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ON κ-LINDELÖF SPACES

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Abstract. We use the Hausdorff pseudocharacter to bound the cardinality and the Lindelöf degree of κ-Lindelöf Hausdorff spaces.

1. Introduction and preliminaries. Like compactness, the Lindelöf property has been generalized in different ways, by several authors: *linearly Lindelöf*, *strongly discretely linearly Lindelöf* and *κ-Lindelöf* spaces have been defined. Of course, for each property of Lindelöf type, $\mathcal{P}$, it is natural to consider the following two general questions:

QUESTION 1.1. *Which additional conditions force a space X which satisfies $\mathcal{P}$ to be Lindelöf?*

QUESTION 1.2. *Which theorems on Lindelöf spaces can be extended to spaces which satisfy $\mathcal{P}$?*

In [2] and [3] Arhangel'skiĭ and Buzyakova make a contribution in both directions for $\mathcal{P}$ = linearly Lindelöf. In this paper we will do it for $\mathcal{P}$ = κ-Lindelöf. In other words, we are interested in the following problems: (1) Which additional conditions force a κ-Lindelöf space to be Lindelöf? and (2) Which theorems on Lindelöf spaces can be extended to κ-Lindelöf spaces? In particular we will prove that: (1) *the cardinality of a κ^+-Lindelöf Hausdorff space with $H\psi(X) \le \kappa$ is at most 2^κ, assuming that every closed subset A of X is a G_{2^κ}-set.* (2) *The cardinality of a κ^+-Lindelöf Hausdorff space with $H\psi(X) \le \kappa$ is at most 2^{2^κ}.* (Here $H\psi(X)$ is the Hausdorff pseudocharacter of X; see Definition 1.3.)

We refer the reader to [7] and [9] for definitions and terminology on cardinal functions not explicitly given here. Let w, nw, L, s, χ, ψ, ψ_c and t denote the following standard cardinal functions: weight, net weight, Lindelöf degree, spread, character, pseudocharacter, closed pseudocharacter and tightness, respectively. If ϕ is a cardinal function, then the hereditary version of ϕ, denoted $h\phi$, is defined by $h\phi(X) = \sup\{\phi(Y) : Y \subset X\}$. It is well known that ϕ is monotone if and only if $\phi = h\phi$.

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DEFINITION 1.3 ([8]). The *Hausdorff pseudocharacter* of X, denoted $H\psi(X)$, is the smallest infinite cardinal κ such that for each $x \in X$, there is a collection $\mathcal{B}_x$ of open neighborhoods of x, such that:

(1) $|\mathcal{B}_x| \leq \kappa$.
(2) If $x \neq y$ there are $V_x \in \mathcal{B}_x$ and $V_y \in \mathcal{B}_y$ such that $V_x \cap V_y = \emptyset$.

Let κ be an infinite cardinal, and let X be a set. Suppose that for each $x \in X$, $\mathcal{V}_x$ is a family of subsets of X which contain x. For every $L \subseteq X$, let $L^* = \{x \in X : V \cap L \neq \emptyset$ for all $V \in \mathcal{V}_x\}$ (see Hodel [8]).

In the proofs of Theorems 2.9, 2.12 and 2.14 we will make use of the following result due to Hodel [8].

THEOREM 1.4 ([8]). *Let κ be an infinite cardinal, and let X be a set. If for each $x \in X$, $\mathcal{V}_x = \{V_\gamma(x) : \gamma < \kappa\}$ is a family of subsets of X which contain x such that for $x \neq y$, there exists $\gamma \in \kappa$ such that $V_\gamma(x) \cap V_\gamma(y) = \emptyset$, then for every $L \subseteq X$:*

(1) $|L^*| \leq |L|^\kappa$.
(2) *If $L = \bigcup_{\alpha<\kappa^+} E_\alpha^*$, where $\{E_\alpha : 0 \leq \alpha < \kappa^+\}$ is a sequence of subsets of X with $\bigcup_{\beta<\alpha} E_\beta^* \subseteq E_\alpha$ for all $\alpha < \kappa^+$, then $L^* = L$.*

2. κ-Lindelöf spaces

DEFINITION 2.1 ([1]). A topological space X is called κ-*Lindelöf* if every open cover $\mathcal{U}$ of X with $|\mathcal{U}| \leq \kappa$ has a countable subcover.

It follows that every Lindelöf space is κ-Lindelöf for every infinite cardinal κ. However, a κ-Lindelöf space need not be Lindelöf (see [4]). On the other hand, every linearly Lindelöf space (*every increasing open cover of X has a countable subcover*) is ω_1-Lindelöf. Of course, every topological space which can be represented as a countable union of subspaces each of which is κ-Lindelöf is itself κ-Lindelöf.

The next result is easy to prove.

THEOREM 2.2. *The following are equivalent for a topological space X and an infinite cardinal number κ:*

(1) *X is κ-Lindelöf.*
(2) *For every collection $\mathcal{F}$ of nonempty closed subsets of X with $|\mathcal{F}| \leq \kappa$ which satisfies the countable intersection property, $\bigcap\{F : F \in \mathcal{F}\} \neq \emptyset$.*
(3) *For every collection $\mathcal{F}$ of nonempty sets of X with $|\mathcal{F}| \leq \kappa$ which satisfies the countable intersection property, $\bigcap\{\overline{F} : F \in \mathcal{F}\} \neq \emptyset$.*

One easily checks that if a subspace F of a topological space X is a κ-Lindelöf space, then for every collection $\mathcal{U}$ of open subsets of X with $F \subseteq \bigcup \mathcal{U}$, there exists $\mathcal{V} \in [\mathcal{U}]^{\leq\omega}$ such that $F \subseteq \bigcup \mathcal{U}$.

Like Lindelöfness, κ-Lindelöfness is preserved by continuous mappings and closed subsets, which is easy to prove:

THEOREM 2.3. *If X is a κ-Lindelöf space, then so is every closed subset and every continuous image of X.*

It is clear from Definition 2.1 that if X is a κ-Lindelöf space for some infinite cardinal κ, then X is γ-Lindelöf for every infinite cardinal $\gamma \leq \kappa$. Now let κ be an infinite cardinal and suppose that X is γ-Lindelöf for every infinite cardinal $\gamma < \kappa$. Is it true that X is κ-Lindelöf? In the next result we give a partial affirmative answer to this question. The proof follows the pattern of Theorem 45 in [10].

THEOREM 2.4. *Suppose that κ is a singular cardinal with $\mathrm{cf}(\kappa) \neq \omega$ and X is a topological space that is θ-Lindelöf for every cardinal number $\omega \leq \theta < \kappa$. Then X is κ-Lindelöf.*

Proof. Let $\mathcal{U} = \{U_\alpha : \alpha < \kappa\}$ be an open cover of X. Choose cardinals $\kappa_\beta < \kappa$, $\beta \in \mathrm{cf}(\kappa)$, for which $\sup\{\kappa_\beta : \beta \in \mathrm{cf}(\kappa)\} = \kappa$. For each $\beta \in \kappa$ let $V_\beta = \bigcup\{U_\alpha : \alpha < \kappa_\beta\}$ and $\mathcal{W} = \{V_\beta : \beta \in \mathrm{cf}(\kappa)\}$.

Clearly $\bigcup\mathcal{W} = X$ and $|\mathcal{W}| \leq \mathrm{cf}(\kappa) < \kappa$; hence, by hypothesis, there is $\mathcal{W}' \in [\mathcal{W}]^{\leq\omega}$ such that $X = \bigcup\mathcal{W}'$. Now, since $\mathrm{cf}(\kappa)$ is regular, there exists $\beta \in \mathrm{cf}(\kappa)$ such that $\bigcup\mathcal{W}' \subseteq \bigcup\{U_\alpha : \alpha < \kappa_\beta\}$. Thus $\{U_\alpha : \alpha < \kappa_\beta\}$ cover X and due to $\kappa_\beta < \kappa$, there exists $\mathcal{V} \in [\{U_\alpha : \alpha < \kappa_\beta\}]^{\leq\omega}$ such that $X = \bigcup\mathcal{V}$. ■

As we mentioned after Definition 2.1, every Lindelöf space is κ-Lindelöf. Now, it is not difficult to show that if X is a κ-Lindelöf space such that $\kappa \geq w(X)$ or $\kappa \geq nw(X)$, then X is Lindelöf. This fact suggests the next question.

QUESTION 2.5. *For which infinite cardinals κ, does κ-Lindelöf imply Lindelöf?*

In connection with the last question we have the following simple result.

PROPOSITION 2.6. *Let X be a κ-Lindelöf space with $s(X) \leq \kappa$ such that $\overline{D}$ is Lindelöf for every discrete subspace D of X. Then X is Lindelöf.*

Proof. Let $\mathcal{U}$ be an open cover of X. Since $s(X) \leq \kappa$, there exists $D \in [X]^{\leq\kappa}$ discrete and $\mathcal{V}_1 \in [\mathcal{U}]^{\leq\kappa}$ such that $X = \bigcup\mathcal{U} = \overline{D} \cup \bigcup\mathcal{V}_1$ (see Proposition 4.8 of [7]). Now, as $\overline{D} \subseteq \bigcup\mathcal{U}$ and $\overline{D}$ is Lindelöf, there exists $\mathcal{V}_2 \in [\mathcal{U}]^{\leq\omega}$ such that $\overline{D} \subseteq \bigcup\mathcal{V}_1$. Then $\mathcal{V} = \mathcal{V}_1 \cup \mathcal{V}_2 \in [\mathcal{U}]^{\leq\kappa}$ and $X = \bigcup\mathcal{V}$. Thus, since X is κ-Lindelöf, there is $\mathcal{W} \in [\mathcal{V}]^{\leq\omega}$ (and therefore $\mathcal{W} \in [\mathcal{U}]^{\leq\omega}$) such that $\bigcup\mathcal{W} = X$. ■

COROLLARY 2.7. *If X is a linearly Lindelöf space with $s(X) \leq \omega_1$ such that $\overline{D}$ is Lindelöf for every discrete subspace D of X, then X is Lindelöf.*

The author does not know the answer to the following question.

QUESTION 2.8. *Let X be a topological space and suppose that $\overline{D}$ is κ-Lindelöf for every discrete subspace D of X. Is X a κ-Lindelöf space?*

Note that if X is a topological space with $s(X) = \omega$ and $\overline{D}$ is κ-Lindelöf for every discrete subspace D of X, then X is κ-Lindelöf.

Arhangel'skiĭ and Buzyakova have proved in [2] that *if X is a Tikhonov space with $t(X) = \kappa$ such that $\overline{D}$ is Lindelöf for every discrete subspace D of X, then X is κ^+-Lindelöf.*

It is of interest whether Arhangel'skiĭ's inequality and its generalizations hold in the class of ω_1-Lindelöf spaces. In [4], Buzyakova proved that *every first countable ω_1-Lindelöf Hausdorff space has cardinality at most 2^{2^κ}*; and countable pseudocharacter can be replaced by countable tigthness plus closed pseudocharacter (see [4]).

In [8], Hodel obtained a very nice generalization of Arhangel'skiĭ's inequality by showing that $|X| \leq 2^{L(X)H\psi(X)}$ for every Hausdorff space. This generalizes Arhangel'skiĭ's inequality in that it replaces χ with $H\psi$ (the Hausdorff pseudocharacter), a local cardinal function that captures the Hausdorff property of X. At the same time $H\psi$ is a strengthening of ψ_c (the closed pseudocharacter) and so tightness can be omittted from the hypotheses. Hence it is natural to ask: *Let X be a κ^+-Lindelöf Hausdorff space with $H\psi(X) \leq \kappa$; is it true that* (a) $|X| \leq 2^\kappa$*;* (b) $|X| \leq 2^{2^\kappa}$?

We will use the elementary submodels technique (see [5] or [6]) to obtain a couple of positive partial answers to (a).

Let κ be an infinite cardinal. Recall that a subset A of a space X is called a *G_κ-set* if there is a family $\mathcal{V}_A$ of open subsets of X with $|\mathcal{V}_A| \leq \kappa$ such that $A = \bigcap \mathcal{V}_A$.

THEOREM 2.9. *Let $X \in T_2$ be a κ^+-Lindelöf space with $H\psi(X) \leq \kappa$ such that every closed subset $A \in [X]^{\leq 2^\kappa}$ is a G_{2^κ}-set in X. Then $|X| \leq 2^\kappa$.*

Proof. For each $x \in X$ fix a collection $\mathcal{B}_x$ of open neighborhoods of x with $|\mathcal{B}_x| \leq \kappa$ such that if $x \neq y$, then there are $V_x \in \mathcal{B}_x$ and $V_y \in \mathcal{B}_y$ which satisfy $V_x \cap V_y = \emptyset$. Consider a chain of elementary submodels $\{\mathcal{M}_\alpha : \alpha \in \kappa^+\}$ such that $\{X, \tau, 2^\kappa\} \cup 2^\kappa \subseteq \mathcal{M}_0$, $\mathcal{M}_\alpha \in \mathcal{M}_{\alpha+1}$, $|\mathcal{M}_\alpha| \leq 2^\kappa$ and $\mathcal{M}_\alpha$ is closed under κ-sequences, for every $\alpha < \kappa$. Let $\mathcal{M} = \bigcup\{\mathcal{M}_\alpha : \alpha \in \kappa^+\}$.

Claim: $X \cap \mathcal{M} = (X \cap \mathcal{M})^* = \{x \in X : V \cap (X \cap \mathcal{M}) \neq \emptyset \text{ for all } V \in \mathcal{B}_x\}$. Indeed, it is clear that $X \cap \mathcal{M} \subseteq (X \cap \mathcal{M})^*$. Now, if $x \in (X \cap \mathcal{M})^*$, then there exists $A \in [X \cap \mathcal{M}]^{\leq\kappa}$ such that $x \in A^*$. Since $|A| \leq \kappa$ and κ^+ is regular, there exists $\alpha \in \kappa^+$ such that $A \subseteq \mathcal{M}_\alpha$. Moreover $A \in \mathcal{M}_\alpha$, hence $A^* \in \mathcal{M}_\alpha$. Now, from Theorem 1.4, $|A^*| \leq 2^\kappa$, so $A^* \subseteq \mathcal{M}_\alpha$. Thus $A^* \subseteq \mathcal{M}_\alpha \subseteq X \cap \mathcal{M}_\alpha \subseteq X \cap \mathcal{M}$; hence $x \in X \cap \mathcal{M}$. Therefore $(X \cap \mathcal{M})^* \subseteq X \cap \mathcal{M}$.

From the claim we know that $X \cap \mathcal{M}$ is closed in X. Therefore $X \cap \mathcal{M}$ is κ^+-Lindelöf. Moreover $X \cap \mathcal{M} = \bigcup\{(X \cap \mathcal{M}_\alpha)^* : \alpha \in \kappa^+\}$.

For every $\alpha \in \kappa$, we fix a family $\mathcal{V}_{(X\cap\mathcal{M}_\alpha)^*}$ of open subsets of X with $|\mathcal{V}_{(X\cap\mathcal{M}_\alpha)^*}| \leq 2^\kappa$ such that $\bigcap \mathcal{V}_{(X\cap\mathcal{M}_\alpha)^*} = (X \cap \mathcal{M}_\alpha)^*$.

The proof will be complete once we show that $X = X\cap\mathcal{M}$. Assume that there is $p \in X\setminus(X\cap\mathcal{M})$. Then for each $\alpha \in \kappa$, we can choose $U_\alpha \in \mathcal{V}_{(X\cap\mathcal{M}_\alpha)^*}$ such that $p \notin U_\alpha$. Then $\mathcal{U} = \{U_\alpha : \alpha \in \kappa^+\}$ is an open covering of $X \cap \mathcal{M}$, so there exists $\mathcal{V} \in [\mathcal{U}]^{\leq\omega}$ such that $X \cap \mathcal{M} \subseteq \bigcup \mathcal{V}$.

Since $\mathcal{V} \subseteq \mathcal{M}$ and $|\mathcal{V}| \leq \kappa$, we have $\mathcal{V} \in \mathcal{M}$, so $\bigcup \mathcal{V} \in \mathcal{M}$. Thus $\bigcup \mathcal{V}$ covers X, which is a contradiction because $p \notin \bigcup \mathcal{V}$. Therefore $X = X\cap\mathcal{M}$. Thus $|X| \leq 2^\kappa$. ∎

Corollary 2.10 ([2]). *Let X be a first countable ω_1-Lindelöf Hausdorff space such that every closed subset $A \in [X]^{\leq 2^\omega}$ is a G_{2^ω}-set in X. Then $|X| \leq 2^\omega$.*

Definition 2.11 ([1]). Let X be a topological space. A subspace $Y \subseteq X$ is *κ-Lindelöf in X* if for each open covering $\mathcal{U}$ of X with $|\mathcal{U}| \leq \kappa$ there is $\mathcal{V} \in [\mathcal{U}]^{\leq\omega}$ such that $Y \subseteq \bigcup \mathcal{V}$.

Theorem 2.12. *Let X be a Hausdorff space with $H\psi(X) \leq \kappa$, and let Y be a dense subspace of X which is 2^κ-Lindelöf in X. Then $|X| \leq 2^\kappa$.*

Proof. Assume that $\mathcal{M}$ is an elementary submodel of some sufficiently large fragment of the universe with $|\mathcal{M}| \leq 2^\kappa$ such that $\mathcal{M}$ is closed under κ-sequences and $\{X, Y, \tau, 2^\kappa\} \cup 2^\kappa \subseteq \mathcal{M}$.

For each $x \in X$ fix a collection $\mathcal{B}_x$ of open neighborhoods of x with $|\mathcal{B}_x| \leq \kappa$ such that if $x \neq y$, then there are $V_x \in \mathcal{B}_x$ and $V_y \in \mathcal{B}_y$ with $V_x \cap V_y = \emptyset$.

Note that $Y \cap \mathcal{M} = (Y \cap \mathcal{M})^* = \{x \in X : V \cap (Y \cap \mathcal{M}) \neq \emptyset$ for all $V \in \mathcal{B}_x\}$; hence $Y \cap \mathcal{M}$ is closed in X, and thus actually in Y. Moreover $Y \cap \mathcal{M}$ is 2^κ-Lindelöf in X.

Claim: $Y \subseteq Y\cap\mathcal{M}$. Indeed, assume that there is $p \in Y \setminus (Y\cap\mathcal{M})$. Then for every $y \in Y\cap\mathcal{M}$, there are $U_y \in \mathcal{B}_y$ and $V_p \in \mathcal{B}_p$ such that $U_y \cap V_p = \emptyset$. Clearly $\mathcal{U} = \{U_y \in \mathcal{B}_y : y \in Y\cap\mathcal{M}\} \cup \{X \setminus (Y \cap \mathcal{M})$ is an open cover of X with cardinality $\leq 2^\kappa$. Since $Y\cap\mathcal{M}$ is 2^κ-Lindelöf in X, there is $\mathcal{V} \in [\mathcal{U}]^{\leq\omega}$ such that $Y \cap \mathcal{M} \subseteq \bigcup\{V : V \in \mathcal{V}\}$. Since $\mathcal{V} \subseteq \mathcal{M}$ (note that $\mathcal{B}_y \subseteq \mathcal{M}$ for every $y \in Y \cap \mathcal{M}$) and $|\mathcal{V}| \leq \kappa$, we have $\mathcal{V} \in \mathcal{M}$, so $\bigcup \mathcal{V} \in \mathcal{M}$. Thus $\mathcal{V}$ covers Y, which is a contradiction because $p \notin \bigcup \mathcal{V}$. Hence $Y \subseteq Y \cap \mathcal{M}$.

Now, from our claim, we see that $|Y| \leq 2^\kappa$ and, in virtue of the fact that $X = Y^*$, Theorem 1.4 implies that $|X| \leq 2^\kappa$. ∎

Corollary 2.13 ([1]). *Let X be a Hausdorff space with $\chi(X) \leq \kappa$, and let Y be a subspace of X which is dense in X and 2^κ-Lindelöf in X. Then $|X| \leq 2^\kappa$.*

The proof of the next theorem is similar to the proof of Lemma 3.1 in [4].

THEOREM 2.14. *Let $X \in T_2$ be a κ^+-Lindelöf space with $H\psi(X) \leq \kappa$. Then $L(X) \leq 2^\kappa$.*

Proof. Let $\mathcal{U}$ be an arbitrary open cover of X. For each $\alpha < \kappa^+$ define a subset A_α of X with $|A_\alpha| \leq \kappa$ as follows:

(1) $A_0 = \emptyset$.
(2) Since $|\bigcup\{A_\beta : \beta < \alpha\}| \leq \kappa$, we have $|(\bigcup\{A_\beta : \beta < \alpha\})^*| \leq 2^\kappa$ (Theorem 1.4), where $Z^* = \{x \in X : V \cap Z \neq \emptyset$ for all $V \in \mathcal{B}_x\}$; hence there exists $\mathcal{U}_\alpha \in [\mathcal{U}]^{\leq 2^\kappa}$ such that $(\bigcup\{A_\beta : \beta < \alpha\})^* \subseteq \bigcup \mathcal{U}_\alpha$. Choose $x_\alpha \in X \setminus \bigcup\{\bigcup \mathcal{U}_\beta : \beta \leq \alpha\}$. If no such point exists then stop the inductive definition. Otherwise, put $A_\alpha = (\bigcup\{A_\beta : \beta < \alpha\}) \cup \{x_\alpha\}$.

To finish the proof, note that for some step $\alpha < \kappa^+$ our process must stop (to see this, assume the contrary and use the fact that if $A = \bigcup\{A_\alpha^* : \alpha < \kappa^+\}$ then $A = A^*$ to obtain a contradiction). Hence, there exists $\alpha < \kappa^+$ such that $X \subseteq \bigcup\{\bigcup \mathcal{U}_\beta : \beta \leq \alpha\}$. ∎

COROLLARY 2.15. *If $X \in T_2$ is a κ^+-Lindelöf space with $H\psi(X) \leq \kappa$, then $|X| \leq 2^{2^\kappa}$.*

COROLLARY 2.16. *Assume* GCH. *If X is a κ^+-Lindelöf Hausdorff space with $H\psi(X) \leq \kappa$, then X is Lindelöf and $|X| \leq 2^\kappa$.*

COROLLARY 2.17. *Assume* CH. *If X is a linearly Lindelöf Hausdorff space with $H\psi(X) \leq \omega$, then X is Lindelöf and $|X| \leq 2^\omega$.*

COROLLARY 2.18 ([4]). *If X is a κ^+-Lindelöf Hausdorff space with $\chi(X) \leq \kappa$, then $L(X) \leq 2^\kappa$ and $|X| \leq 2^{2^\kappa}$.*

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