ORBIT ALGEBRAS AND PERIODICITY

BY

PETTER ANDREAS BERGH (Trondheim)

Abstract. Given an object in a category, we study its orbit algebra with respect to an endofunctor. We show that if the object is periodic, then its orbit algebra modulo nilpotence is a polynomial ring in one variable. This specializes to a result on Ext-algebras of periodic modules over Gorenstein algebras. We also obtain a criterion for an algebra to be of wild representation type.

1. Introduction. Given a suitably nice category and an endofunctor, we may construct the orbit algebra of a given object (cf. [Len]). These algebras are well suited for studying various types of periodicity. For example, when the endofunctor is an equivalence, then we show that, modulo nilpotence, the orbit algebra of an indecomposable periodic object is the polynomial ring in one variable. Using this, we generalize [GSS, Proposition 1.3] and obtain a periodicity result for modules over Gorenstein algebras. Moreover, we use this theory to study modules over selfinjective algebras, in particular modules which are periodic with respect to the Auslander–Reiten translate. Namely, we show that, modulo nilpotence, the Ext-orbit algebra of such a module with respect to the Nakayama automorphism is the polynomial ring in one variable. As an application of the latter, we provide a criterion for an algebra to be of wild representation type.

2. Orbit algebras. Throughout this paper, we let k be an algebraically closed field and Λ a finite-dimensional k-algebra. We denote by $\operatorname{mod}\Lambda$ the category of finitely generated left Λ-modules. Whenever we deal with Λ-modules, we assume they belong to $\operatorname{mod}\Lambda$.

Recall that a category $\mathcal{C}$ is *k-linear* if for all objects $X, Y, Z \in \mathcal{C}$ the set $\operatorname{Hom}_{\mathcal{C}}(X, Y)$ is a k-vector space, and the composition

$$\operatorname{Hom}_{\mathcal{C}}(X, Y) \times \operatorname{Hom}_{\mathcal{C}}(Y, Z) \to \operatorname{Hom}_{\mathcal{C}}(X, Z)$$

is k-bilinear. Furthermore, such a category is Hom-*finite* if all the morphism spaces are finite-dimensional over the ground field. Now let $\mathcal{C} \xrightarrow{F} \mathcal{C}$ be an

endofunctor. Then we may define a graded algebra

$$\operatorname{Hom}_{\mathcal{C}}(F^*(X), X) := \bigoplus_{i=0}^{\infty} \operatorname{Hom}_{\mathcal{C}}(F^i(X), X),$$

with ring structure given as follows: for $f \in \operatorname{Hom}_{\mathcal{C}}(F^i(X), X)$ and $g \in \operatorname{Hom}_{\mathcal{C}}(F^j(X), X)$, the product fg is the composition $f \circ F^i(g)$, which is an element in $\operatorname{Hom}_{\mathcal{C}}(F^{i+j}(X), X)$.

The algebra just defined is the *orbit algebra* of M with respect to F (cf. [Len]). As an example, suppose Λ is selfinjective, and denote its enveloping algebra $\Lambda \otimes_k \Lambda^{\mathrm{op}}$ by Λ^{e}. Furthermore, let $\underline{\operatorname{mod}} \Lambda^{\mathrm{e}}$ be the stable module category of bimodules, and denote by $\tau_{\Lambda^{\mathrm{e}}}$ the Auslander–Reiten translate of Λ^{e}. The orbit algebra

$$\mathbb{A}(\Lambda, \tau_{\Lambda^{\mathrm{e}}}) := \bigoplus_{i=0}^{\infty} \underline{\operatorname{Hom}}_{\Lambda^{\mathrm{e}}}(\tau_{\Lambda^{\mathrm{e}}}^i(\Lambda), \Lambda)$$

is called the *Auslander–Reiten orbit algebra* of Λ. In [Po1] it was shown that these algebras are invariant under stable equivalences of Morita type between symmetric algebras, and this was generalized in [Po2] to arbitrary finite-dimensional selfinjective algebras. In [Po4] the Auslander–Reiten orbit algebras of a class of finite-dimensional basic connected selfinjective Nakayama algebras were computed. In particular, it was shown that if Γ is such an algebra over a field K, and $\tau_{\Gamma^{\mathrm{e}}}(\Gamma) \simeq \Gamma$, then there are two possibilities. Namely, if Γ is a radical square zero algebra then $\mathbb{A}(\Gamma, \tau_{\Gamma^{\mathrm{e}}}) \simeq K[x]$, and if not then there exists a natural number t such that $\mathbb{A}(\Gamma, \tau_{\Gamma^{\mathrm{e}}}) \simeq K[x, y]/(y^t)$. In [Po3], τ-periodicity was investigated using similar techniques to those used in [Sc1] to study syzygy-periodicity.

We end this section with the following result. It shows that if the endofunctor F is an equivalence and M is "indecomposable" and F-periodic, then the orbit algebra of M modulo nilpotence is a polynomial ring. The result and its proof are inspired by [GSS, Proposition 1.3].

THEOREM 2.1. *Let $\mathcal{C}$ be a k-linear* Hom-*finite category, and let $\mathcal{C} \xrightarrow{F} \mathcal{C}$ be an equivalence. Furthermore, let $M \in \mathcal{C}$ be an object whose endomorphism ring is local, and suppose $F^n(M) \simeq M$ for some $n \geq 1$, where n is minimal with this property. Then*

$$\operatorname{Hom}_{\mathcal{C}}(F^*(M), M)/I \simeq k[x],$$

where I is the ideal in $\operatorname{Hom}_{\mathcal{C}}(F^*(M), M)$ generated by the homogeneous nilpotent elements, and x is a homogeneous element in degree n.

Proof. Let $f \in \operatorname{Hom}_{\mathcal{C}}(F^u(M), M)$ be a homogeneous nilpotent element. We first show that for any homogeneous element $g \in \operatorname{Hom}_{\mathcal{C}}(F^v(M), M)$, the products fg and gf are also nilpotent in $\operatorname{Hom}_{\mathcal{C}}(F^*(M), M)$. Choose

a positive number p with the property that $p(u + v)$ is a multiple of n. If the element $(fg)^p$ is is an isomorphism, then there is a morphism $h \in \mathrm{Hom}_{\mathcal{C}}(M, F^u(M))$ such that $f \circ h$ is the identity on M. This implies that the map $h \circ f$ is not nilpotent in $\mathrm{End}_{\mathcal{C}}(F^u(M))$, and therefore an isomorphism since this endomorphism ring is local. But then f is an isomorphism, contradicting the assumption that it is nilpotent in $\mathrm{Hom}_{\mathcal{C}}(F^*(M), M)$. Thus the element $(fg)^p$ cannot be an isomorphism in $\mathrm{Hom}_{\mathcal{C}}(F^{p(u+v)}(M), M)$. Since $\mathrm{Hom}_{\mathcal{C}}(F^{p(u+v)}(M), M)$ is isomorphic to $\mathrm{End}_{\mathcal{C}}(M)$, we see that $(fg)^p$ corresponds to an element in the radical of $\mathrm{End}_{\mathcal{C}}(M)$, and the same holds for $F^{ni}((fg)^p)$ for any $i \geq 0$. However, the radical is nilpotent, hence $(fg)^p$, and therefore also fg, is nilpotent. A similar argument shows that gf is also nilpotent in $\mathrm{Hom}_{\mathcal{C}}(F^*(M), M)$.

Next, let u be a multiple of n, and let f and g be homogeneous nilpotent elements in $\mathrm{Hom}_{\mathcal{C}}(F^*(M), M)$ of degree u. We show that $f + g$ is also nilpotent. Both f and g correspond to radical elements of $\mathrm{End}_{\mathcal{C}}(M)$, and the same holds for $F^{ni}(f)$ and $F^{ni}(g)$ for every $i \geq 0$. Since the radical of $\mathrm{End}_{\mathcal{C}}(M)$ is nilpotent, we see that $f + g$ must be nilpotent in $\mathrm{Hom}_{\mathcal{C}}(F^*(M), M)$.

We now show that if u is not a multiple of n, then any element of $\mathrm{Hom}_{\mathcal{C}}(F^u(M), M)$ is nilpotent in $\mathrm{Hom}_{\mathcal{C}}(F^*(M), M)$. Let f be such an element, and choose a number p with the property that pu is a multiple of n. Using the same arguments as above, we see that the element f^p cannot be an isomorphism, hence it corresponds to a radical element of $\mathrm{End}_{\mathcal{C}}(M)$. Again using arguments as above, we see that f^p, and therefore also f, must be nilpotent in $\mathrm{Hom}_{\mathcal{C}}(F^*(M), M)$.

Finally, let $F^n(M) \xrightarrow{f} M$ be an isomorphism, and suppose $f^u \in I$ for some u. Then by the first part of the proof, we can write $f^u = f_1 + \cdots + f_t$, where each f_i is an element of $\mathrm{Hom}_{\mathcal{C}}(F^{un}(M), M)$ nilpotent in $\mathrm{Hom}_{\mathcal{C}}(F^*(M), M)$. By the second part of the proof, the element f^u, and therefore also f, is nilpotent, an obvious contradiction. Therefore f^u does not belong to I for any $u \geq 0$. Since k is algebraically closed, the ring $\mathrm{End}_{\mathcal{C}}(M)$ modulo its radical is just k itself. Therefore, up to scalars, for any $i \geq 0$ there is only one non-nilpotent element in $\mathrm{Hom}_{\mathcal{C}}(F^{in}(M), M)$, namely f^i. This shows that $\mathrm{Hom}_{\mathcal{C}}(F^*(M), M)/I$ is isomorphic to the polynomial ring $k[f]$. ∎

3. Applications. As a first application of Theorem 2.1, we generalize [GSS, Proposition 1.3] on extension algebras of periodic modules. Suppose the algebra Λ is Gorenstein, that is, the injective dimension of Λ as a module over itself is finite. Denote by $\mathrm{MCM}(\Lambda)$ the category of maximal Cohen–Macaulay Λ-modules, i.e.

$$\mathrm{MCM}(\Lambda) = \{M \in \mathrm{mod}\, \Lambda \mid \mathrm{Ext}^i_\Lambda(M, \Lambda) = 0 \text{ for all } i > 0\}.$$

It follows from general cotilting theory that this is a Frobenius exact category, in which the projective-injective objects are the projective Λ-modules, and the injective envelopes are the left $(\operatorname{add} \Lambda)$-approximations. Therefore the stable category $\underline{\operatorname{MCM}}(\Lambda)$, which is obtained by factoring out all morphisms which factor through projective Λ-modules, is a triangulated category. Its shift functor is given by cokernels of left $(\operatorname{add} \Lambda)$-approximations, the inverse shift is the usual syzygy functor.

THEOREM 3.1. *Suppose that Λ is Gorenstein, and let M be an indecomposable Λ-module such that $\Omega_\Lambda^n(M) \simeq M$ for some $n \geq 1$, where n is minimal with this property. Then*

$$\operatorname{Ext}_\Lambda^*(M, M)/I \simeq k[x],$$

where I is the ideal in $\operatorname{Ext}_\Lambda^*(M, M)$ generated by the homogeneous nilpotent elements, and x is a homogeneous element of degree n.

Proof. Since M is periodic, it must be a maximal Cohen–Macaulay module. Consider the equivalence Ω_Λ on $\underline{\operatorname{MCM}}(\Lambda)$. It follows from Theorem 2.1 that

$$\operatorname{Hom}_{\underline{\operatorname{MCM}}(\Lambda)}(\Omega_\Lambda^*(M), M)/J \simeq k[x],$$

where J is the ideal in $\operatorname{Hom}_{\underline{\operatorname{MCM}}(\Lambda)}(\Omega_\Lambda^*(M), M)$ generated by the homogeneous nilpotent elements. Now when i is positive, we may identify the module $\operatorname{Hom}_{\underline{\operatorname{MCM}}(\Lambda)}(\Omega_\Lambda^i(M), M)$ with $\operatorname{Ext}_\Lambda^i(M, M)$. Furthermore, modulo nilpotence the rings $\operatorname{End}_{\underline{\operatorname{MCM}}(\Lambda)}(M)$ and $\operatorname{End}_\Lambda(M)$ are isomorphic. ∎

Now let Λ be arbitrary. Let $\operatorname{mod} \Lambda \xrightarrow{F} \operatorname{mod} \Lambda$ be an exact functor such that $F(\Lambda)$ is a projective left module (as happens for example when F is an equivalence). Then F preserves projective modules (cf. [Ber, Section 2]). For a positive integer t, we then define a graded algebra

$$\operatorname{Ext}_\Lambda^{t*}(F^*(M), M) := \bigoplus_{i=0}^{\infty} \operatorname{Ext}_\Lambda^{ti}(F^i(M), M),$$

with ring structure given as follows: for $\eta \in \operatorname{Ext}_\Lambda^{tu}(F^u(M), M)$ and $\theta \in \operatorname{Ext}_\Lambda^{tv}(F^v(M), M)$, the product $\eta\theta$ is the Yoneda product $\eta \circ F^u(\theta)$, which is an element in $\operatorname{Ext}_\Lambda^{t(u+v)}(F^{u+v}(M), M)$. This ring structure is well defined by [Ber, Lemma 2.1], and in the following result we apply Theorem 2.1 to these algebras.

THEOREM 3.2. *Suppose that Λ is selfinjective, and let $\operatorname{mod} \Lambda \xrightarrow{F} \operatorname{mod} \Lambda$ be an equivalence. Let M be an indecomposable Λ-module, and suppose that $\Omega_\Lambda^{tn}(F^n(M)) \simeq M$ for some $n \geq 1$, where n is minimal with this property. Then*

$$\operatorname{Ext}_\Lambda^{t*}(F^*(M), M)/I \simeq k[x],$$

where I is the ideal in $\mathrm{Ext}_{\Lambda}^{t*}(F^*(M), M)$ generated by the homogeneous nilpotent elements, and x is a homogeneous element of degree tn.

Proof. The proof is analogous to that of Theorem 3.1. ∎

Recall that if Λ is selfinjective, then the *Nakayama automorphism* $\mathcal{N}$ is the equivalence $D \circ \mathrm{Hom}_{\Lambda}(-, \Lambda)$, where D is the usual k-dual. The composition $\Omega_{\Lambda}^2 \circ \mathcal{N}$ is isomorphic to the *Auslander–Reiten translate* τ of Λ (cf. [ARS, Proposition IV.3.7]). An immediate application of Theorem 3.2 is the following result on τ-periodic modules.

COROLLARY 3.3. *Suppose that Λ is selfinjective, and let M be an indecomposable Λ-module. Suppose $\tau^n(M) \simeq M$ for some $n \geq 1$, where n is minimal with this property. Then*

$$\mathrm{Ext}_{\Lambda}^{2*}(\mathcal{N}^*(M), M)/I \simeq k[x],$$

where I is the ideal in $\mathrm{Ext}_{\Lambda}^{2*}(\mathcal{N}^*(M), M)$ generated by the homogeneous nilpotent elements, and x is a homogeneous element of degree $2n$. ∎

We now look at orbit algebras of modules whose minimal projective resolutions are not "too big". Let M be a Λ-module with minimal projective resolution

$$\cdots \to P_2 \to P_1 \to P_0 \to M \to 0,$$

say. Recall that the *complexity* of M, denoted $\mathrm{cx}\, M$, is defined as

$$\mathrm{cx}\, M := \inf\{t \in \mathbb{N} \cup \{0\} \mid \exists a \in \mathbb{R} \text{ such that } \dim_k P_n \leq an^{t-1} \text{ for } n \gg 0\}.$$

In general, the complexity of a module may be infinite. From the definition, we see that $\mathrm{cx}\, M = 0$ if and only if M has finite projective dimension, and that $\mathrm{cx}\, M = 1$ precisely when $\{\dim_k P_n\}_{n=0}^{\infty}$ is bounded. The following lemma shows that any equivalence of $\mathrm{mod}\, \Lambda$ preserves the complexity of a module.

LEMMA 3.4. *If $\mathrm{mod}\, \Lambda \xrightarrow{F} \mathrm{mod}\, \Lambda$ is an equivalence, then:*

(i) $\mathrm{cx}\, M = \mathrm{cx}\, F(M)$ *for every $M \in \mathrm{mod}\, \Lambda$,*

(ii) *if $\{M_i\}_{i=1}^{\infty}$ is a set of modules such that $\{\dim_k M_i\}_{i=1}^{\infty}$ is bounded, then $\{\dim_k F^{t_i}(M_i)\}_{i=1}^{\infty}$ is also bounded, where each t_i is an arbitrary number.*

Proof. Since F is an equivalence, it maps a projective resolution of M to a projective resolution of $F(M)$. Moreover, it maps the minimal projective resolution $\mathbb{P}_M$ of M to that of $F(M)$. Namely, if $F(\mathbb{P}_M)$ contains a direct summand of the form $Q \xrightarrow{\mathrm{id}} Q$, where Q is a projective module, then $F^{-1}(F(\mathbb{P}_M))$ contains $F^{-1}(Q) \xrightarrow{\mathrm{id}} F^{-1}(Q)$ as a summand. However, since $F^{-1}(F(\mathbb{P}_M))$ is isomorphic to $\mathbb{P}_M$, the projective module $F^{-1}(Q)$ must be the zero module. Then Q is the zero module, hence $F(\mathbb{P}_M)$ contains no

non-trivial direct summands of the form $Q \xrightarrow{\text{id}} Q$. This shows that $F(\mathbb{P}_M)$ is the minimal projective resolution of $F(M)$, and so $\operatorname{cx} M = \operatorname{cx} F(M)$. The second part follows from the fact that F preserves the length of a module. ∎

With the help of this lemma, we obtain the following converse to Theorem 3.2, for modules of complexity one.

THEOREM 3.5. *Suppose that Λ is selfinjective, and let* $\operatorname{mod} \Lambda \xrightarrow{F} \operatorname{mod} \Lambda$ *be an equivalence. Let M be an indecomposable Λ-module of complexity one. If*

$$\operatorname{Ext}_{\Lambda}^{t*}(F^*(M), M)/I$$

is a polynomial ring in one variable, where I is the ideal in $\operatorname{Ext}_{\Lambda}^{t*}(F^*(M), M)$ generated by the homogeneous nilpotent elements, then M is $(\Omega_{\Lambda}^t \circ F)$-periodic.

Proof. Suppose that $\operatorname{Ext}_{\Lambda}^{t*}(F^*(M), M)/I$ is a polynomial ring $k[x]$. Let η be a homogeneous element in $\operatorname{Ext}_{\Lambda}^{t*}(F^*(M), M)$ corresponding to x in the factor algebra, and choose a map $\Omega_{\Lambda}^{tn}(F^n(M)) \xrightarrow{f_\eta} M$ representing this element. Note that by Dickson's lemma, the module $\Omega_{\Lambda}^{ti}(M)$, and therefore also $\Omega_{\Lambda}^{ti}(F^i(M))$, is indecomposable for all i. Moreover, since M is of complexity one, it follows from Lemma 3.4 that $\{\dim_k \Omega_{\Lambda}^{ti}(F^i(M))\}_{i=1}^{\infty}$ is bounded. Therefore, if f_η is not an isomorphism, then the Harada–Sai lemma and a proof analogous to that of [Sc2, Theorem 2] shows that η is nilpotent. This clearly cannot be the case, hence f_η must be an isomorphism. ∎

Using Lemma 3.4, we now show that, under certain conditions, given a number t and an equivalence F on $\operatorname{mod} \Lambda$, there exists an infinite set $\{M_i\}_{i=1}^{\infty}$ of modules satisfying the following: the modules are of the same dimension and pairwise non-isomorphic, and each of them is indecomposable and not periodic with respect to $\Omega_{\Lambda}^t \circ F$.

THEOREM 3.6. *Suppose that Λ is selfinjective, and let* $\operatorname{mod} \Lambda \xrightarrow{F} \operatorname{mod} \Lambda$ *be an equivalence. Furthermore, suppose there exists an indecomposable Λ-module M such that*:

  (i) $\operatorname{cx} M = 1$,
  (ii) $\operatorname{Ext}_{\Lambda}^{t*}(F^*(M), M)/I$ *is not a polynomial ring, where I is the ideal generated by the homogeneous nilpotent elements.*

Then M is not periodic with respect to $\Omega_{\Lambda}^t \circ F$. Moreover, there exists a positive integer d and infinitely many non-isomorphic indecomposable Λ-modules of dimension d, none of which are $(\Omega_{\Lambda}^t \circ F)$-periodic.

Proof. That M is not $(\Omega_{\Lambda}^t \circ F)$-periodic is an immediate consequence of Theorem 3.2. Now the modules $\{\Omega_{\Lambda}^{tn}(F^n(M))\}_{n=0}^{\infty}$ are all pairwise non-isomorphic, and not $(\Omega_{\Lambda}^t \circ F)$-periodic. Furthermore, from Lemma 3.4 we see that $\{\dim_k \Omega_{\Lambda}^{tn}(F^n(M))\}_{n=0}^{\infty}$ is bounded. ∎

Applying this theorem to the Nakayama automorphism of a selfinjective algebra, we obtain the following result, which provides infinitely many modules of the same dimension, none of which are τ-periodic.

COROLLARY 3.7. *Let Λ be selfinjective, and suppose there exists an indecomposable Λ-module M such that*:

    (i) $\operatorname{cx} M = 1$,
    (ii) $\operatorname{Ext}_\Lambda^{2*}(\mathcal{N}^*(M), M)/I$ *is not a polynomial ring, where I is the ideal generated by the homogeneous nilpotent elements.*

Then M is not τ-periodic, and there exists a positive integer d and infinitely many non-isomorphic indecomposable Λ-modules of dimension d, none of which are τ-periodic.

Recall that an algebra is of *wild representation type* if its representation theory is at least as complicated as the classification of finite-dimensional vector spaces together with two non-commuting endomorphisms (see [Ben], [C-B] or [SiS] for a precise definition). We end this paper with the following application of Corollary 3.7, providing a criterion for an algebra to be of wild representation type. See also [BGL] for a related result on hereditary algebras.

COROLLARY 3.8. *Let Λ be selfinjective, and suppose there exists an indecomposable Λ-module M such that*:

    (i) $\operatorname{cx} M = 1$,
    (ii) $\operatorname{Ext}_\Lambda^{2*}(\mathcal{N}^*(M), M)/I$ *is not a polynomial ring, where I is the ideal generated by the homogeneous nilpotent elements.*

Then Λ has wild representation type.

Proof. By Corollary 3.7 and [C-B, Theorem D], the algebra is not of tame representation type, hence it is wild. ∎

REFERENCES

[ARS] M. Auslander, I. Reiten and S. O. Smalø, *Representation Theory of Artin Algebras*, Cambridge Stud. Adv. Math. 36, Cambridge Univ. Press, 1995.
[BGL] D. Baer, W. Geigle and H. Lenzing, *The preprojective algebra of a tame hereditary Artin algebra*, Comm. Algebra 15 (1987), 425–457.
[Ben] D. J. Benson, *Representations and Cohomology*, Vol. I, Cambridge Stud. Adv. Math. 30, Cambridge Univ. Press, 1991.
[Ber] P. A. Bergh, *Twisted support varieties*, J. Pure Appl. Algebra 212 (2008), 753–773.
[C-B] W. W. Crawley-Boevey, *On tame algebras and bocses*, Proc. London Math. Soc. 56 (1988), 451–483.
[GSS] E. Green, N. Snashall and Ø. Solberg, *The Hochschild cohomology ring of a selfinjective algebra of finite representation type*, Proc. Amer. Math. Soc. 131 (2003), 3387–3393.

[Len]   H. Lenzing, *Wild canonical algebras and rings of automorphic forms*, in: Finite-Dimensional Algebras and Related Topics (Ottawa, ON, 1992), NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci. 424, Kluwer, Dordrecht, 1994, 191–212.

[Po1]   Z. Pogorzały, *Invariance of Hochschild cohomology algebras under stable equivalences of Morita type*, J. Math. Soc. Japan 53 (2001), 913–918.

[Po2]   —, *A new invariant of stable equivalences of Morita type*, Proc. Amer. Math. Soc. 131 (2003), 343–349.

[Po3]   —, *On the Auslander–Reiten periodicity of self-injective algebras*, Bull. London Math. Soc. 36 (2004), 156–168.

[Po4]   —, *Auslander–Reiten orbit algebras for self-injective Nakayama algebras*, Algebra Colloq. 12 (2005), 351–360.

[Sc1]   R. Schulz, *Boundedness and periodicity of modules over QF rings*, J. Algebra 101 (1986), 450–469.

[Sc2]   —, *Finite generation of the extension algebra* $\mathrm{Ext}_R^*(M, M)$, J. Austral. Math. Soc. Ser. A 59 (1995), 366–374.

[SiS]   D. Simson and A. Skowroński, *Elements of the Representation Theory of Associative Algebras*, Vol. 3, London Math. Soc. Student Texts 72, Cambridge Univ. Press, 2007.

Institutt for matematiske fag
NTNU
N-7491 Trondheim, Norway
E-mail: bergh@math.ntnu.no