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ACTIONS OF HOPF ALGEBRAS ON PRO-SEMISIMPLE NOETHERIAN ALGEBRAS AND THEIR INVARIANTS

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Abstract. Let H be a Hopf algebra over a field k such that every finite-dimensional (left) H-module is semisimple. We give a counterpart of the first fundamental theorem of the classical invariant theory for locally finite, finitely generated (commutative) H-module algebras, and for local, complete H-module algebras. Also, we prove that if H acts on the k-algebra $A = k[[X_1, \ldots, X_n]]$ in such a way that the unique maximal ideal in A is invariant, then the algebra of invariants A^H is a noetherian Cohen–Macaulay ring.

Introduction and the main results. Let k be a field and let H be a Hopf algebra over k. By analogy with the invariant theory of algebraic groups, the following is of importance.

Question. Let A be a commutative, finitely generated (resp., noetherian) H-module algebra. When is the algebra of invariants A^H also finitely generated (resp., noetherian)?

It is known that for rational actions of an algebraic group G the answer is "yes" whenever the group G is linearly reductive, i.e. whenever each finite-dimensional, rational G-module is semisimple. An important property used in the proof of this result is that every rational G-module is a sum of its finite-dimensional submodules. We say that the Hopf algebra H is *finitely semisimple* if every finite-dimensional (left) H-module is semisimple; this is an analogue of a linearly reductive algebraic group. An H-module algebra A is said to be *locally finite* if A, as an H-module, is a sum of its finite-dimensional submodules; this is a good analogue of the rational actions of algebraic groups on algebras. So, a precise counterpart of the above mentioned classical result is the following.

Theorem 1. *Suppose that the Hopf algebra H is finitely semisimple*, *and that A is a commutative*, *finitely generated* (*resp.*, *noetherian*), *locally finite H-module algebra. Then A^H is a finitely generated* (*resp.*, *noetherian*) *algebra.*

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This theorem is a consequence of our Corollary 2.9 and Theorem 3.2, and, as we mentioned in [2, p. 220], for cocommutative H it can be proved exactly in the same manner as for the rational actions of linearly reductive algebraic groups, using a Reynolds operator.

However, there are interesting noetherian H-module algebras which are not locally finite. For example, if A is a noetherian H-module algebra and I is an invariant ideal in A, then the induced action of H on the completion $\widehat{A} = \varprojlim A/I^n$ is not, in general, locally finite even if H is finitely semisimple and A is locally finite.

Example 2. Let $k = \mathbb{C}$ and let $L = \mathrm{sl}(2, k)$. Then the universal enveloping algebra $U(L)$ is a finitely semisimple Hopf algebra and we have the well known (locally finite) action of H on $A = k[X, Y]$ determined by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} aX + bY \\ cX + dY \end{pmatrix}.$$

Obviously, the induced action of H on the completion $\widehat{A} = k[[X, Y]]$ of A in the maximal (invariant) ideal (X, Y) is given by the same formula. In particular, the matrix $\left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right) \in L \subset H$ acts on $\widehat{A}$ via the derivation $D : \widehat{A} \to \widehat{A}$ such that $D(X) = X$ and $D(Y) = -Y$. It turns out that the induced action is not locally finite. In order to see this, it clearly suffices to show that there is an $f \in \widehat{A}$ such that the set $\{D^j(f) : j \geq 0\}$ is linearly independent over k. Put $f = \sum_{i=1}^{\infty} X^i$ and suppose that $\sum_{r=0}^{s} t_r D^r(f) = 0$ for some s and $t_0, \ldots, t_s \in k$. Then

$$0 = \sum_{r=0}^{s} t_r \Big(\sum_{i=1}^{\infty} i^r X^i \Big) = \sum_{i=1}^{\infty} \Big(\sum_{r=0}^{s} t_r i^r \Big) X^i,$$

whence in particular, $\sum_{r=0}^{s} t_r i^r = 0$, $i = 1, \ldots, s+1$. But the determinant of this system of linear equations (with respect to t_r's) is the Vandermonde determinant $V(1, \ldots, s+1)$, which is clearly different from 0. Therefore, $t_0 = t_1 = \ldots = t_s = 0$. This means that the set $\{D^j(f) : j \geq 0\}$ is linearly independent.

Another type of interesting noetherian H-module algebras which need not be locally finite arises in the following situation. Suppose that the Hopf algebra H is pointed [7, 9] (for instance, in characteristic 0 every cocommutative Hopf algebra is pointed) and that A is a commutative H-module algebra. Furthermore, let S be a multiplicative system in A such that $gs \in S$ for any group-like element $g \in H$ and any $s \in S$. Then, as shown in [11], there exists a unique action of H on the localization A_S such that the natural homomorphism of algebras $A \to A_S$ is a morphism of H-module algebras. So, if A is noetherian, then we obtain a noetherian H-module algebra A_S. Again it turns out that A_S, in general, is not locally finite.

EXAMPLE 3. Let H, A and f be as in Example 2, and let $S = \{(1-X)^n : n \geq 0\}$. Since 1 is the unique group-like element in H, we have the action of H on A_S. It is easy to see that if we look at A_S as a subalgebra of $k[[X,Y]]$, then the action of H on the localization A_S is the restriction of the action of H on $k[[X,Y]]$ considered in Example 2. Moreover, $f = \sum_{i\geq 1} X^i = (1-X)^{-1} \in A_S$. This implies that f does not belong to any finite-dimensional H-submodule of A_S, that is, the action of H on A_S is not locally finite.

The main goal of this paper is to find a counterpart of Theorem 1 for H-module algebras arising from locally finite H-module algebras by means of the operation of completion. The case of localizations will be investigated elsewhere.

Observe that the H-module algebra $\widehat{A}$ from Example 2 is, as an H-module, the inductive limit of the system $\{k[X,Y]/(X,Y)^n : n \geq 1\}$ of semisimple H-modules. The same is obviously true for the induced actions of a finitely semisimple Hopf algeba H on the completion $\widehat{A} = \varprojlim A/I^n$, where A is a locally finite H-module algebra and I is an invariant ideal in A. This suggests the following.

DEFINITION. A *pro-semisimple H-module algebra* is an H-module algebra A (not necessarily commutative) provided with a linear topology defined by a family $\{I_i\}$ of (two-sided) invariant ideals in A satisfying the conditions:

(1) A/I_i is a semisimple H-module for all i,

(2) the natural homomorphism of H-module algebras $p : A \to \varprojlim A/I_i$ is an isomorphism.

If the Hopf algebra H is finitely semisimple and A is a locally finite H-module algebra with an invariant ideal I, then the completion of A in the I-adic topology is a pro-semisimple H-module algebra. In particular, A itself with the discrete topology is a pro-semisimple H-module algebra. More generally, if $\{I_n : n \geq 0\}$ is any admissible sequence of invariant ideals in A (see Section 1), then the completion $\widehat{A} = \varprojlim A/I_n$ endowed with the natural action of H is also a pro-semisimple H-module algebra. The main objective of this paper is to prove the following.

THEOREM 4. *If A is a pro-semisimple, right noetherian H-module algebra, then so is the algebra of invariants A^H.*

THEOREM 5. *If I is an invariant ideal in a commutative, noetherian, pro-semisimple H-module algebra A such that all its powers I^n, $n \geq 1$, are closed (as subsets of A), then the induced topology in A^H given by the set of ideals $\{(I^n)^H : n \geq 0\}$ is equivalent to the I^H-adic topology in A^H.*

As corollaries from these theorems we get

Theorem 6. *Suppose that A is a noetherian H-module algebra which is semisimple as an H-module. Then, for each invariant ideal I in A and the induced action of H on the completion $\widehat{A} = \varprojlim A/I^n$, the algebra $(\widehat{A})^H$ is noetherian and the natural inclusion $i : A^H \to A$ induces an isomorphism of algebras $\widehat{A^H} \simeq (\widehat{A})^H$, where $\widehat{A^H}$ is the completion of A^H in the I^H-adic topology.*

Theorem 7. *Suppose that the Hopf algebra H is finitely semisimple and (A, m) is a local, complete, noetherian H-module algebra satisfying the conditions*:

(1) *the unique maximal ideal m in A is invariant*,
(2) *the quotient field A/m is a finite field extension of k.*

Then A^H is a local, complete, noetherian algebra with the unique maximal ideal m^H. In particular, if A is of the form $k[[X_1, \ldots, X_n]]/J$ for some $n \geq 1$ and some ideal J, then A^H is of the same form.

The last statement in the above theorem can be vieved as a counterpart of Theorem 1 for complete, local H-module algebras. Under the assumptions of Theorem 7, we also prove that the ring A^H is Cohen–Macaulay, whenever $A = k[[X_1, \ldots, X_n]]$.

In the proof of Theorems 1 and 3, an essential role is played by a Reynolds operator.

The content of the paper can be summarized as follows. Preliminaries are presented in Section 1. In Section 2 we prove the above mentioned Theorem 3 (in a more general setting and not only for commutative H-module algebras). In Section 3, given a commutative noetherian ring A, we present a description of all admissible sequences $\boldsymbol{I} = \{I_j : j \geq 0\}$ of ideals in A satisfying the second Artin–Rees property, i.e., $I_0 = A$, $I_j \supset I_{j+1}$, $I_i I_j \subset I_{i+j}$ for $i, j \geq 0$, and the graded algebra $G(\boldsymbol{I}) = \bigoplus_{j\geq 0} I_j$ is noetherian. From this description it follows that the topology in A defined by any such sequence $\boldsymbol{I}$ is equivalent to the I_1-adic topology. Hence one gets Theorem 5.

The definition of an admissible sequence satisfying the second Artin–Rees property and its application in the proof of Theorem 5 come from [3, Section 1]. Also, if H is the group algebra kG of some group G, then Theorems 4 and 6 were proved in [3] for H-module algebras that are semisimple as H-modules. Section 2 of the paper was patterned upon [10, Section 2].

1. Preliminaries. Throughout the paper k denotes a field which will serve as the ground field for all vector spaces and algebras under consideration. All tensor products (unless otherwise stated) are defined over k. By H we denote a fixed Hopf algebra with comultiplication $\Delta : H \to H \otimes H$ and counity $\varepsilon : H \to k$. As in [9], we write $\Delta(h) = \sum h_{(1)} \otimes h_{(2)}$ for $h \in H$. An H-module is meant to be a left H-module. Given an H-module V, V^H will stand for the submodule of invariants $V^H = \{v \in V : hv = \varepsilon(h)v,\ h \in H\}$. We say that V is *trivial* when $V = V^H$. If $f : V \to U$ is a homomorphism of H-modules, then $f^H : V^H \to U^H$ denotes the restriction of f to V^H. An H-module V is called *locally finite* if it is a sum of its finite-dimensional submodules.

Definition 1.1. The Hopf algebra H is called (left) *finitely semisimple* if each finite-dimensional H-module is semisimple.

Examples of finitely semisimple Hopf algebras are:

(a) Any H which is semisimple (e.g., $H = kG$, where G is a finite group with $(|G|, \operatorname{char} k) = 1$).

(b) $H = kG_p$, where p is a prime different from the characteristic of k and G_p is the group $\{x \in \mathbb{C} : \exists_n x^{p^n} = 1\}$ (an easy exercise).

(c) $H = U(L)$, the universal enveloping algebra of a finite-dimensional, semisimple Lie algebra L (k is supposed to have characteristic 0).

(d) $H = U_q(\mathrm{sl}(2, k))$, the quantum enveloping algebra of the Lie algebra $\mathrm{sl}(2, k)$, where $k = \mathbb{C}$ and q is not a root of unity (see [5, Theorem VII.2.2]).

Notice that if the Hopf algebra H is finitely semisimple and V is a locally finite H-module, then every submodule and every quotient module of V is semisimple.

Recall that a (left) *action* of H on a k-algebra A is an H-module structure $\gamma : H \otimes A \to A$ on A as a vector space (we write $\gamma(h \otimes a) = h.a$) such that $h.1_A = \varepsilon(h)1_A$ and $h.(xy) = \sum(h_{(1)}.x)(h_{(2)}.y)$ for all $h \in H$, $x, y \in A$, and $\sum h_{(1)} \otimes h_{(2)} = \Delta(h)$. In other words, A together with γ is an H-module algebra (see [7, 9]). The action γ (or the corresponding H-module algebra A) is called *locally finite* if A is locally finite as an H-module. If H is a finite-dimensional vector space, then clearly every action of H on a k-algebra A is locally finite.

Given H-module algebras A and B, a homomorphism of algebras $f : A \to B$ is called a *homomorphism of H-module algebras* if $f(h.a) = h.f(a)$ for all $h \in H$ and $a \in A$. An H-module algebra A is said to be *semisimple* when A is semisimple as an H-module. If A is an H-module algebra, then A^H is a subalgebra in A called the *algebra of invariants of* A. We say that an ideal I in A is *invariant* if $h.x \in I$ for all $h \in H$ and $x \in I$, i.e., if I is a submodule of A, as an H-module. One readily checks that if an ideal I in A is invariant, then all its powers I^j are also invariant, and so we have the quotient H-modules A/I^j, $j \geq 1$.

By a *topological H-module* we mean an H-module V provided with the topology given by a family $\{V_t\}$ of submodules of V (as a fundamental system of neighborhoods of 0). When we want to indicate the topology of V we write $(V, \{V_t\})$. The trivial H-module k will be treated as a topological H-module with the discrete topology. A *morphism of topological H-modules* is a continuous homomorphism of H-modules. All submodules and quotient modules of a topological H-module will be viewed as topological H-modules with the induced topology and quotient topology, respectively.

If $(V, \{V_t\})$ and $(W, \{W_j\})$ are topological H-modules, then the tensor product $V \otimes W$ will be considered as a topological H-module with the topology defined by the family $\{V_t \otimes W + V \otimes W_j\}$ (precisely, their images in $V \otimes W$). If $(V, \{V_t\})$ is a topological H-module, then its *completion* $\widehat{V}$ is defined to be the inductive limit $\varprojlim V/V_t$ provided with the topology inherited from the product topology in $\prod V/V_t$ (notice that V/V_t's have the discrete topology). A topological H-module V is said to be *complete* if the canonical homomorphism $p : V \to \widehat{V}$ is an isomorphism of H-modules. It is easy to see that the topology in $\widehat{V}$ is given by the family of submodules $\{\widehat{V}_t\}$, p induces an isomorphism $V/V_t \simeq \widehat{V}/\widehat{V}_t$ for all t, and $\widehat{V}$ is complete. The category of all complete H-modules will be denoted by cMod. Since for every H-module U the topological H-module $(U, \{0\})$ is complete, the category of H-modules will be identified with the full subcategory of *cMod* consisting of all discrete H-modules. Observe that for any complete H-module V the trivial submodule V^H is also complete.

A *topological H-module algebra* is an H-module algebra A (not necessarily commutative) provided with a topology given by a family of invariant (two-sided) ideals. In the obvious manner, any topological H-module algebra is a topological H-module. It is not difficult to see that a topological algebra is nothing else (up to equivalence of topologies) than a triple (A, m, η), where $m : A \otimes A \to A$ and $\eta : A \to k$ are morphisms of topological H-modules satisfying the appropriate associativity and unity axioms. Such an algebra A is said to be *complete* if A is complete as a topological H-module. If A is a topological H-module algebra, then its completion $\widehat{A}$ is a complete H-module algebra in the obvious manner.

In order to give examples of topological H-module algebras let us recall that a sequence $\boldsymbol{I} = \{I_0, I_1, \ldots\}$ of ideals in a ring A is called *admissible* if $I_0 = A$, $I_1 \supset I_2 \supset \ldots$, and $I_i I_j \subset I_{i+j}$ for all $i, j \geq 0$. Now if A is an H-module algebra and $\boldsymbol{I}$ is an admissible sequence of invariant ideals in A, then $(A, \boldsymbol{I})$ and its completion are topological H-module algebras of special interest for us. An important special case is when $\boldsymbol{I} = \{I^m : m \geq 0\}$, where I is an invariant ideal in A. Then the corresponding topology is the I-adic topology.

If $\boldsymbol{I} = \{I_i : i \geq 0\}$ is an arbitrary admissible sequence of ideals in a ring A, then we denote by $G(\boldsymbol{I})$ the graded ring $\bigoplus_{i=0}^{\infty} I_i$ with the multiplication

"." defined as follows: if $a \in I_i,\ b \in I_j$, then $a.b = ab \in I_{i+j}$ (see [3]). In the case where $\boldsymbol{I} = \{I^i : i \geq 0\}$ for some ideal I in A we write $G(I)$ instead of $G(\boldsymbol{I})$. If A is an H-module algebra and $\boldsymbol{I}$ is an admissible sequence of invariant ideals in A, then the algebra $G(\boldsymbol{I})$ is an H-module algebra in a natural way.

Definition 1.2(see [3]). An admissible sequence $\boldsymbol{I}$ of ideals in a ring A has *right AR 2* (the *second right Artin–Rees property*) if the ring $G(\boldsymbol{I})$ is right noetherian. If A is commutative, then clearly $G(\boldsymbol{I})$ is commutative and we say that A has *AR 2* whenever $G(\boldsymbol{I})$ is noetherian. An ideal I in the ring A has right AR 2 if the sequence $\{I^i\}$ has right AR 2.

It is obvious that if $\boldsymbol{I}$ is an admissible sequence with right AR 2, then the ring A is right noetherian. The significance of AR 2 is expressed by the following.

Theorem 1.3 ([3, Corollary 1.4]). *If A is a ring and $\boldsymbol{I}$ is an admissible sequence of ideals which has right AR 2, then the completion of A in the topology given by $\boldsymbol{I}$ is a right noetherian ring.*

If A is a commutative, noetherian ring, then any ideal in A has AR 2 [1, Chap. 10]. In Section 3 we give other examples of admissible sequences of ideals with AR 2 (see Examples 3.7 and 3.10). If L is a finite-dimensional, nilpotent Lie algebra and $A = U(L)$ is the universal enveloping algebra of L, then the augmentation ideal $I = LA$ has right AR 2 (see [8]).

2. The category of pro-semisimple H-modules

Definition 2.1. A *pro-semisimple H-module* is a complete topological H-module $(V, \{V_t\})$ such that V/V_t is a semisimple H-module for all t.

It is not difficult to show that any closed submodule of a pro-semisimple H-module is also pro-semisimple. If the Hopf algebra H is finitely semisimple, then the completion of any locally finite, topological H-module is a pro-semisimple H-module. Let $\mathrm{p}(H)$ denote the full subcategory of the category of topological H-modules whose objects are pro-semisimple H-modules. Notice that the category $\mathrm{s}(H)$ of all semisimple H-modules equipped with the discrete topology is a subcategory of the category $\mathrm{p}(H)$. Observe also that if V is a pro-semisimple H-module, then the trivial submodule V^H is also a pro-semisimple H-module. Moreover, if $f : V \to W$ is a morphism in $\mathrm{p}(H)$, then so is $f^H : V^H \to W^H$.

Now we define the category of pro-semisimply graded H-modules which plays an important role in what follows. A *pro-semisimply graded H-module* is a pair $(W, \{W_q\})$, where W is an H-module and $\{W_q\}$ is a family of pro-semisimple H-modules (indexed by an arbitrary set) such that the H-module W is the direct sum of the H-modules $\{W_q\}$. The family $\{W_q\}$ is called a *pro-semisimple grading* of W. As usual, we write $W = \bigoplus W_q$ instead of the pair $(W, \{W_q\})$. If $W = \bigoplus W_q$ and $U = \bigoplus U_j$ are pro-semisimply graded H-modules and $i_q : W_q \to W$ ($p_j : U \to U_j$) denote the natural injections (natural projections), then a *morphism from* $W = \bigoplus W_q$ *to* $U = \bigoplus U_j$ is a morphism of H-modules $f : W \to U$ such that all the compositions $p_j f i_q : W_q \to U_j$ are morphisms of pro-semisimple H-modules. The category of pro-semisimply graded H-modules will be denoted by $\mathrm{pg}(H)$. Providing each pro-semisimple H-module V with the trivial grading $\{V_1 = V\}$ one can consider the category $\mathrm{p}(H)$ as a subcategory of the category $\mathrm{pg}(H)$. If $W = \bigoplus W_q$ is an object of $\mathrm{pg}(H)$, then $W^H = \bigoplus W_q^H$ is also an object of $\mathrm{pg}(H)$, because, as we mentioned above, V^H is a pro-semisimple H-module if V is. Moreover, if $f : W \to U$ is a morphism in $\mathrm{pg}(H)$, then $f^H : W \to U$ is.

Definition 2.2. Let $\mathcal{C}$ be a subcategory of the category $\mathrm{pg}(H)$-modules having the property: if $f : V \to W$ is a morphism in $\mathcal{C}$, then $f^H : V^H \to W^H$ is. We say that on the category $\mathcal{C}$ there exists a *Reynolds operator* R if for each $V \in \mathrm{ob}\,\mathcal{C}$ a morphism $R(V) : V \to V^H$ in $\mathcal{C}$ is given such that the following conditions hold:

(1) if $V \in \mathrm{ob}\,\mathcal{C}$ and $v \in V^H$, then $R(V)(v) = v$,
(2) if $f : V \to W$ is a morphism in $\mathcal{C}$, then the diagram

$$\begin{tikzcd} V \arrow[r, "R(V)"] \arrow[d, "f"] & V^H \arrow[d, "f^H"] \\ W \arrow[r, "R(W)"] & W^H \end{tikzcd}$$

is commutative.

Remark. It is easy to see that on every $\mathcal{C}$ there exists at most one Reynolds operator.

Theorem 2.3. *On the category* $\mathrm{pg}(H)$ *there exists a Reynolds operator R.*

Proof. We shall construct R in three steps using the inclusions $\mathrm{s}(H) \subset \mathrm{p}(H) \subset \mathrm{pg}(H)$. The construction is a simple modification of what has been done in [10, proof of Theorem 3.11].

First we show that there exists a Reynolds operator R on $\mathrm{s}(H)$. Let $H^+ = \{h \in H : \varepsilon(h) = 0\}$. If U is a simple H-module, then clearly $H^+U = 0$ if U is trivial and $H^+U = U$ otherwise. Hence $U = U^H \oplus H^+U$ for each semisimple H-module U. This in turn implies that the natural projections $R(U) : U \to U^H$, $U \in \mathrm{s}(H)$, define a Reynolds operator R on $\mathrm{s}(H)$. Now exactly in the same manner as in [10] one shows that the Reynolds operator R on $\mathrm{s}(H)$ can be extended first to $\mathrm{p}(H)$ and then to $\mathrm{pg}(H)$.

Corollary 2.4. *If* $f : W \to U$ *is a surjective morphism in* $\mathrm{pg}(H)$, *then so is* $f^H : W \to U$.

Proof. This follows easily from the definition of a Reynolds operator.

Corollary 2.5. *Let* $(V, \{V_t\})$ *be a topological* H*-module which is semisimple as an* H*-module. Then the natural morphism of complete* H*-modules* $f : \widehat{V^H} \to (\widehat{V})^H$, $f((v_t + V_t^H)) = (v_t + V_t)$, *is an isomorphism* (*i.e., the operations of completion and taking invariants commute*).

Proof. It is obvious that f is injective. Let $\widehat{v} = (v_t + V_t) \in (\widehat{V})^H$. This means that $v_t + V_t \in (V/V_t)^H$ for each t. Applying Corollary 2.4 to the natural projections $V \to V/V_t$, we can assume that $v_t \in V^H$ for all t. Hence $\widehat{v} \in \operatorname{Im} f$.

Definition 2.6. A *pro-semisimple* H*-module algebra* is a topological H-module algebra which is pro-semisimple as a topological H-module. A $\mathrm{pg}(H)$*-algebra* is an H-module algebra A together with a pro-semisimple grading of A as an H-module such that for each $y \in A^H$ the map $\widetilde{y} : A \to A$, $\widetilde{y}(a) = ya$, is a morphism in the category $\mathrm{pg}(H)$ (it is easy to see that $\widetilde{y}$ is always a homomorphism of H-modules).

If A is a $\mathrm{pg}(H)$-algebra, then A^H is a $\mathrm{pg}(H)$-algebra in a natural way. Also it is clear that each pro-semisimple H-module algebra A provided with the trivial grading $\{A_1 = A\}$ is a $\mathrm{pg}(H)$-algebra.

Any semisimple H-module algebra equipped with the discrete topology is a pro-semisimple H-module algebra. In order to give other examples of pro-semisimple H-module algebras and $\mathrm{pg}(H)$-algebras, assume that the Hopf algebra H is finitely semisimple. It is easy to verify that the following statements hold:

(1) Any locally finite H-module algebra (with the discrete topology) is a pro-semisimple H-module algebra.

(2) The completion $\widehat{A}$ of a locally finite, topological H-module algebra A is a pro-semisimple H-module algebra. Moreover, if the topology in A is given by an admissible sequence of ideals with right AR 2, then $\widehat{A}$ is right noetherian.

(3) Any *linearly compact* H-module algebra, i.e., a complete topological H-module algebra $(A, \{I_t\})$ such that A/I_t is a finite-dimensional vector space for all t, is a pro-semisimple H-module algebra. For instance, if H acts on the algebra of formal power series $A = k[[X_1, \dots, X_n]]$ in such a way that its unique maximal ideal m is invariant, then A together with the m-adic topology is a linearly compact H-module algebra.

(4) If $\boldsymbol{I} = \{I_n : n \geq 0\}$ is any admissible sequence of closed and invariant ideals in a pro-semisimple H-module algebra A, then the graded H-module

algebra $G(\boldsymbol{I}) = \bigoplus_{i=0}^{\infty} I_i$ is a pg(H)-algebra. Moreover, it is right noetherian whenever $\boldsymbol{I}$ has right AR 2.

COROLLARY 2.7. *Let R denote the Reynolds operator on the category* pg(H) *and let A be a* pg(H)*-algebra. Then for $R = R(A)$ and all $y \in A^H$, $a \in A$ we have $R(ya) = yR(a)$, that is, $R : A \to A^H$ is a homomorphism of* (*left*) *A^H-modules.*

Proof. Apply condition (2) of Definition 2.2 to the morphism of pro-semisimply graded H-modules $f = \widetilde{y} : A \to A$.

THEOREM 2.8. *If A is a right noetherian* pg(H)*-algebra, then A^H is also a right noetherian* pg(H)*-algebra.*

Proof. Let $R = R(A)$. Since, by Corollary 2.7, $R(ya) = yR(a)$ and $R(y) = y$ for $y \in A^H$ and $a \in A$, $IA \cap A^H = I$ for any right ideal I in A^H. Hence A^H is right noetherian, because so is A.

An immediate consequence of the above theorem is the following.

COROLLARY 2.9. *If A is a right noetherian*, *semisimple H-module algebra*, *then A^H is right noetherian. In particular*, *if H is finitely semisimple and A is a locally finite*, *right noetherian H-module algebra*, *then A^H is right noetherian.*

From Theorem 2.8 we also get generalizations of Donkin's results [3, Corollary 2.2, Theorem 2.3, and Corollary 2.4].

COROLLARY 2.10. *Let A be a pro-semisimple H-module algebra and let $\boldsymbol{I} = \{I_r : r \geq 0\}$ be an admissible sequence of closed invariant ideals with right AR 2. Then the admissible sequence $\boldsymbol{I}^H = \{I_r^H : r \geq 0\}$ of ideals in A^H also has right AR 2.*

Proof. In view of the assumptions, $G(\boldsymbol{I})$ is a right noetherian pg(H)-algebra. From Theorem 2.8 it follows that $G(\boldsymbol{I}^H) = G(\boldsymbol{I})^H$ is also right noetherian. This means that $\boldsymbol{I}^H$ has right AR 2, as was to be proved.

THEOREM 2.11. *Let A be a semisimple H-module algebra*, *and let $\boldsymbol{I} = \{I_j : j \geq 0\}$ be an admissible sequence of invariant ideals with right AR 2. Furthermore*, *let $\widehat{A}$ denote the completion of A in the topology determined by $\boldsymbol{I}$.*

(1) *The natural homomorphism of* (*complete*) *topological algebras $f : \widehat{A^H} \to (\widehat{A})^H$, $f((a_j + I_j^H)) = (a_j + I_j)$, is an isomorphism.*

(2) *The ring $(\widehat{A})^H$ is right noetherian.*

In particular, *if H is finitely semisimple and A is locally finite*, *then $(\widehat{A})^H$ is right noetherian.*

Proof. From Corollary 2.5 we know that f is an isomorphism. Part (2) follows from part (1), by Corollary 2.10 and Theorem 1.3.

3. Commutative H-module algebras. Let V be an H-module. Then the tensor algebra $T(V)$ is an H-module algebra via

$$h.(v_1 \otimes \ldots \otimes v_n) = \sum h_{(1)}v_1 \otimes \ldots \otimes h_{(n)}v_n.$$

It is obvious that the action of H on $T(V)$ preserves the natural grading of $T(V)$. Let $I = I(V)$ denote the ideal in $T(V)$ generated by the set

$$\{h.(v \otimes v' - v' \otimes v) : h \in H,\ v, v' \in V\}.$$

Then I is an invariant homogeneous ideal in $T(V)$. Set $S_H(V) = T(V)/I$ (the definition of $S_H(V)$ comes from [12]). Recall that a graded algebra $A = \bigoplus_{i\geq 0} A_i$ is called *connected* if $A_0 = k$. With the above notation, one has the following.

LEMMA 3.1. (1) *$S_H(V)$ is a graded, connected, commutative H-module algebra such that all its homogeneous components $S_H(V)_i$, $i \geq 0$, are H-submodules of A and $S_H(V)_1 = V$. Furthermore, if H is cocommutative, then $S_H(V)$ is the ordinary symmetric H-module algebra $S(V)$.*

(2) *If V is finite-dimensional, then the algebra $S_H(V)$ is finitely generated and all its homogeneous components are finite-dimensional H-submodules of $S_H(V)$. In particular, $S_H(V)$ is a locally finite H-module algebra.*

(3) *Let A be any commutative H-module algebra and let $g : V \to A$ be a homomorphism of H-modules. Then there exists a unique homomorphism of H-module algebras $\widetilde{g} : S_H(V) \to A$ (called the induced homomorphism) such that its restriction to $V = S_H(V)_1$ equals g. Moreover, if the set $g(V)$ generates the algebra A, then the morphism $\widetilde{g}$ is surjective.*

Proof. This is a straightforward computation.

From now on, we assume that all H-module algebras under consideration are commutative.

THEOREM 3.2. *Suppose that the Hopf algebra H is finitely semisimple and A is a finitely generated, locally finite H-module algebra. Then the algebra A^H is finitely generated.*

Proof. As A is locally finite and finitely generated, there exist linearly independent generators $y_1, \ldots, y_n$ of the algebra A such that $V = ky_1 + \ldots + ky_n$ is an H-submodule of A. From Lemma 3.1(3) it follows that the inclusion $g : V \to A$ induces a surjective morphism of H-module algebras $\widetilde{g} : S_H(V) \to A$. Moreover, $S_H(V)$ is locally finite, by Lemma 3.1(2). Hence both A and $S_H(V)$ are semisimple H-module algebras, because H is finitely semisimple. Applying Corollary 2.4 (to $\widetilde{g}$) and Theorem 2.8, we see that the homomorphism of algebras $\widetilde{g}^H : S_H(V)^H \to A^H$ is surjective and that $S_H(V)^H$ is a noetherian ring. Since $S_H(V)^H$ is a connected graded algebra, the latter implies that $S_H(V)^H$ is a finitely generated algebra. Since $A^H = \widetilde{g}(S_H(V)^H)$, this shows that A^H is finitely generated.

Let A' be a subring of a commutative ring A and let $i : A' \to A$ be the natural inclusion. Recall that A is called *pure over* A' if the map $i \otimes_{A'} M : A' \otimes_{A'} A \to A \otimes_{A'} M$ is injective for any A'-module M. It is obvious that A is pure over A', whenever i splits over A', i.e., whenever $ti = \mathrm{id}_{A'}$ for some homomorphism of A'-modules $t : A \to A'$. In particular, if A is a pro-semisimple H-module algebra, then A is pure over A^H. In fact, by Corollary 2.7, the Reynolds operator $R = R(A) : A \to A^H$ is a homomorphism of A^H-modules such that $R(a) = a$ for $a \in A^H$.

THEOREM 3.3. *Let A be a finitely generated H-module algebra which is a regular integral domain. Then A^H is a Cohen–Macaulay ring in each of the following cases*:

(1) *H is finitely semisimple and A is locally finite.*

(2) *$A = \bigoplus_{i \ge 0} A_i$ is a connected graded algebra such that all A_i's are semisimple H-submodules of A.*

Proof. In both cases A is a semisimple H-module algebra. If condition (1) holds, then, according to Theorem 3.2, A^H is a finitely generated algebra. Moreover, A is pure over A^H. Hence A^H is a Cohen–Macaulay ring, by [6, Theorem 0.2]. Now suppose that (2) holds. It follows from Theorem 2.8 that the ring A^H is noetherian. Furthermore, A^H is obviously a connected graded algebra. Therefore, A^H is finitely generated. As A is pure over A^H, we conclude that A^H is a Cohen–Macaulay ring, again by [6, Theorem 0.2].

We now describe all admissible sequences with AR 2 in any commutative, noetherian ring.

LEMMA 3.4. *Let A be a commutative noetherian ring and let $\boldsymbol{I} = \{I_i : i \ge 0\}$ be an admissible sequence of ideals in A. Then I has AR 2 if and only if I satisfies the following condition*:

$$(*) \qquad \textit{There exists an } n \ge 1 \textit{ such that } I_{n+j} = \sum_{i=1}^{n} I_i I_{n+j-i} \textit{ for all } j \ge 1.$$

Proof. Let $J = \bigoplus_{i \ge 1} I_i \subset G(\boldsymbol{I}) = \bigoplus_{i \ge 0} I_i$ and let J_q denote the ideal in $G(\boldsymbol{I})$ generated by $\bigoplus_{i=1}^{q} I_i \subset G(\boldsymbol{I})$, $q = 1, 2, \ldots$ Then $J_q \subset J_{q+1}$ for all q and J is the union of all J_q's. If the algebra $G(\boldsymbol{I})$ is noetherian, then there exists an n such that $J = J_n$. But J_n is a graded ideal in $G(\boldsymbol{I})$ whose $(n + j)$th component is equal to $\sum_{i=1}^{n} I_i I_{n+j-i}$, $j \ge 1$. This proves the implication

"$\Rightarrow$". If $\boldsymbol{I}$ satisfies the condition $(*)$, then clearly $J = J_n$. Hence the ideal J is finitely generated, because $I_1 \oplus \ldots \oplus I_n$ is a finitely generated A-module. Let $a_1, \ldots, a_s$ be homogeneous generators of J. Then $G(\boldsymbol{I}) = A[a_1, \ldots, a_s]$. Consequently, $G(\boldsymbol{I})$ is noetherian, because so is A.

COROLLARY 3.5. *Let $\boldsymbol{I} = \{I_i\}$ be an admissible sequence of ideals in A which has AR 2.*

(1) *There exists an n such that $I_{nj} \subset I_1^j \subset I_j$ for all $j \geq 0$. In particular, the topology determined by the sequence $\boldsymbol{I}$ is equivalent to the I_1-adic topology.*

(2) *The completion of A in the topology determined by the sequence $\boldsymbol{I}$ is isomorphic to the completion of A in the I_1-adic topology.*

Proof. By Lemma 3.4, the sequence $\boldsymbol{I}$ satisfies the condition $(*)$. This implies that

$$I_{nj} \subset \sum_{i=1}^{n} I_{n(j-1)+n-i} I_i \subset \sum_{i=1}^{n} I_{n(j-1)} I_i \qquad \text{for all } j \geq 1,$$

whence, by induction on j,

$$I_{jn} \subset \sum_{j_1+\ldots+j_n=j} I_1^{j_1} \ldots I_n^{j_n} \qquad \text{for all } j \geq 0.$$

Since all the ideals I_i are contained in I_1, it follows that $I_{jn} \subset I_1^j$ for $j \geq 0$. Obviously, $I_1^j \subset I_j$, because the sequence $\boldsymbol{I}$ is admissible. This proves part (1). Part (2) is a consequence of (1).

The following theorem gives a description of all admissible sequences of ideals with AR 2.

THEOREM 3.6. *Let $(I_0, I_1, \ldots, I_n)$ be a sequence of ideals in a commutative ring A satisfying the condition*:

(i) $I_0 = A$, $I_1 \supset I_2 \supset \ldots \supset I_n$ *and* $I_i I_s \subset I_{i+s}$ *for* $i + s \leq n$.

Moreover, let $I_{n+j} = \sum_{i=1}^{n} I_i I_{n+j-i}$ for $j \geq 1$ (inductive formula). Then the sequence $\boldsymbol{I}(I_1, \ldots, I_n) = \{I_q : q \geq 0\}$ is admissible and has AR 2. Conversely, if $\boldsymbol{I} = \{I_j\}$ is an admissible sequence of ideals with AR 2, then there exists an n such that the sequence of ideals $(I_0, I_1, \ldots, I_n)$ satisfies the condition (i) *and $\boldsymbol{I} = \boldsymbol{I}(I_0, \ldots, I_n)$.*

Proof. Two simple inductions show that $\boldsymbol{I}(I_0, \ldots, I_n)$ is an admissible sequence. The rest of the theorem follows from Lemma 3.4.

EXAMPLE 3.7. Let A be a commutative ring. If I is an ideal in A, then clearly the sequence $(I_0 = A, I_1 = I)$ satisfies condition (i) in the above theorem and $\boldsymbol{I}(I_0, I_1) = \{I^j : j \geq 0\}$. If I_1, I_2 are ideals in A such that $I_1 \supset I_2$ and $I_1^2 \subset I_2$, then the sequence $(I_0 = A, I_1, I_2)$ also satisfies condition (i), and one easily checks that $\boldsymbol{I}(I_0, I_1, I_2) = \{I_j\}$, where $I_j = I_1^i I_2^r$ for $j = 2i + r,\ 0 \le r \le 1$.

The next example shows that if $\boldsymbol{I}$ is an admissible sequence of ideals and the topology defined by $\boldsymbol{I}$ is equivalent to the I_1-adic topology, then $\boldsymbol{I}$ need not have AR 2.

EXAMPLE 3.8. Let $A = k[X, Y]$ and let $J_1 = (X, Y)$, $J_2 = (X, Y^2)$. Further, let $I_0 = A$ and let $I_i = J_2^i J_1$ for $i \ge 1$. Then $\boldsymbol{I} = \{I_i : i \ge 0\}$ is obviously an admissible sequence of ideals in A and the topology defined by $\boldsymbol{I}$ and the I_1-adic topology are equivalent, because $I_{2j} \subset I_1^j \subset I_j$ for all $j \ge 0$. Suppose that $\boldsymbol{I}$ satisfies the condition $(*)$ from Lemma 3.4, i.e., there exists an $n \ge 1$ such that $I_{n+j} = I_1 I_{n+j-1} + \ldots + I_n I_j$ for $j \ge 1$. This means that $I_2^{n+j} I_1 = I_2^{n+j} I_1^2$, which is impossible. By Lemma 3.4, $\boldsymbol{I}$ does not have AR 2.

REMARK 3.9. Let $\boldsymbol{I} = \{I_i\}$ be an admissible sequence of ideals in a commutative ring A which has AR 2. Since $I_1 I_i \subset I_{i+1}$ for all i, $\boldsymbol{I}$ is an I_1-filtration of the ring A in the sense of [1, Chap. 10]. However, in general, it is not a stable I_1-filtration, i.e., there does not exist an s such that $I_1 I_i = I_{i+1}$ for $i \ge s$. This is illustrated by the following

EXAMPLE 3.10. Let A, I_1, and I_2 be as in Example 3.8. Set $I_{2i+r} = I_2^i I_1^r$ for $i \ge 1$ and $r = 0, 1$. It is easy to verify that $\boldsymbol{I} = \{I_i : i \ge 0\}$ $(I_0 = A)$ is an admissible sequence satisfying the condition $(*)$ from Lemma 3.4 for $n = 2$, and so $\boldsymbol{I}$ has AR 2. But $I_1 I_{2i+1} \ne I_{2(i+1)}$ for all i, because the first ideal equals $(X, Y)^2 (X, Y^2)^i$ and the second one $(X, Y^2)^{i+1}$.

Now we show some applications of the above results. By a *local ring* we mean a commutative, noetherian ring with the unique maximal ideal.

THEOREM 3.11. *Let A be a noetherian, pro-semisimple H-module algebra and let I be an invariant ideal in A such that all the ideals $I^i, i \ge 0$, are closed. Then the I^H-adic topology in A^H is equivalent to the topology defined by the admissible sequence of ideals $\{(I^r)^H : r \ge 0\}$.*

Proof. Let $\boldsymbol{I} = \{I^r : r \ge 0\}$. By Corollary 2.10, the admissible sequence $\boldsymbol{I}^H = \{(I^r)^H\}$ has AR 2. Now the theorem follows from Corollary 3.5.

THEOREM 3.12. *Suppose that A is a noetherian H-module algebra and I is an invariant ideal in A such that A is complete in the I-adic topology and the H-modules A/I^i are semisimple for all $i \ge 0$. Then A^H is a noetherian ring, complete in the I^H-adic topology.*

Proof. The algebra A together with the I-adic topology is a pro-semisimple H-module algebra. Therefore, according to Theorem 2.8, A^H is a noetherian ring, complete in the topology given by the admissible sequence

$\{(I_i)^H\}$. Furthermore, all the ideals I^i are closed (in the I-adic topology), by [1, Proposition 10.15,(II)]. Hence we get the assertion, by Theorem 3.11.

THEOREM 3.13. *Suppose that the Hopf algebra H is finitely semisimple and that A is a noetherian H-module algebra.*

(1) *If A is locally finite, I is an invariant ideal in A, and $\widehat{A}$ is the completion of A in the I-adic topology, then the algebra $(\widehat{A})^H$ is noetherian and the natural inclusion $i : A^H \to A$ induces an isomorphism of the completion of A^H in the I^H-adic topology with $(\widehat{A})^H$.*

(2) *If A is a complete local ring with the invariant maximal ideal m and A/m is a finite field extension of k, then A^H is a complete local ring with the unique maximal ideal m^H. In particular, if $A = k[[X_1, \dots, X_n]]/J$ (for some n and an ideal J) and the ideal $m = (X_1 + J, \dots, X_n + J)$ is invariant, then A^H is of the same form.*

Proof. In the situation of (1), we know from Corollary 2.5 that the inclusion $i : A^H \to A$ induces an isomorphism of the completion of A^H in the topology given by the ideals $\{(I^i)^H : i \geq 0\}$ with the algebra $(\widehat{A})^H$. The conclusion now follows from Theorem 3.11 applied to A with the discrete topology. As for part (2), a simple induction shows that the H-modules A/m^j, $j \geq 1$, are finite-dimensional vector spaces. Hence A is a pro-semisimple H-module algebra, because H is finitely semisimple. By Theorem 3.12, this implies that A^H is a noetherian ring, complete in the m^H-adic topology. Furthermore, one easily verifies that m^H is the unique maximal ideal in A^H. Thus we obtain the first statement in (2). The second one is a consequence of the Cohen classification of complete local rings.

THEOREM 3.14. *Fix $n \geq 0$ and suppose that H acts on the algebra $A = k[[X_1, \dots, X_n]]$ in such a way that the (unique) maximal ideal m in A is invariant and the H-modules m/m^j, $j \geq 1$, are semisimple. Then A^H is a complete, local Cohen–Macaulay ring.*

Proof. In view of Theorem 3.13, A^H is a complete local ring. So, it remains to prove that A^H is Cohen–Macaulay. By [2, Thm. 4(2)], we can assume (possibly changing variables) that $h.X_i \in kX_1 + \ldots + kX_n$ for all $h \in H$ and $i = 1, \dots, n$. It follows that the action of H on the algebra A preserves the subalgebra $B = k[X_1, \dots, X_n]$, so that we have the induced action of H on B. Moreover, if $B = \bigoplus_{j \geq 0} B_j$ is the natural grading in B (given by degree), then all B_j's are H-submodules of B. We show that B is semisimple as an H-module. First observe that for each $j \geq 0$ the H-module m^j/m^{j+1} is semisimple, because it is a submodule of the semisimple H-module m/m^{j+1}. On the other hand, the natural inclusion $B_j \subset m^j$ induces an isomorphism of H-modules $B_j \simeq m^j/m^{j+1}$. Hence $B = \bigoplus_{j \geq 0} B_j$ is a semisimple H-module. Now making use of Theorem 3.3(2), we see that B^H

is a Cohen–Macaulay ring. It is clear that A^H is the completion of B^H in the topology defined by the admissible sequence of ideals $\{(m'^j)^H\}$, where m' is the (maximal) ideal in B generated by the variables $X_1, \ldots, X_n$. From Theorem 3.11 (applied to B with the discrete topology and $I = m'$) it follows that the topology in B^H given by the sequence $\{(m'^i)^H\}$ is equivalent to the m'^H-adic topology. Hence A^H is isomorphic to the completion of B^H in the m'^H-adic topology. The conclusion now follows from [4, Theorem 18.8], because m'^H is the maximal ideal in the Cohen–Macaulay ring B^H.

Corollary 3.15. *If the Hopf algebra H is finitely semisimple and H acts on the algebra $A = k[[X_1, \ldots, X_n]]$ in such a way that the maximal ideal m in A is invariant*, *then A^H is a complete local Cohen–Macaulay ring.*

Proof. The corollary is a consequence of the theorem, because the H-modules m/m^j, $j \geq 1$, are finite-dimensional.

Remark 3.16. Part (2) of Theorem 3.11 together with Theorem 3.14 can be viewed as an analogue of Theorem 3.2 for complete local H-module algebras.

The following example is an application of Corollary 3.15.

Example 3.17. Assume that the field k is algebraically closed and fix a prime $p \neq \operatorname{char} k$. Moreover, let $a^{(1)}, \ldots, a^{(n)}$ be arbitrary p-adic numbers, and let $\Omega = \{\alpha = (\alpha_1, \ldots, \alpha_n) \in \mathbb{N}^n : \alpha_1 a^{(1)} + \ldots \alpha_n a^{(n)} = 0\}$. It turns out that $A' = \{\sum_{\alpha \in \Omega} t_\alpha X^\alpha \in k[[X_1, \ldots, X_n]]\}$, $X^\alpha = X_1^{\alpha_1} \ldots X_n^{\alpha_n}$, is a complete, local Cohen–Macaulay subring of $k[[X_1, \ldots, X_n]]$. To see this, let $H = kG_p$ be the finitely semisimple Hopf algebra from example (b) of Section 1, and let ζ_j be the primitive root of unity of degree p^{j+1}, $j \geq 0$. Then the formulas

$$\zeta_j . X_i = \zeta_j^{a_j^{(i)}} X_j, \qquad i = 1, \ldots, n, \ j \geq 0,$$

where $a^{(i)} = (a_0^{(i)}, a_1^{(i)}, \ldots)$, $a_j^{(i)} \in \mathbb{Z}/p^{j+1}$, determine an action of H on the algebra $A = k[[X_1, \ldots, X_n]]$ such that the maximal ideal in A is invariant. One simply checks that $A^H = A'$. So, we are done, by Corollary 3.15.

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