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ADDENDUM TO "NECESSARY CONDITION FOR KOSTYUCHENKO TYPE SYSTEMS TO BE A BASIS IN LEBESGUE SPACES"

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Abstract. It is well known that if $\varphi(t) \equiv t$, then the system $\{\varphi^n(t)\}_{n=0}^{\infty}$ is not a Schauder basis in $L_2[0,1]$. It is natural to ask whether there is a function φ for which the power system $\{\varphi^n(t)\}_{n=0}^{\infty}$ is a basis in some Lebesgue space L_p. The aim of this short note is to show that the answer to this question is negative.

1. Introduction. It is well known that if $\varphi(t) \equiv t$, then the system $\{\varphi^n(t)\}_{n=0}^{\infty}$ is not a Schauder basis in $L_2[0,1]$ (see, for example, [AG, p. 52]). It is natural to ask whether there is a function φ for which the power system $\{\varphi^n(t)\}_{n=0}^{\infty}$ is a basis in some Lebesgue space L_p. The aim of this short note is to demonstrate that the answer to this question is a direct consequence of the author's paper [Sh].

2. Main result

THEOREM. *Let $\varphi(t)$ be any measurable, a.e. finite function on $[a,b]$. Then $\{\varphi^n(t)\}_{n=0}^{\infty}$ is not a basis in $L_p[a,b]$.*

Proof. Assume the contrary: $\{\varphi^n(t)\}_{n=0}^{\infty}$ is a basis in $L_p[a,b]$ for some $p \geq 1$. Then every $f \in L_p[a,b]$ has a unique expansion (in L_p norm)

$$f(t) = a_0 + a_1\varphi(t) + \cdots + a_n\varphi^n(t) + \cdots. \tag{1}$$

By [Sh, Theorem 4.1] we have $|\varphi(t)| = c = \text{const}$ a.e. on $[a,b]$. The case $c = 0$ is trivial. If $c \neq 0$, then the mapping $f \mapsto \varphi(t)f$ is surjective. Therefore multiplying (1) by $\varphi(t)$ we see that $\varphi^0(t) \equiv 1$ has at least two different representations of the form (1), which implies that $\{\varphi^n(t)\}_{n=0}^{\infty}$ is not a basis.

This contradiction proves the theorem.

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