to \mathbb{R}$ by $A^v(X)_\eta = \langle \eta, F(\Phi^X_x)(v)\rangle$, $\eta \in (F_xM)^*$, $x \in M$, where $\Phi^X_x : (\varepsilon, \varepsilon) \to M$, $\Phi^X_x(t) = \mathrm{Exp}(tX)_x$, $t \in (-\varepsilon, \varepsilon)$, $\varepsilon > 0$. The correspondence $A^v : T_{|\mathcal{M}f_n} \rightsquigarrow T^{(0,0)}(F_{|\mathcal{M}f_n})^*$ is a natural operator.

PROPOSITION 1. *Let $v_1, \ldots, v_L$ be a basis of the vector space $F_0\mathbb{R}$. Every natural operator $A : T_{|\mathcal{M}f_n} \rightsquigarrow T^{(0,0)}(F_{|\mathcal{M}f_n})^*$ is of the form*

$$A = H(A^{v_1}, \ldots, A^{v_L})$$

for a unique smooth map $H \in \mathcal{C}^\infty(\mathbb{R}^L)$.

Proof. Let $v_1^*, \ldots, v_L^* \in (F_0\mathbb{R})^*$ be the dual basis. Let $q : \mathbb{R}^n \to \mathbb{R}$ be the projection onto the first factor.

For A as above we define $H : \mathbb{R}^L \to \mathbb{R}$ by

$$H(t_1, \ldots, t_L) = A(\partial/\partial x^1)_{(F_0 q)^*(\sum_{s=1}^L t_s v_s^*)}.$$

We prove that $A = H(A^{v_1}, \ldots, A^{v_L})$. Since any non-vanishing vector field X is locally $\partial/\partial x^1$ in some local coordinates on M, it is sufficient to show that

$$A(\partial/\partial x^1)_\eta = H(A^{v_1}(\partial/\partial x^1)_\eta, \ldots, A^{v_L}(\partial/\partial x^1)_\eta) \quad \text{for any } \eta \in (F_0\mathbb{R}^n)^*.$$

Using the invariance of A and A^{v_s} with respect to $(x^1, (1/t)x^2, \ldots, (1/t)x^n) : \mathbb{R}^n \to \mathbb{R}^n$ for $t \neq 0$ and next letting $t \to 0$, we can assume that $\eta = (F_0 q)^*(\sum_{s=1}^L t_s v_s^*)$. Now, it remains to observe that $A^{v_s}(\partial/\partial x^1)_\eta = t_s$ for $s = 1, \ldots, L$.

The uniqueness of H is clear as $(A^{v_s}(\partial/\partial x^1))_{s=1}^L$ is a surjection onto $\mathbb{R}^L$. ■

By [2], we can choose a basis $v_1, \ldots, v_L \in F_0\mathbb{R}$ such that v_s is homogeneous of weight $n_s \in \mathbb{N} \cup \{0\}$, i.e. $F(\tau\, \mathrm{id}_\mathbb{R})(v_s) = \tau^{n_s} v_s$ for any $\tau \in \mathbb{R}$.

$(*)$ By a permutation we can assume that $v_1, \ldots, v_{k_1}$ are of weight 0, $v_{k_1+1}, \ldots, v_{k_2}$ are of weight 1, etc.

Then $A^{v_1}(X), \ldots, A^{v_{k_1}}(X)$ do not depend on X, i.e. $A^{v_1}, \ldots, A^{v_{k_1}}$ are natural functions on $(FM)^*$. Moreover $A^{v_{k_1+1}}(X), \ldots, A^{v_{k_2}}(X)$ depend linearly on X, i.e. $A^{v_{k_1+1}}, \ldots, A^{v_{k_2}}$ are linear operators.

Corollary 1. *Every natural function G on $(F_{|\mathcal{M}f_n})^*$ is of the form*

$$G = K(A^{v_1}, \ldots, A^{v_{k_1}})$$

for a unique $K \in \mathcal{C}^\infty(\mathbb{R}^{k_1})$. If F has the point property, *i.e.* $F\,\mathrm{pt} = \mathrm{pt}$, *then* $G = \mathrm{const}$.

Corollary 2. *Every natural linear operator $A : T_{\mathcal{M}f_n} \rightsquigarrow T^{(0,0)}(F_{|\mathcal{M}f_n})^*$ is of the form*

$$A = \sum_{s=k_1+1}^{k_2} K_s(A^{v_1}, \ldots, A^{v_{k_1}})A^{v_s}$$

for some unique $K_s \in \mathcal{C}^\infty(\mathbb{R}^{k_1})$.

Proof. The corollaries are consequences of Proposition 1 and the homogeneous function theorem [4]. ■

2. A decomposition proposition. Let F and $v_1, \ldots, v_L$ be as in Section 1 with the assumption $(*)$.

Example 2. If $\omega : TM \to \mathbb{R}$ is a 1-form on an n-manifold M, we have its vertical lifting $B^V(\omega) = \omega \circ T\pi : T(FM)^* \to \mathbb{R}$ to $(FM)^*$, where $\pi : (FM)^* \to M$ is the bundle projection. The correspondence $B^V : T^*_{|\mathcal{M}f_n} \rightsquigarrow T^*(F_{|\mathcal{M}f_n})^*$ is a natural operator.

Proposition 2 (Decomposition Proposition). *Consider a natural operator $B : T^*_{|\mathcal{M}f_n} \rightsquigarrow T^*(F_{|\mathcal{M}f_n})^*$. Then there exists a unique natural function a on $(F_{|\mathcal{M}f_n})^*$ such that*

$$B = aB^V + \lambda$$

for some canonical 1*-form λ on $(F_{|\mathcal{M}f_n})^*$.*

Lemma 1. (a) *We have $(B(\omega) - B(0))|(V(F\mathbb{R}^n)^*)_0 = 0$ for any $\omega \in \Omega^1(\mathbb{R}^n)$, where $(V(F\mathbb{R}^n)^*)_0$ is the fiber over $0 \in \mathbb{R}^n$ of the vertical subbundle in $T(F\mathbb{R}^n)^*$.*

(b) *If F has the point property then $B(\omega)|(V(F\mathbb{R}^n)^*)_0 = 0$ for any $\omega \in \Omega^1(\mathbb{R}^n)$.*

Proof. (a) There is a basis in $(V(F\mathbb{R}^n)^*)_0 \cong (F_0\mathbb{R}^n)^* \times (F_0\mathbb{R}^n)^*$ of homogeneous elements with weight from $\{0, -1, -2, \ldots,\}$ with respect to the action of $\mathbb{R}_+$ on $(V(F\mathbb{R}^n)^*)_0 \cong (F_0\mathbb{R}^n)^* \times (F_0\mathbb{R}^n)^*$ by lifting homotheties (see [2]). We use the invariance of $(B(\omega) - B(0))|(V(F\mathbb{R}^n)^*)_0$ with respect to the homotheties $(1/t)\,\mathrm{id}_{\mathbb{R}^n}$ for $t \neq 0$ and apply the homogeneous function theorem. We find that $(B(\omega) - B(0))|(V(F\mathbb{R}^n)^*)_0$ is independent of ω. This ends the proof of (a).

(b) We observe that if F has the point property then $(F_0\mathbb{R}^n)^*$ has no non-zero homogeneous elements of weight 0. Next, we use the invariance of the restriction $B(\omega)|(V(F\mathbb{R}^n)^*)_0$ with respect to the homotheties $(1/t)\,\mathrm{id}_{\mathbb{R}^n}$ for $t \neq 0$ and let $t \to 0$. ■

Proof of Proposition 2. Replacing B by $B - B(0)$ we can assume that $B(0) = 0$ and $B(\omega)|(V(F\mathbb{R}^n)^*)_0 = 0$ for any $\omega \in \Omega^1(\mathbb{R}^n)$. Then B is uniquely determined by the values $\langle B(\omega)_\eta, F^*(\partial/\partial x^1)_\eta\rangle$ for any $\omega = \sum \omega_i dx^i \in \Omega^1(\mathbb{R}^n)$ and $\eta \in (F_0\mathbb{R}^n)^*$, where $F^*(\partial/\partial x^1)$ is the complete lifting (flow prolongation) of $\partial/\partial x^1$ to $(F\mathbb{R}^n)^*$.

Using the invariance of B with respect to the homotheties $(1/t)\,\mathrm{id}_{\mathbb{R}^n}$ for $t \neq 0$ we get the homogeneity condition

$$t\left\langle B(\omega)_\eta, F^*\left(\frac{\partial}{\partial x^1}\right)_\eta\right\rangle = \left\langle B((t\,\mathrm{id}_{\mathbb{R}^n})^*\omega)_{F(\frac{1}{t}\,\mathrm{id}_{\mathbb{R}^n})^*(\eta)}, F^*\left(\frac{\partial}{\partial x^1}\right)_{F(\frac{1}{t}\,\mathrm{id}_{\mathbb{R}^n})^*(\eta)}\right\rangle.$$

Then by the non-linear Peetre theorem [4], the homogeneous function theorem and $B(0) = 0$ we deduce that $\langle B(\omega)_\eta, F^*(\partial/\partial x^1)_\eta\rangle$ is a linear combination of $\omega_1(0), \ldots, \omega_n(0)$ with coefficients being smooth maps in the homogeneous coordinates of η of weight 0.

Then using the invariance of B with respect to $(x^1, (1/t)x^2, \ldots, (1/t)x^n) : \mathbb{R}^n \to \mathbb{R}^n$ for $t \neq 0$ and letting $t \to 0$ we end the proof.

3. On canonical 1-forms on $(F_{|\mathcal{M}f_n})^*$

PROPOSITION 3. *Every canonical* 1*-form* λ *on* $(F_{|\mathcal{M}f_n})^*$ *induces a linear natural operator* $A^{(\lambda)} : T_{|\mathcal{M}f_n} \rightsquigarrow T^{(0,0)}(F_{|\mathcal{M}f_n})^*$ *such that* $A^{(\lambda)}(X)_\eta = \langle\lambda_\eta, F^*(X)_\eta\rangle$, $\eta \in (FM)^*$, $X \in \mathcal{X}(M)$, *where* $F^*(X)$ *is the complete lifting* (*flow operator*) *of* X *to* $(FM)^*$. *If* F *has the point property*, *then the correspondence* $\lambda \mapsto A^{(\lambda)}$ *is a linear injection.*

Proof. The injectivity is a consequence of Lemma 1(b). ■

4. A corollary

COROLLARY 3. *Assume that* F *has the point property and there are no non-zero elements in* $F_0\mathbb{R}$ *of weight* 1. (*For example*, *let* $F = F_1 \otimes F_2 : \mathcal{M}f \to \mathcal{VB}$ *be a tensor product of two vector bundle functors* $F_1, F_2 : \mathcal{M}f \to \mathcal{VB}$ *with the point property.*) *Then every natural operator* $B : T^*_{|\mathcal{M}f_n} \rightsquigarrow T^*(F_{|\mathcal{M}f_n})^*$ *is a constant multiple of the vertical lifting.*

Proof. Since there are no non-zero elements in $F_0\mathbb{R}$ of weight 1, we see that every canonical 1-form on $(F_{|\mathcal{M}f_n})^*$ is zero because of Corollary 2 and Proposition 3. Then Proposition 2 together with Corollary 1 ends the proof. ■

5. Applications

5.1. *The bundle functor T^{r*}_k of (k,r)-covelocities.* Let $T^{(r)}_k : \mathcal{M}f \to \mathcal{VB}$ be the bundle functor sending every manifold M to a vector bundle $T^{(r)}_k M = (J^r(M,\mathbb{R}^k)_0)^*$ over M, and every map $f : M \to N$ to a vector bundle map $T^{(r)}_k f : T^{(r)}_k M \to T^{(r)}_k N$ covering f such that $\langle T^{(r)}_k f(\eta), j^r_{f(x)}\gamma\rangle = \langle \eta, j^r_x(\gamma\circ f)\rangle$ for $\eta \in (T^{(r)}_k M)_x$, $j^r_{f(x)}\gamma \in J^r_{f(x)}(N,\mathbb{R}^k)_0$, $x \in M$, $\gamma = (\gamma^1,\ldots,\gamma^k) : N \to \mathbb{R}^k$, $\gamma(f(x)) = 0$. Then $T^{r*}_k = (T^{(r)}_k{}_{|\mathcal{M}f_n})^* : \mathcal{M}f_n \to \mathcal{VB}$ is the well known vector bundle functor of (k,r)-covelocities.

We have k canonical 1-forms $\lambda^r_1,\ldots,\lambda^r_k$ on $T^{r*}_k M$ such that

$$\langle \lambda^r_j, v\rangle = \langle d_x\gamma^j, T\pi(v)\rangle$$

for $v \in T_w T^{r*}_k M$, $w = j^r_x\gamma$, $x \in M$, $\gamma = (\gamma^1,\ldots,\gamma^k) : M \to \mathbb{R}^k$, $\gamma(x) = 0$, $j = 1,\ldots,k$, where $\pi : T^{r*}_k M \to M$ is the bundle projection.

COROLLARY 4. *Every natural operator $B : T^*_{|\mathcal{M}f_n} \rightsquigarrow T^*T^{r*}_k$ is a linear combination of the vertical lifting B^V and $\lambda^r_1,\ldots,\lambda^r_k$ with real coefficients.*

Proof. The vector bundle functor $T^{(r)}_k$ has the point property and the subspace of elements in $(T^{(r)}_k\mathbb{R})_0$ of weight 1 is k-dimensional. Then by Proposition 3 together with Corollaries 1 and 2, the space of canonical forms on T^{r*}_k is at most k-dimensional. Now, Proposition 2 ends the proof. ∎

In the special case $k = 1$, $T^{r*}_1 = T^{r*}$ is the r-cotangent bundle functor. So, we have the following result.

COROLLARY 5. *Every natural operator $B : T^*_{|\mathcal{M}f_n} \rightsquigarrow T^*T^{r*}$ is a linear combination of the vertical lifting B^V and the canonical r-cotangent bundle 1-form λ^r with real coefficients.*

In the case $r = 1$, $T^{1*} \cong T^*$ is the cotangent bundle and we recover the result mentioned in the introduction.

5.2. *The kernel of the jet projection $\pi^r_1 : T^{r*} \to T^{1*}$.* Let $\overline{T}^{(r)} : \mathcal{M}f \to \mathcal{VB}$ be the bundle functor sending every manifold M to the factor vector bundle $\overline{T}^{(r)}M = T^{(r)}M/TM$ over M, and every map $f : M \to N$ to the factor vector bundle map $\overline{T}^{(r)}f : \overline{T}^{(r)}M \to \overline{T}^{(r)}N$ covering f. Then $(\overline{T}^{(r)}_{|\mathcal{M}f_n})^*$ can be identified with the kernel $\ker\pi^r_1$ of the jet projection $\pi^r_1 : T^{r*} \to T^{1*}$.

COROLLARY 6. *Every natural operator $B : T^*_{|\mathcal{M}f_n} \rightsquigarrow T^*(\ker\pi^r_1)$ is a constant multiple of the vertical lifting.*

Proof. In $\overline{T}^{(r)}_0\mathbb{R}$ there are no non-zero elements of weight 1. ∎

5.3. *The r-jet prolongation J^rT^* of T^*.* Let $(J^rT^*)^* : \mathcal{M}f \to \mathcal{VB}$ be the bundle functor sending every manifold M to the vector bundle $(J^rT^*M)^*$

over M, and every map $f : M \to N$ to the vector bundle map $(J^rT^*)^*f : (J^rT^*M)^* \to (J^rT^*N)^*$ covering f such that $\langle (J^rT^*)^*f(\eta), j^r_{f(x)}\omega\rangle = \langle \eta, j^r_x(f^*\omega)\rangle$ for $\eta \in (J^rT^*M)^*_x$, $j^r_{f(x)}\omega \in J^r_{f(x)}T^*N$, $x \in M$, $\omega \in \Omega^1(N)$.

We have a canonical 1-form θ^r on J^rT^*M such that

$$\langle \theta^r, v\rangle = \langle \omega_x, T\pi(v)\rangle$$

for $v \in T_w(J^rT^*M)$, $w = j^r_x\omega$, $x \in M$, $\omega \in \Omega^1(M)$, where $\pi : J^rT^*M \to M$ is the bundle projection.

COROLLARY 7 ([5]). *Every natural operator $B : T^*_{|\mathcal{M}f_n} \rightsquigarrow T^*(J^rT^*)$ is a linear combination of the vertical lifting B^V and θ^r with real coefficients.*

Proof. The vector bundle functor $(J^rT^*)^* : \mathcal{M}f \to \mathcal{VB}$ has the point property and the subspace of elements in $(J^rT^*\mathbb{R})^*_0$ of weight 1 is 1-dimensional. ∎

In the case $r = 0$, $J^0T^* = T^*$ and we again recover the result mentioned in the introduction.

5.4. *The r-jet prolongation $J^r(\otimes^pT^*)$ of the tensor power $\otimes^pT^*$.* Let $(J^r(\otimes^pT^*))^* : \mathcal{M}f \to \mathcal{VB}$ be the bundle functor sending every manifold M to the vector bundle $(J^r(\otimes^pT^*M))^*$ over M, and every map $f : M \to N$ to the vector bundle map $(J^r(\otimes^pT^*))^*f : (J^r(\otimes^pT^*M))^* \to (J^r(\otimes^pT^*N))^*$ covering f such that $\langle (J^r(\otimes^pT^*))^*f(\eta), j^r_{f(x)}\tau\rangle = \langle \eta, j^r_x(f^*\tau)\rangle$ for $x \in M$, $\tau \in \mathcal{T}^{(0,p)}(N)$, $\eta \in (J^r(\otimes^pT^*M))^*_x$, $j^r_{f(x)}\tau \in J^r_{f(x)}(\otimes^pT^*N)$.

COROLLARY 8 ([5]). *For $p \geq 2$ every natural operator $B : T^*_{|\mathcal{M}f_n} \rightsquigarrow T^*(J^r(\otimes^pT^*))$ is a constant multiple of the vertical lifting.*

Proof. The vector bundle functor $(J^r(\otimes^pT^*))^*$ has the point property and the subspace of elements from $(J^r(\otimes^pT^*\mathbb{R}))^*_0$ of weight 1 is 0-dimensional. ∎

Similarly, replacing the tensor power $\otimes^p$ by the symmetric tensor power $\odot^p$ or by the skew-symmetric tensor power Λ^p we have

COROLLARY 9 ([5]). *For $p \geq 2$ every natural operator $B : T^*_{|\mathcal{M}f_n} \rightsquigarrow T^*(J^r(\odot^pT^*))$ is a constant multiple of the vertical lifting.*

COROLLARY 10 ([5]). *For $p \geq 2$ every natural operator $B : T^*_{|\mathcal{M}f_n} \rightsquigarrow T^*(J^r(\Lambda^pT^*))$ is a constant multiple of the vertical lifting.*

5.5. *The tensor pover $\otimes^rT^*$.* From Corollary 3 we obtain immediately

COROLLARY 11. *For $r \geq 2$ every natural operator $B : T^*_{|\mathcal{M}f_n} \rightsquigarrow T^*(\otimes^rT^*)$ is a constant multiple of the vertical lifting.*

Quite similarly we have

COROLLARY 12. *For $r \geq 2$ every natural operator $B : T^*_{|\mathcal{M}f_n} \rightsquigarrow T^*(\Lambda^r T^*)$ is a constant multiple of the vertical lifting.*

COROLLARY 13. *For $r \geq 2$ every natural operator $B : T^*_{|\mathcal{M}f_n} \rightsquigarrow T^*(\odot^r T^*)$ is a constant multiple of the vertical lifting.*

REMARK. It is clear that the presented list of applications of Propositions 1, 2 and 3 is not complete. Other applications are left to the reader.

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