COLLOQUIUM MATHEMATICUM

VOL. 96 2003 NO. 2

REPRESENTATION-TAME INCIDENCE ALGEBRAS OF FINITE POSETS

BY

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Abstract. Continuing the paper [Le], we give criteria for the incidence algebra of an arbitrary finite partially ordered set to be of tame representation type. This completes our result in [Le], concerning completely separating incidence algebras of posets.

1. Introduction. Throughout, we assume that K is an algebraically closed field. We continue the study of representation-tame incidence K-algebras of finite posets, that is, partially ordered sets, started in [Le]. We use the terminology and notation introduced in [Le]. In particular, given a finite-dimensional K-algebra A, we denote by $\operatorname{mod} A$ the category of all finite-dimensional right A-modules. The algebra A is said to be of *tame representation type* (or *representation-tame*) if, for each dimension d, the isomorphism classes of indecomposable modules in $\operatorname{mod} A$ of dimension d form at most finitely many one-parameter families. The reader is referred to [Dr] and [S; Sections 14.2–4] for precise definitions of representation-tame and representation-wild algebras.

Throughout, we denote by $Q = (Q_0, Q_1)$ a finite connected quiver with Q_0 being the set of vertices and Q_1 the set of arrows. We assume that Q has no oriented cycles and no arrows having the same starting and ending vertex with another path. In particular Q has no multiple arrows. We view the quiver Q as a poset with respect to the partial order relation $\preceq$ on Q_0 defined by the formula:

$$x \preceq y \text{ if there is an oriented path from } x \text{ to } y \text{ in } Q.$$

Following [Le], the *incidence K-algebra* of the poset Q is the bound quiver algebra

$$A(Q) = KQ/I, \tag{1.1}$$

where I is the ideal of the path K-algebra KQ of Q generated by all com-

2000 *Mathematics Subject Classification*: 16G20, 16G60, 16G70.

Supported by the Polish Scientific Grant KBN No. 5 PO3A 008 21 and the Foundation for Polish Science.

mutativity relations in Q, that is, by all elements $\omega_1 - \omega_2$ in KQ, where ω_1 and ω_2 are paths in Q having the same starting vertex and the same ending vertex.

Note that $A(Q)$ is the incidence K-algebra KS of a finite poset $(S, \preceq)$ whose Hasse quiver is Q. Conversely, the incidence K-algebra KS of any finite poset $(S, \preceq)$ is of the form $A(Q)$ (see [S; Section 14.1]).

In [Le], we give a characterization of representation-tame completely separating incidence K-algebras $A(Q)$, by means of the weak nonnegativity of the associated Tits quadratic form $q_{A(Q)}$ (see (1.2) below). In the present paper we study representation-tame algebras $A(Q)$ which are not completely separating.

We recall from [Bo1] that the *Tits quadratic form* of an arbitrary finite-dimensional bound quiver K-algebra $A = KQ/I$, where Q is a finite quiver and I is an admissible ideal of KQ, is the integral quadratic form $q_A : \mathbb{Z}^n \to \mathbb{Z}$ defined by the formula

$$(1.2) \qquad q_A(x) = \sum_{i \in Q_0} x_i^2 - \sum_{\alpha \in Q_1} x_{s(\alpha)} x_{t(\alpha)} + \sum_{i,j \in Q_0} r_{ij} x_i x_j,$$

where $n = |Q_0|$, $s(\alpha)$ and $t(\alpha)$ are the source and target of the arrow $\alpha \in Q_1$, r_{ij} is the cardinality of $R \cap KQ(i,j)$, R is a minimal set of relations which generate the ideal I, and $KQ(i,j)$ is the vector space spanned by the paths from i to j. We also recall that if $A = KQ/I$ is of tame representation type, then q_A is *weakly non-negative*, that is, $q_A(x) \geq 0$ for any $x \in \mathbb{Z}$ with non-negative coordinates (see [P]).

It is well known that every incidence algebra $A(Q)$ admits a universal Galois covering

$$(1.3) \qquad \widetilde{A}(Q) \to \widetilde{A}(Q)/G = A(Q),$$

where $\widetilde{A}(Q)$ is a (strongly) simply connected locally bounded K-category and $G = \pi_1(Q)$ is the fundamental group of the bound quiver (Q, I). We recall from [MP] that the group G is trivial or finitely generated free.

Our main result is the following theorem proved in Section 5.

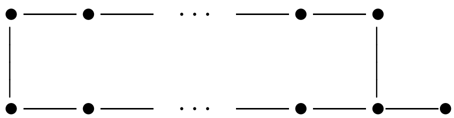
THEOREM 1.4. *Suppose that K is an algebraically closed field. Let $A(Q)$ be the incidence K-algebra of a finite partially ordered set with the Hasse quiver Q and let $\widetilde{A}(Q)$ be the simply connected locally bounded K-category in the universal Galois covering of $A(Q)$. The following conditions are equivalent.*

(i) *The algebra $A(Q)$ is of tame representation type.*

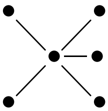
(ii) *The Tits form q_B of any finite convex subcategory B of $\widetilde{A}(Q)$ is weakly non-negative.*

(iii) *The category $\widetilde{A}(Q)$ does not contain, as a convex subcategory, a concealed algebra of any of the following six types*: $\widetilde{\widetilde{\mathbb{A}}}_{m,n}$ *with* $m \geq 1$, T_5, $\widetilde{\widetilde{\mathbb{D}}}_n$, *with* $n \geq 4$, $\widetilde{\widetilde{\mathbb{E}}}_6$, $\widetilde{\widetilde{\mathbb{E}}}_7$ *or* $\widetilde{\widetilde{\mathbb{E}}}_8$, *where*

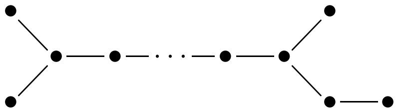
$\widetilde{\widetilde{\mathbb{A}}}_{m,n}$:



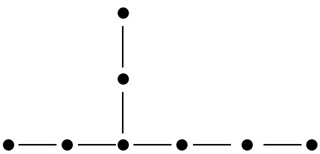
T_5 :



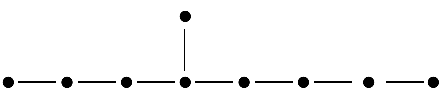
$\widetilde{\widetilde{\mathbb{D}}}_n$:



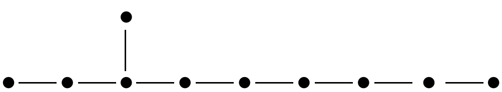
$\widetilde{\widetilde{\mathbb{E}}}_6$:



$\widetilde{\widetilde{\mathbb{E}}}_7$:

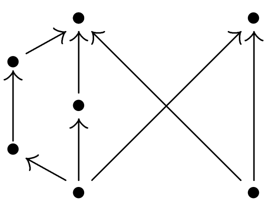


$\widetilde{\widetilde{\mathbb{E}}}_8$:



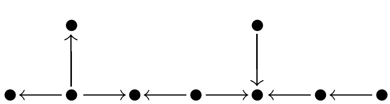
Here $\widetilde{\widetilde{\mathbb{D}}}_n$ *has* $n+2$ *vertices with* $4 \leq n \leq 8$, *and* $\bullet\!-\!\bullet$ *means either* $\bullet \to \bullet$ *or* $\bullet \leftarrow \bullet$. *Moreover*, $\widetilde{\widetilde{\mathbb{A}}}_{m,n}$ *is the poset of a minimal wild hereditary algebra* (see [U]) *with* $m+n$ *vertices and* $m+n$ *arrows in its cycle*, *where* m *of the arrows have clockwise orientation and* n *have counterclockwise orientation.*

For illustration, consider the following poset Q:



It is easy to check that the Tits form $q_{A(Q)}$ is non-negative and the poset of the universal Galois covering $\widetilde{A}(Q)$ contains a convex subcategory $A(Q')$, where

Q' :

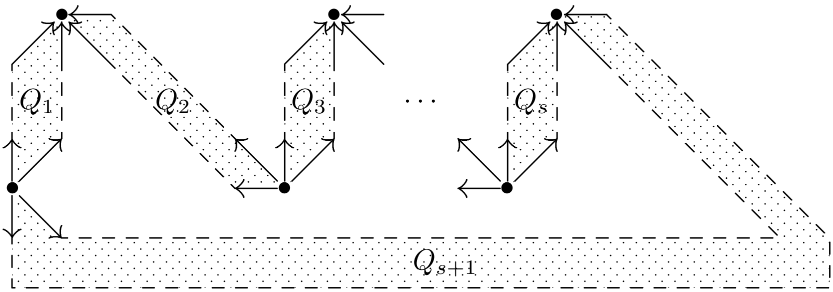


is of type $\widetilde{\widetilde{\widetilde{\mathbb{D}}}}_7$. It follows that $\widetilde{A}(Q)$ is of wild representation type and, by [DS], so is $A(Q)$.

The example shows that the study of the universal Galois coverings $\widetilde{A}(Q)$ of the incidence algebras $A(Q)$ is of importance.

We show below that if $A(Q)$ is of tame representation type then $\widetilde{A}(Q)$ is not trivial only in the case when Q is a *crown*, that is, has the form (see [D])

(1.5)



where $s \geq 3$, each of the posets $Q_1, \ldots, Q_{s+1}$ has exactly one minimal and one maximal point, and for $i \neq j$ any points $x \in Q_i$, $y \in Q_j$ are incomparable, except the case $\{x, y\} \cap Q_i \cap Q_j \neq \emptyset$.

We recall that the completely separating incidence algebras $A(Q)$ of tame representation type are described in [Le]. Thus it remains to describe the non-completely separating ones. This is done in Corollary 5.2.

The results of this paper were presented at the International Conference ICRA-X in Toronto, August 2002.

2. Preliminaries. Let Q be a finite poset as in Section 1. We denote by $\omega(Q)$ the *width* of Q, that is, the greatest number of pairwise incomparable points in Q. Moreover, we denote by $\operatorname{rep}_K(Q)$ the category of K-linear representations of Q (see [Le]). Throughout this paper, we identify $\operatorname{rep}_K(Q)$ with the category $\operatorname{mod} A(Q)$ of finite-dimensional modules over the incidence algebra $A(Q)$ (see [GR] and [S; Chapter 14] for details).

We recall from [Le] and [Lo] that with a given arrow $\alpha : a \to b$ in Q_1 we associate a new poset Q^α, called the *contraction* of Q at α. It is obtained from Q by contracting the arrow α to the vertex $a = b$. More precisely,

$$Q^\alpha = Q \setminus \{a, b\} \cup \{\overline{\{a, b\}}\} \tag{2.1}$$

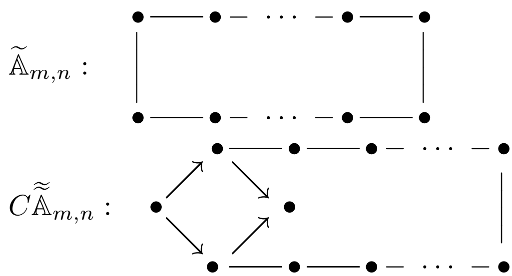
with the following partial order: $x \preceq y$ (in Q^α) if $\{x, y\} \subseteq Q \setminus \{a, b\}$ and $x \preceq y$ in Q; $x \preceq \overline{\{a, b\}}$ if $x \preceq a$ or $x \preceq b$ in Q; and $\overline{\{a, b\}} \preceq y$ if $a \preceq y$ or $b \preceq y$ in Q (see [Le] for an example).

In the proof of our main result we frequently use the critical and hypercritical algebras. We recall that a finite-dimensional K-algebra A is said to be *critical* (resp. *hypercritical*) if A is a concealed algebra of type $\widetilde{\mathbb{D}}_n$, $\widetilde{\mathbb{E}}_6$, $\widetilde{\mathbb{E}}_7$ or $\widetilde{\mathbb{E}}_8$ (resp. T_5, $\widetilde{\widetilde{\mathbb{D}}}_n$, $\widetilde{\widetilde{\mathbb{E}}}_6$, $\widetilde{\widetilde{\mathbb{E}}}_7$ or $\widetilde{\widetilde{\mathbb{E}}}_8$; see [ASS] and [R2]).

The critical (resp. hypercritical) algebras have been completely classified in [Bo2], [HV] (resp. in [U], see also [L] and [Wi]). In particular, it was shown in [HV] that there are only four families of critical algebras of type $\widetilde{\mathbb{D}}_n$, given by the quivers (1)–(4) in [Le; p. 247].

We also freqently use the quivers

(2.2)



(without commutativity relation), where •—• means either • → • or • ← • and $\widetilde{\mathbb{A}}_{m,n}$ has $m+n$ vertices and $m+n$ arrows: m of the arrows have clockwise orientation and n have counterclockwise orientation.

We note that if $\widetilde{\mathbb{A}}_{m,n}$ has at least two minimal vertices (or equivalently, at least two maximal vertices), then it is a crown (1.5). The incidence algebra $A(C\widetilde{\widetilde{\mathbb{A}}}_{m,n})$ is a concealed algebra of $A(\widetilde{\widetilde{\mathbb{A}}}_{m,n})$.

Let $A = KQ/I$ be an arbitrary bound quiver algebra. Following [D], we call a module V in $\operatorname{mod} A$ *thin* if $\dim_K V_x \le 1$ for any vertex x.

The following easy fact is very useful.

Assume that B is the incidence algebra of a poset and $A = B[V]$ (resp. $A = [V]B$) is the one-point extension (resp. coextension) of the algebra B by a module V. If A is the incidence algebra of some poset, then the module V is thin.

We recall that among the four families of concealed algebras of type $\widetilde{\mathbb{D}}_n$ there are three families of incidence algebras of posets. For concealed incidence algebras from our three families, the simple regular thin modules and the indecomposable regular thin modules of regular length 2 are known [NS].

We recall from [Sk] that a bound quiver algebra KQ/I is *strongly simply connected* if for each convex full subquiver Q' of Q the associated algebra KQ'/I' is simply connected.

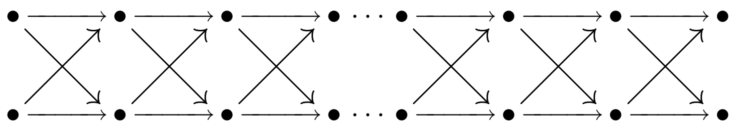
All strongly simply connected representation-tame algebras which are minimal of non-polynomial growth are listed in [NS]. In particular, pg-critical incidence algebras of finite posets are listed in [Le; Lemma 2.4] (see also [NS; Theorem 3.2]).

Let $A(Q)$ be the incidence algebra of a finite poset Q. For every $x \in Q$, we denote by P_x the indecomposable projective $A(Q)$-module associated with x. The module P_x is said to have a *separated radical* if the supports of any two non-isomorphic indecomposable direct summands of $\operatorname{rad} P_x$ are contained in different connected components of the subposet Q_x of Q obtained by deleting all those points y such that there is a path with source y and target x.

If all the indecomposable projective $A(Q)$-modules have separated radical, then $A(Q)$ is said to satisfy the *separating condition* [BLS] (see also [ASS]). The incidence algebra $A(Q)$ is called *completely separating* if, for any convex subposet Q' of Q, the associated incidence algebra KQ'/I' also satisfies the separating condition [D]. For example, the pg-critical algebras (2.4c) and (2.4d) of [Le; Lemma 2.4] are not completely separating.

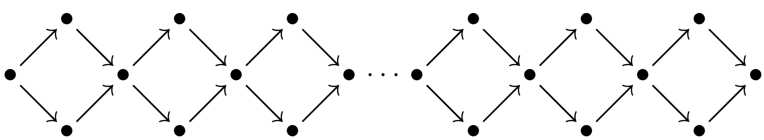
For any natural number $n \geq 1$, we consider the poset

(2.3) G_n :



called a *garland* [S], having $2(n+1)$ points, and the poset

(2.4) F_n :



having n commutative squares.

3. Families of non-completely separating posets. In the class of the incidence algebras $A(Q)$ of posets Q we distinguish the following four subclasses.

($\mathcal{A}$) The crowns (see (1.5)).

($\mathcal{B}$) The incidence algebras $A(Q)$ where Q has a convex proper subposet which is a crown different from a poset $\widetilde{\mathbb{A}}_{m,n}$ (see (2.2)). That is, $\omega(Q_i) \geq 2$ for some $i \in \{1, \ldots, s+1\}$.

($\mathcal{C}$) The incidence algebras $A(Q)$ for which any crown contained in Q as a convex subposet is equal to $\widetilde{\mathbb{A}}_{m,n}$ (for some m, n) and Q contains a proper subposet which is a crown different from $\widetilde{\mathbb{A}}_{2,2}$.

($\mathcal{D}$) The incidence algebras $A(Q)$ for which any convex subposet of Q which is a crown is of type $\widetilde{\mathbb{A}}_{2,2}$ and Q contains a proper subposet isomorphic to a crown.

The following simple lemma is very useful.

LEMMA 3.0. (a) *If a K-algebra $A(Q)$ belongs to any of the classes* ($\mathcal{A}$)–($\mathcal{D}$), *then $A(Q)$ is not completely separating.*

(b) *The classes* $(\mathcal{A})$–$(\mathcal{D})$ *are pairwise disjoint.*

(c) *Any incidence K-algebra $A(Q)$ of a non-completely separating poset Q belongs to one of the classes* $(\mathcal{A})$–$(\mathcal{D})$.

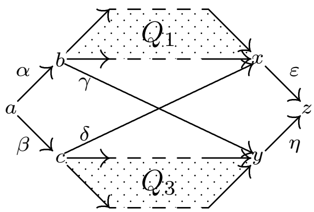
Proof. The crowns can be split into three classes: (i) the crowns different from $\widetilde{\mathbb{A}}_{m,n}$, (ii) the posets of type $\widetilde{\mathbb{A}}_{m,n}$ such that $(m,n) \neq (2,2)$, and (iii) the poset $\widetilde{\mathbb{A}}_{2,2}$. On the other hand, we know from [D] that any non-completely separating incidence algebra $A(Q)$ contains a convex subposet isomorphic to a crown. Hence the lemma follows. ■

The proof of the following easy proposition is left to the reader.

PROPOSITION 3.1. *Suppose that Q is a crown such that the K-category $\widetilde{A}(Q)$ in the universal Galois covering of $A(Q)$ does not contain*, *as a convex subcategory*, *a concealed algebra of any of the following six types*: $\widetilde{\widetilde{\mathbb{A}}}_{m,n}$, *with* $m \geq 1$, T_5, $\widetilde{\widetilde{\mathbb{D}}}_n$ *with* $n \geq 4$, $\widetilde{\widetilde{\mathbb{E}}}_6$, $\widetilde{\widetilde{\mathbb{E}}}_7$ *or* $\widetilde{\widetilde{\mathbb{E}}}_8$. *Then each of the posets* $Q_1, \ldots, Q_{s+1}$ (*in the notation of* (1.5)) *is a subposet of some garland.*

We consider the following family of posets:

(3.2)



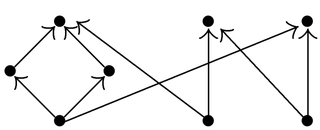
where Q_i (for $i = 1, 3$) are subposets of the garland G_n.

PROPOSITION 3.3. *If $A(Q) \in (\mathcal{B})$ and contains no convex subcategory which is a concealed algebra of a minimal wild hereditary algebra* (see [U]), *then $A(Q)$ is simply connected and Q is a subposet of* (3.2).

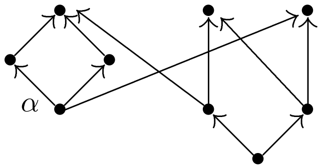
Proof. Let Q' denote a crown which is a proper subposet of Q different from $\widetilde{\mathbb{A}}_{m,n}$ (for any m, n). If $A(Q)$ contains as a convex subalgebra an algebra $A(Q'')$ which is an extension (or coextension) of $A(Q')$ by a module which is not a thin sincere indecomposable module, then this extension (or coextension) is of wild type and contains a concealed algebra of a minimal wild hereditary algebra. Moreover, this concealed algebra is a full subcategory (in general not convex) and it is a contraction of Q'' (see (2.1)).

We illustrate this fact by the following example:

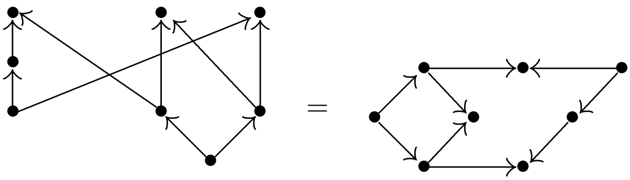
(3.4) Q' :



The poset Q' contains its contraction equal to $\widetilde{\mathbb{A}}_{4,3}$. The extension Q'' of Q' of the form



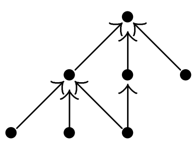
contains its contraction at the arrow α:



which is of type $C\widetilde{\widetilde{\mathbb{A}}}_{4,3}$ (see (2.2)).

Observe that the extensions (resp. coextensions) by a sincere indecomposable module of a crown are simply connected algebras with one minimal (resp. one maximal) vertex in the poset.

For our crown Q', we have $s = 3$ (in the notation of (1.5)). Let us look at the smallest case with $s = 5$, that is, Q' of the form (3.4). If we take the coextension by a thin sincere indecomposable module then the poset Q has a convex subposet of the form



which is of type $\widetilde{\widetilde{\mathbb{D}}}_5$.

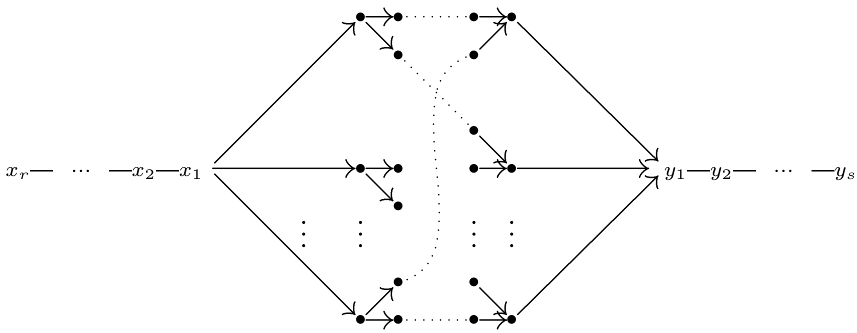
One can show that if either

(i) $\omega(Q_1) = \omega(Q_2) = 2$, or
(ii) $\omega(Q_i) \geq 3$, for some $i = 1, 2, 3, 4$, or
(iii) Q_i contains a subposet isomorphic to $\bullet\!\longrightarrow\bullet\quad\bullet$,

then $A(Q)$ has a hypercritical convex subalgebra. Observe that if $A(Q)$ contains a coextension (or extension) of a crown then the next coextension (resp. extension) has a hypercritical convex subalgebra. Hence Q is either (3.2) or $(3.2)\setminus\{a\}$ or $(3.2)\setminus\{z\}$. ■

We next consider the posets

(3.5)



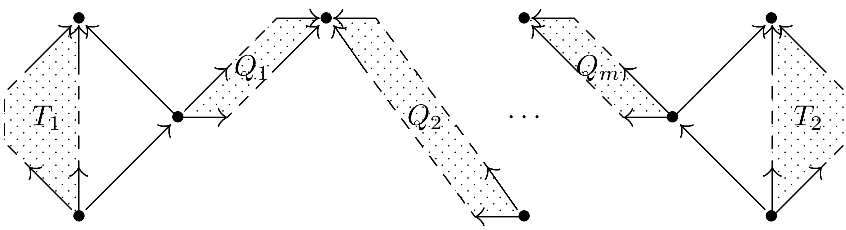
which are coil enlargements of some quiver $\widetilde{\mathbb{A}}_{m,n}$, using thin sincere indecomposable regular modules in the sense of [AST].

Applying the arguments of Ringel [R1], one can prove the following useful result.

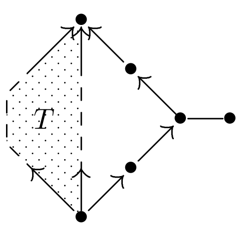
PROPOSITION 3.6. *If $A(Q) \in (\mathcal{C})$ and $A(Q)$ does not contain a convex subcategory which is a concealed algebra of a minimal wild hereditary algebra, then $A(Q)$ is simply connected and Q is a subposet of* (3.5).

Now, we consider the following posets:

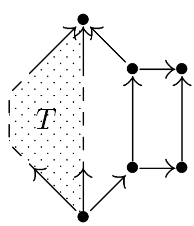
(3.7a)



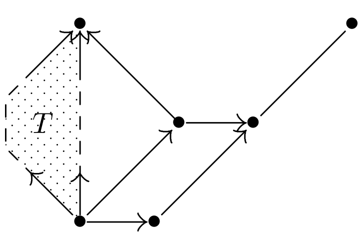
(3.7b)



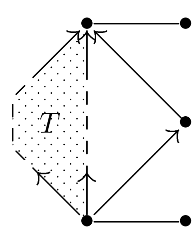
(3.7c)



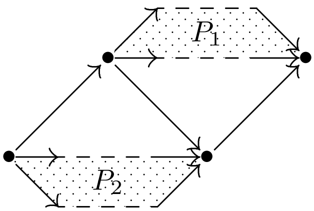
(3.7d)



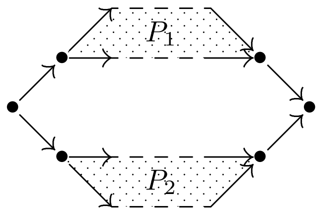
(3.7e)



(3.7f)



(3.7g)



where T_1, T_2, P_1, P_2, $Q_1, \ldots, Q_m$ and T are subposets of (2.4), with $\omega(T) = \omega(P_i) = 2$; possibly $m = 1$, and Q_1 or Q_m may be points.

Note that the incidence algebra $A(Q)$ of a poset Q of one of the forms (3.7a)–(3.7g) is completely separating if and only if each of T_1, T_2, T, $Q_1, \ldots, Q_m$, P_1, P_2 is a completely separating subposet of (2.4).

By repeating the method of the proof of Proposition 3.4 in [Le], one can prove the following proposition.

PROPOSITION 3.8. *If $A(Q) \in (\mathcal{D})$ and $A(Q)$ does not contain a convex hypercritical subalgebra, then $A(Q)$ is simply connected and either Q or Q^{op} is a subposet of one of the posets* (3.7a)–(3.7g)*.*

4. The tameness. One of the main aims of this section is to prove the following result.

PROPOSITION 4.1. *If Q has one of the forms* (3.2) *or* (3.7a)–(3.7g)*, then $A(Q)$ is of tame representation type.*

Proof. Suppose that Q is of the form (3.2). In view of [Le; 2.10(a)], it is enough to prove that $A(Q)$ is representation-tame if Q is of the shape (3.2) with Q_1, Q_3 equal to some F_n, F_m (see (2.4)).

We use the degeneration arguments of [G] and [CB]. For each $\lambda \in K$, we consider the bound quiver algebra $A_\lambda = KQ/I_\lambda$, where I_λ is the ideal generated by the commutativity relations in Q_1, Q_3 and the following relations:

$$\gamma\alpha - \lambda q_3\beta, \quad \delta\beta - \lambda q_1\alpha, \quad \varepsilon\delta - \lambda\eta q_3, \quad \eta\gamma - \lambda\varepsilon q_1,$$

where q_1, q_3 denote the paths from b to x and from c to y, respectively.

Note that $A(Q) = A_1$. If we map the class of q_1 in A_1 to the class of $\lambda^{-1}q_1$ in A_λ and the class of q_3 to the class of $\lambda^{-1}q_3$ and identify the other classes of generators (vertices and arrows, respectively) of A_1 and A_λ, we obtain a K-algebra isomorphism $A_1 \cong A_\lambda$ for $\lambda \neq 0$. We know that if an algebra A_0 is a degeneration of the algebra A (in the sense of [BC] or [G]) and A_0 is of tame representation type, then so is A. Hence, it is enough to show that A_0 is representation-tame.

Similarly to [Le; Lemma 4.5], we can degenerate the algebras Q_1, Q_3 (see [G]), and then degenerate the whole algebra A_0 to a biserial algebra. Hence, by [CB] and [WW], the algebras A_0 and $A(Q)$ are of tame representation type. One can also prove the tameness of $A(Q)$, with Q from the family (3.2), by showing that $A(Q)$ is a kit algebra (see [Br]).

By the same method we have proved in [Le] the tameness of $A(Q)$ for Q of one of the forms (3.7a)–(3.7g). ■

The same type of argument yields the following result.

PROPOSITION 4.2. *If Q is a crown such that each of the posets $Q_1, \ldots$ $\ldots, Q_{s+1}$ (in the notation (1.5)) is a subposet of a garland, then $A(Q)$ is of tame representation type.*

PROPOSITION 4.3. *If Q of the form (3.5) is such that $A(Q)$ has no convex subalgebra which is a concealed algebra of type T_5, $\widetilde{\widetilde{\mathbb{D}}}_n$, $\widetilde{\widetilde{\mathbb{E}}}_6$, $\widetilde{\widetilde{\mathbb{E}}}_7$ or $\widetilde{\widetilde{\mathbb{E}}}_8$, then $A(Q)$ is of tame representation type.*

Proof. By [AST], $A(Q)$ is a coil enlargement of $A(\widetilde{\mathbb{A}}_{m,n})$ (see (2.2)) using a thin sincere indecomposable regular module from a tube of rank 1. Since $A(Q)$ has no convex subalgebra which is a concealed algebra of any of the types T_5, $\widetilde{\widetilde{\mathbb{D}}}_n$, $\widetilde{\widetilde{\mathbb{E}}}_6$, $\widetilde{\widetilde{\mathbb{E}}}_7$ or $\widetilde{\widetilde{\mathbb{E}}}_8$, the types $(m, n, r+1)$ and $(m, n, s+1)$ are tame tubular (in the sense of Ringel [R2]) and, according to [AST; Corollary 4.2], the algebra $A(Q)$ is of tame representation type. ■

5. Proof of Theorem 1.4. Now we are able to prove our main result, Theorem 1.4.

(i)⇒(ii). If $A(Q)$ is of tame representation type then, according to [DS], so is $\widetilde{A}(Q)$. Hence (ii) follows from [P].

(ii)⇒(iii). This is a direct consequence of the fact that the Tits form of a concealed algebra of wild representation type is not weakly non-negative (see [K]).

(iii)⇒(i). Assume that $\widetilde{A}(Q)$ does not contain, as a convex subcategory, a concealed algebra of any of the six types listed in Theorem 1.4.

First, assume that $A(Q)$ is completely separating. Then, according to [Le; Lemma 2.7], the universal Galois covering (1.3) is an isomorphism and, in view of (iii) and [Le; Theorem], $A(Q)$ is of tame representation type.

Next, assume that $A(Q)$ is not completely separating. It follows from Lemma 3.0 that it belongs to one of the classes $(\mathcal{A})$–$(\mathcal{D})$.

Then, by Propositions 3.1, 3.3, 3.6 and 3.8, the poset Q or Q^{op} is of one of the forms (3.2), (3.5), (3.7a)–(3.7g), or Q is a crown such that each of the subposets $Q_1, \ldots, Q_{s+1}$ of Q (in the notation of (1.5)) is a subposet of some garland. If Q is a crown then $A(Q)$ is not simply connected and we construct the universal Galois covering $\widetilde{A}(Q)$ by unrolling the bound quiver (Q, I). If Q is not a crown then $A(Q)$ is simply connected and the covering map is an isomorphism.

Now, by applying Propositions 4.1–4.3, we conclude that $A(Q)$ is of tame representation type. ■

The arguments given above yield the following useful observation.

COROLLARY 5.1. *Suppose that $A(Q)$ is a non-completely separating incidence K-algebra such that each of the statements* (i)–(iii) *of Theorem* 1.4 *holds. Then either*

(i) *the quiver Q is a crown and to make the universal Galois covering $\widetilde{A}(Q)$ of $A(Q)$ means to unroll Q, or*

(ii) *the algebra $A(Q)$ is simply connected and the universal Galois covering map is a K-algebra isomorphism.*

From the proof of Theorem 1.4 presented above, together with our results of Sections 3 and 4, we easily conclude the following classification result.

COROLLARY 5.2. *Suppose that $A(Q)$ is a non-completely separating incidence K-algebra of a finite connected poset Q. If $A(Q)$ is of tame representation type, then Q or Q^{op} is a subposet of one of the posets* (3.2), (3.5), (3.7a)–(3.7g), *or Q is a crown of the form* (1.5) *such that each of the subposets $Q_1, \ldots, Q_{s+1}$ of Q is a garland.*

Acknowledgements. The author is grateful to A. Skowroński for helpful discussions and to D. Simson for friendly care during the preparation of the manuscript.

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Received 26 June 2002;
revised 24 April 2003 (4260)