COLLOQUIUM MATHEMATICUM

VOL. 93 2002 NO. 2

KOROVKIN-TYPE THEOREMS FOR ALMOST PERIODIC MEASURES

BY

SILVIA-OTILIA CORDUNEANU (Iaşi)

Abstract. Some Korovkin-type theorems for spaces containing almost periodic measures are presented. We prove that some sets of almost periodic measures are test sets for some particular nets of positive linear operators on spaces containing almost periodic measures. We consider spaces which contain almost periodic measures defined by densities and measures which can be represented as the convolution between an arbitrary measure with finite support (or an arbitrary bounded measure) and a fixed almost periodic measure. We also give a Korovkin-type result for the space of almost periodic measures; in this case the net of linear operators has a certain contraction property.

1. Introduction. Several Korovkin-type theorems are known for various spaces of functions on a compact or locally compact abelian group G ([1], [2], [10]). In this paper we give some Korovkin-type results for spaces containing almost periodic measures. We consider the notion of "test set" ([10]), which is usually used in Korovkin approximation-type theory on spaces of functions, and we transfer it to the context of almost periodic measures. So, our results have the following structure: *if a particular net* $(T_j)_j$ *of positive linear operators has the property that* $T_j\mu \to \mu$ *on a set of almost periodic measures*, *then* $T_j\mu \to \mu$ *on a larger space of almost periodic measures.*

We denote by $\widehat{G}$ the dual of the locally compact abelian group G, by $\mathrm{AP}(G)$ the space of all almost periodic functions on G and by $\mathrm{ap}(G)$ the space of all almost periodic measures on G. In 1982 F. Altomare proved that each set S of generators of $\widehat{G}$ is a test set for every net $(T_j)_j$ of positive linear operators on the space $\mathrm{AP}(G)$. This means that $T_jf \to f$ for $f \in S$ implies that $T_jf \to f$ for $f \in \mathrm{AP}(G)$ ([2]). We replace $\mathrm{AP}(G)$ with a space which contains almost periodic measures defined by densities. These measures have a fixed positive almost periodic measure μ as base. We use $\{\gamma\mu : \gamma \in \widehat{G}\}$ as test set. The convergence of the net of positive linear operators is in the topology of the locally convex space $\mathrm{ap}(G)$; this topology is given by a certain family $\{\|\cdot\|_f\}_{f\in K(G)}$ of seminorms and is called the *product topology*. Here $K(G)$ is the space of continuous complex-valued functions on G with

2000 *Mathematics Subject Classification*: Primary 43A60.

compact support. Next we indicate the test sets for another two spaces containing almost periodic measures. These spaces are $\{\nu*\mu : \nu$ is a measure with finite support$\}$ and $\{\nu * \mu : \nu$ is a bounded measure$\}$. The measure μ is also a fixed almost periodic measure.

Finally we give a Korovkin-type result for the space of almost periodic measures. This time the net $(T_j)_j$ has the property that every T_j is a *contraction* in the sense of the product topology on $\mathrm{ap}(G)$; we use the density of $\mathrm{AP}(G)$ in $\mathrm{ap}(G)$.

2. Preliminaries. Consider a Hausdorff locally compact abelian group G and let λ be the Haar measure on G. Denote by $\mathcal{C}(G)$ the set of all bounded continuous complex-valued functions on G, and by $\mathcal{C}_{\mathrm{U}}(G)$ the subset of $\mathcal{C}(G)$ containing the uniformly continuous functions. The sets $\mathcal{C}(G)$ and $\mathcal{C}_{\mathrm{U}}(G)$ are Banach algebras with pointwise multiplication and the supremum norm.

Throughout this paper, $\|\cdot\|$ denotes the supremum norm on $\mathcal{C}(G)$. For $f \in \mathcal{C}(G)$ and $a \in G$, the translate of f by a is the function $f_a(x) = f(xa)$ for all $x \in G$. Denote by $K(G)$ the linear space of all continuous complex-valued functions on G with compact support, and by $m(G)$ the space of complex *Radon measures* on G, that is, complex linear functionals μ on $K(G)$ satisfying the following: for each compact subset A of G there exists a positive number $m_{\mu,A}$ such that $|\mu(f)| \le m_{\mu,A}\|f\|$ whenever $f \in K(G)$ and the support of f is contained in A. We use $m_{\mathrm{F}}(G)$ to denote the subspace of $m(G)$ consisting of all *bounded measures*, that is, all linear functionals which are continuous with respect to the supremum norm on $K(G)$. The action of a measure $\mu \in m(G)$ on a function $f \in K(G)$ will be denoted by either $\mu(f)$ or $\int_G f(x)\,d\mu(x)$.

For $\mu \in m(G)$, one defines the *variation measure* $|\mu| \in m(G)$ by $|\mu|(f) = \sup\{|\mu(g)| : g \in K(G),\, |g| \le f\}$ for all $f \in K(G)$, $f \ge 0$. For Borel functions f, g and measures $\mu, \nu \in m(G)$, we can define their convolutions $f * g$, $f * \mu$, $\nu * \mu$, when that is possible ([6]). In [3] L. Argabright and J. Gil de Lamadrid defined a *translation-bounded measure* to be a measure $\mu \in m(G)$ with $m_\mu(A) = \sup_{x\in G} |\mu|(xA) < \infty$ for every compact set $A \subseteq G$. The linear space of translation-bounded measures will be denoted by $m_{\mathrm{B}}(G)$. We identify an arbitrary measure $\mu \in m_{\mathrm{B}}(G)$ with an element of the space $[\mathcal{C}_{\mathrm{U}}(G)]^{K(G)}$ in the following way: $\mu \equiv \{f*\mu\}_{f\in K(G)}$. From this identification we have the inclusion $m_{\mathrm{B}}(G) \subset [\mathcal{C}_{\mathrm{U}}(G)]^{K(G)}$. The space $[\mathcal{C}_{\mathrm{U}}(G)]^{K(G)}$ has the *product topology* defined by the Banach space structure on $\mathcal{C}_{\mathrm{U}}(G)$, hence, $m_{\mathrm{B}}(G)$ is a locally convex space of measures with the relative topology. A system of seminorms for the product topology on $m_{\mathrm{B}}(G)$ is given by the family $\{\|\cdot\|_f\}_{f\in K(G)}$, where $\|\mu\|_f = \|f * \mu\|$ for $f \in K(G)$ and $\mu \in m_{\mathrm{B}}(G)$. In [8] the almost periodic functions are defined:

DEFINITION 2.1. A function $g \in \mathcal{C}(G)$ is called *almost periodic* if the family of translates of g, $\{g_a : a \in G\}$, is relatively compact in the sense of uniform convergence on G.

The set $\mathrm{AP}(G)$ of all almost periodic functions on G is a Banach algebra with respect to the supremum norm, closed under conjugation. Denote by $\widehat{G}$ the dual group of G and by $[\widehat{G}]$ the linear subspace of $\mathcal{C}(G)$ generated by $\widehat{G}$. It is easy to see that $[\widehat{G}] \subset \mathrm{AP}(G)$. The almost periodic measures were introduced and studied by L. Argabright and J. Gil de Lamadrid ([4], [7]).

DEFINITION 2.2. A measure $\mu \in m_{\mathrm{B}}(G)$ is said to be *almost periodic* if $f * \mu \in \mathrm{AP}(G)$ for every $f \in K(G)$.

The set $\mathrm{ap}(G)$ of all almost periodic measures is a locally convex space with respect to the product topology. If $\nu \in m_{\mathrm{F}}(G)$ and $\mu \in \mathrm{ap}(G)$ then $\nu * \mu \in \mathrm{ap}(G)$. Also, if $f \in \mathrm{AP}(G)$ and $\mu \in \mathrm{ap}(G)$, then the measure $f\mu$ defined by $f\mu(g) = \mu(gf)$ for $g \in K(G)$, is almost periodic ([7]).

3. Korovkin-type results for spaces containing almost periodic measures. Let μ be a positive almost periodic measure. Consider the set

$$S(\mu) = \{f\mu : f \in \mathrm{AP}(G)\}.$$

Then $S(\mu)$ is a locally convex space containing the almost periodic measures and its topology is determined by the family $\{\|\cdot\|_f\}_{f \in K(G)}$ of seminorms.

In what follows we consider linear operators $T : \mathrm{ap}(G) \to \mathrm{ap}(G)$ which are *positive* in the sense that if ν is a positive almost periodic measure then $T(\nu)(h) \geq 0$ for all $h \in K(G)$, $h \geq 0$.

LEMMA 3.1. *Let $T : S(\mu) \to \mathrm{ap}(G)$ be a positive linear operator. Then for all $g \in K(G)$,*

$$\|T(f\mu)\|_g \leq 2\|f\| \cdot \|T\mu\|_{|g|}.$$

Proof. If $g \in K(G)$, then $|g| \in K(G)$. For $f \in \mathrm{AP}(G)$ we have

$$\|T(f\mu)\|_g = \|g * T(f\mu)\| = \sup_{x \in G} \Big| \int_G g(xy^{-1})\, d[T(f\mu)](y) \Big|. \tag{1}$$

Consider $x \in G$. It is clear that

$$\Big| \int_G g(xy^{-1})\, d[T(f\mu)](y) \Big| \leq \int_G |g|(xy^{-1})\, d|T(f\mu)|(y). \tag{2}$$

Using the properties of the variation measure we find

$$|T(f\mu)| = |T(f_1\mu + if_2\mu)| \leq |T(f_1\mu)| + |iT(f_2\mu)|.$$

It is easy to see that $|iT(f_2\mu)| = |T(f_2\mu)|$.

Using the properties of the variation measure and the fact that T is positive we obtain

$$|T(f_k\mu)| \le T(|f_k|\mu) \le \|f\|T(\mu), \qquad k = 1, 2.$$

Therefore

$$|T(f\mu)| \le 2\|f\|T(\mu).$$

Hence

$$\int_G |g|(xy^{-1})\, d|T(f\mu)|(y) \le 2\|f\| \int_G |g|(xy^{-1})\, d|T(\mu)|(y) = 2\|f\|[|g| * T(\mu)](x). \tag{3}$$

Therefore combining (1)–(3) it follows that

$$\|T(f\mu)\|_g \le 2\|f\| \cdot \||g| * T(\mu)\| = 2\|f\| \cdot \|T(\mu)\|_{|g|}. \ \blacksquare$$

THEOREM 3.1. *Let $(T_j)_{j\in J}$ be a net of positive linear operators $T_j : S(\mu) \to \mathrm{ap}(G)$ such that $T_j(\mu) = \mu$ for $j \in J$ and $T_j(f\mu) \to f\mu$ for all $f \in \widehat{G}$. Then $T_j(f\mu) \to f\mu$ for all $f \in \mathrm{AP}(G)$.*

Proof. Consider $f \in [\widehat{G}]$, $f = \sum_{i=1}^n a_i\gamma_i$, $\gamma_i \in \widehat{G}$, $i = 1, \dots, n$. For all $j \in J$ and $g \in K(G)$ we have

$$\|T_j(f\mu) - f\mu\|_g \le \sum_{i=1}^n |a_i| \cdot \|T_j(\gamma_i\mu) - \gamma_i\mu\|_g, \quad \text{thus} \quad T_j(f\mu) \to f\mu.$$

Consider $f \in \mathrm{AP}(G)$. For all $h \in \mathrm{AP}(G)$, $g \in K(G)$ and $j \in J$ we obtain

$$\|T_j(f\mu) - f\mu\|_g \le \|T_j(f\mu) - T_j(h\mu)\|_g + \|T_j(h\mu) - h\mu\|_g + \|h\mu - f\mu\|_g, \tag{4}$$

$$\|h\mu - f\mu\|_g = \|g * (h\mu - f\mu)\| = \sup_{x\in G}\Big|\int_G g(xy^{-1})[h(y) - f(y)]\, d\mu(y)\Big| \le \|h - f\| \cdot \|g\| m_\mu(A^{-1}), \tag{5}$$

where $A = \operatorname{supp} g$. From Lemma 3.1 it follows that

$$\|T_j(f\mu) - T_j(h\mu)\|_g \le 2\|f - h\| \cdot \|\mu\|_{|g|}. \tag{6}$$

On the other hand $\overline{[\widehat{G}]} = \mathrm{AP}(G)$ in the sense of uniform convergence ([9]) and we can find $h_0 \in [\widehat{G}]$ such that

$$\|f - h_0\| \le \min\left(\frac{\varepsilon}{3\|g\| m_\mu(A^{-1})}, \frac{\varepsilon}{6\|\mu\|_{|g|}}\right). \tag{7}$$

From $T_j(h_0\mu) \to h_0\mu$ it results that there exists $j_0 \in J$ with

$$\|T_j(h_0\mu) - h_0\mu\|_g < \varepsilon/3 \quad \text{for } j \ge j_0. \tag{8}$$

Combining (4)–(8) we conclude that $\|T_j(f\mu) - f\mu\|_g < \varepsilon$ for $j \ge j_0$. $\blacksquare$

For all $x \in G$ we denote by δ_x the Dirac measure concentrated at x.

PROPOSITION 3.1. *Consider $\mu \in \mathrm{ap}(G)$ and let $(T_j)_{j\in J}$ be a net of positive linear operators $T_j : \mathrm{ap}(G) \to \mathrm{ap}(G)$ such that $T_j(\mu) \to \mu$.*

(a) *If $T_j(\delta_x * \mu) = \delta_x * T_j(\mu)$ for all $x \in G$ and $j \in J$ then $T_j(\nu*\mu) \to \nu*\mu$ for all measures ν with finite support.*

(b) *If $T_j(\nu*\mu) = \nu*T_j(\mu)$ for all $\nu \in m_{\mathrm{F}}(G)$ and $j \in J$ then $T_j(\nu*\mu) \to \nu * \mu$ for all $\nu \in m_{\mathrm{F}}(G)$.*

Proof. (a) Consider a measure ν with finite support. We can represent it as $\nu = \sum_{i=1}^n a_i\delta_{x_i}$. It is easy to see that for all $j \in J$,

$$T_j(\nu * \mu) = \sum_{i=1}^n a_i\delta_{x_i} * T_j(\mu).$$

Therefore $T_j(\nu * \mu) \to \nu * \mu$.

(b) Let $\nu \in m_{\mathrm{F}}(G)$. For all $g \in K(G)$ and $j \in J$ we obtain

$$\|T_j(\nu * \mu) - \nu * \mu\|_g = \|\nu * T_j(\mu) - \nu * \mu\|_g = \|\nu * g * [T_j(\mu) - \mu]\|.$$

Taking into account that

$$\|\nu * g * [T_j(\mu) - \mu]\| = \sup_{x\in G}\Big|\int_G g * [T_j(\mu) - \mu](x)\, d\nu(x)\Big|,$$

we obtain $\|\nu * g * [T_j(\mu) - \mu]\| \le |\nu|(G)\|g * [T_j(\mu) - \mu]\|$. Hence

$$\|T_j(\nu * \mu) - \nu * \mu\|_g \le |\nu|(G)\|T_j(\mu) - \mu\|_g,$$

and therefore $T_j(\nu * \mu) \to \nu * \mu$, for all $\nu \in m_{\mathrm{F}}(G)$. ∎

THEOREM 3.2. *Let $(T_j)_{j\in J}$ be a net of positive linear operators $T_j : \mathrm{ap}(G) \to \mathrm{ap}(G)$ such that $T_j(\gamma\lambda) \to \gamma\lambda$ for all $\gamma \in \widehat{G}$, and $\|T_j(\mu)\|_f \le \|\mu\|_f$ for all $f \in K(G)$, $j \in J$ and $\mu \in \mathrm{ap}(G)$. Then $T_j(\mu) \to \mu$ for all $\mu \in \mathrm{ap}(G)$.*

Proof. First we see that if $g \in [\widehat{G}]$ then the hypothesis implies that $T_j(g\lambda) \to g\lambda$. Now consider $g \in \mathrm{AP}(G)$. There exists a sequence $(g_n)_n$ of trigonometric polynomials such that $\|g_n - g\| \to 0$ ([9]). Then for all $f \in K(G)$ we obtain

$$\begin{aligned}
(9)\qquad & \|T_j(g\lambda) - g\lambda\|_f \\
&\le \|T_j(g\lambda) - T_j(g_n\lambda)\|_f + \|T_j(g_n\lambda) - g_n\lambda\|_f + \|g_n\lambda - g\lambda\|_f \\
&\le \|T_j(g_n\lambda) - g_n\lambda\|_f + 2\|g_n\lambda - g\lambda\|_f \\
&= \|T_j(g_n\lambda) - g_n\lambda\|_f + 2\|f * g_n\lambda - f * g\lambda\| \\
&\le \|T_j(g_n\lambda) - g_n\lambda\|_f + 2\|f\| \cdot \|g_n - g\|\lambda(A^{-1}),
\end{aligned}$$

where $A = \operatorname{supp} f$. By the first part of the proof and the hypothesis it results from (9) that $T_j(g\lambda) \to g\lambda$.

Consider $\mu \in \mathrm{ap}(G)$. We prove that $\|T_j(\mu) - \mu\|_f \to 0$ for all $f \in K(G)$. Suppose that $(\varphi_i)_{i\in I}$ is an approximate identity for the convolution ([6, p. 450]). This means that $\varphi_i \in K(G)$, $\varphi_i \geq 0$, $\operatorname{supp}\varphi_i = V_i$, $i \in I$. Also, assume that $(V_i)_{i\in I}$ is a decreasing net of sets such that $\bigcap_{i\in I} V_i = \{e\}$ and $\int_G \varphi_i(x)\, d\lambda(x) = 1$ for all $i \in I$.

Let $f_i = \varphi_i * \mu$ for all $i \in I$; it is obvious that every $f_i \in \mathrm{AP}(G)$. If $f \in K(G)$ then $f * \mu \in \mathrm{AP}(G)$ and $f * \mu = \lim_i(\varphi_i * f * \mu) = \lim_i(f * f_i)$ in $\mathrm{AP}(G)$. Hence there exists $(f_i)_i$ such that $\|f * f_i - f * \mu\| \to 0$. Further we obtain

$$\begin{aligned} (10)\quad \|T_j(\mu) - \mu\|_f &\leq \|T_j(\mu) - T_j(f_i\lambda)\|_f + \|T_j(f_i\lambda) - f_i\lambda\|_f + \|f_i\lambda - \mu\|_f \\ &\leq \|T_j(f_i\lambda) - f_i\lambda\|_f + 2\|f_i\lambda - \mu\|_f \\ &= \|T_j(f_i\lambda) - f_i\lambda\|_f + 2\|f * f_i\lambda - f * \mu\|. \end{aligned}$$

Consider $\varepsilon > 0$. There exists i_0 such that $\|f * f_i - f * \mu\| < \varepsilon/4$ for all $i \geq i_0$. Further there exists j_0 such that $\|T_j(f_{i_0}\lambda) - f_{i_0}\lambda\|_f < \varepsilon/2$ for all $j \geq j_0$. Therefore (10) implies that $\|T_j(\mu) - \mu\|_f < \varepsilon$ for all $j \geq j_0$. ∎

Remark 3.1. We have proved in Theorem 3.2 that if $\mu \in \mathrm{ap}(G)$ then there exists a net $(f_i)_i$, $f_i \in \mathrm{AP}(G)$ for all $i \in I$, such that $f_i\lambda \to \mu$ in the sense of the product topology of $\mathrm{ap}(G)$. In other words $\overline{\mathrm{AP}(G)} = \mathrm{ap}(G)$.

Theorem 3.3. *Consider $\nu \in m_{\mathrm{F}}(G)$ and let $(T_j)_{j\in J}$ be a net of positive linear operators $T_j : \mathrm{ap}(G) \to \mathrm{ap}(G)$ such that $T_j(\nu * \gamma\lambda) \to \nu * \gamma\lambda$ for all $\gamma \in \widehat{G}$. Moreover, suppose that $\|T_j(\mu)\|_f \leq \|\mu\|_f$ for all $\mu \in \mathrm{ap}(G)$, $f \in K(G)$ and $j \in J$. Then $T_j(\nu * \mu) \to \nu * \mu$ for all $\mu \in \mathrm{ap}(G)$.*

Proof. First we see that if $g \in [\widehat{G}]$ then the hypothesis implies that $T_j(\nu * g\lambda) \to \nu * g\lambda$. Now consider $g \in \mathrm{AP}(G)$. There exists a sequence $(g_n)_n$ of trigonometric polynomials such that $\|g_n - g\| \to 0$ ([9]). Then for all $f \in K(G)$ and $j \in J$ we obtain

$$\begin{aligned} (11)\quad \|T_j(\nu * g\lambda) - \nu * g\lambda\|_f &\leq \|T_j(\nu * g\lambda) - T_j(\nu * g_n\lambda)\|_f \\ &\quad + \|T_j(\nu * g_n\lambda) - \nu * g_n\lambda\|_f + \|\nu * g_n\lambda - \nu * g\lambda\|_f \\ &\leq \|T_j(\nu * g_n\lambda) - \nu * g_n\lambda\|_f + 2\|\nu * g_n\lambda - \nu * g\lambda\|_f \\ &= \|T_j(\nu * g_n\lambda) - \nu * g_n\lambda\|_f + 2\|f * \nu * g_n\lambda - f * \nu * g\lambda\| \\ &\leq \|T_j(\nu * g_n\lambda) - \nu * g_n\lambda\|_f + 2\|f * \nu\| \cdot \|g_n - g\|\lambda(A^{-1}), \end{aligned}$$

where $A = \operatorname{supp} f$. By the first part of the proof and the hypothesis it results from (11) that $T_j(\nu * g\lambda) \to \nu * g\lambda$.

Consider $\mu \in \mathrm{ap}(G)$. We prove that $\|T_j(\nu * \mu) - \nu * \mu\|_f \to 0$ for all $f \in K(G)$. Let $f \in K(G)$. Since $\mu \in \mathrm{ap}(G)$ and according to Remark 3.1 we can select a net $(f_i)_i$, $f_i \in \mathrm{AP}(G)$, such that $f_i\lambda \to \mu$. Further we obtain

$$\begin{aligned}(12)\qquad \|T_j(\nu * \mu) - \nu * \mu\|_f &\\ &\le \|T_j(\nu * \mu) - T_j(\nu * f_i\lambda)\|_f \\ &\quad + \|T_j(\nu * f_i\lambda) - \nu * f_i\lambda\|_f + \|\nu * f_i\lambda - \nu * \mu\|_f \\ &\le \|T_j(\nu * f_i\lambda) - \nu * f_i\lambda\|_f + 2\|\nu * f_i\lambda - \nu * \mu\|_f \\ &= \|T_j(\nu * f_i\lambda) - \nu * f_i\lambda\|_f + 2\|f * \nu * f_i\lambda - f * \nu * \mu\|,\end{aligned}$$

for all $j \in J$ and $i \in I$. We easily obtain

$$\|f * \nu * f_i\lambda - f * \nu * \mu\| \le |\nu|(G)\|f * f_i\lambda - f * \mu\|.$$

Consider $\varepsilon > 0$. There exists i_0 such that

$$\|f * f_i - f * \mu\| < \frac{\varepsilon}{4|\nu|(G)}$$

for all $i \ge i_0$. Further there exists j_0 such that for all $j \ge j_0$,

$$\|T_j(\nu * f_{i_0}\lambda) - \nu * f_{i_0}\lambda\|_f < \varepsilon/2.$$

Therefore (12) implies that $\|T_j(\mu) - \mu\|_f < \varepsilon$ for all $j \ge j_0$. ■

REFERENCES

[1] F. Altomare, *Quelques remarques sur les ensembles de Korovkin dans les espaces des fonctions continues complexes*, in: Séminaire G. Choquet, M. Rogalski, J. Saint Raymond, 20e Année 1980–1981, no. 4, Publ. Math. Univ. Pierre et Marie Curie, Paris, 1980.

[2] —, *Frontières abstraites et convergence de familles filtrées de formes linéaires sur les algèbres de Banach commutatives*, in: Séminaire G. Choquet, M. Rogalski, J. Saint Raymond, 21e Année 1981-1982, no. 6, Publ. Math. Univ. Pierre et Marie Curie, Paris, 1982.

[3] L. N. Argabright and J. Gil de Lamadrid, *Fourier analysis of unbounded measures on locally compact abelian groups*, Mem. Amer. Math. Soc. 145 (1974).

[4] —, —, *Almost periodic measures*, ibid. 428 (1990).

[5] N. Dinculeanu, *Vector Measures*, Deutscher Verlag Wiss., Berlin, 1966.

[6] —, *Integrarea pe Spaţii Local Compacte*, Editura Academiei, Bucureşti, 1965 (in Romanian).

[7] J. Gil de Lamadrid, *Sur les mesures presque périodiques*, in: Séminaire KGB sur les marches aléatoires, Astérisque 4 (1973), 61–89.

[8] E. Hewitt and K. A. Ross, *Abstract Harmonic Analysis*, Vol. I, Springer, Berlin, 1963.

[9] W. Rudin, *Fourier Analysis on Groups*, Interscience Tracts Pure Appl. Math. 12, Interscience–Wiley, New York, 1962.

[10] J. Sussich, *Korovkin's theorem for locally compact abelian groups*, thesis, Murdoch Univ., Perth, 1982.

Department of Mathematics
Gh. Asachi Technical University, Iaşi
Bd. Copou 11
6600 Iaşi, Romania
E-mail: scord@tuiasi.ro

Received 7 February 2001;
revised 18 February 2002 (4028)