COLLOQUIUM MATHEMATICUM

VOL. 100 2004 NO. 2

A NEW VERSION OF LOCAL-GLOBAL PRINCIPLE FOR ANNIHILATIONS OF LOCAL COHOMOLOGY MODULES

BY

K. KHASHYARMANESH (Tehran), M. YASSI (Mashhad) and A. ABBASI (Mashhad)

Abstract. Let R be a commutative Noetherian ring. Let $\mathfrak{a}$ and $\mathfrak{b}$ be ideals of R and let N be a finitely generated R-module. We introduce a generalization of the $\mathfrak{b}$-finiteness dimension of $f_{\mathfrak{a}}^{\mathfrak{b}}(N)$ relative to $\mathfrak{a}$ in the context of generalized local cohomology modules as

$$f_{\mathfrak{a}}^{\mathfrak{b}}(M,N) := \inf\{i \geq 0 \mid \mathfrak{b} \subseteq \sqrt{(0 :_R H_{\mathfrak{a}}^i(M,N))}\,\},$$

where M is an R-module. We also show that $f_{\mathfrak{a}}^{\mathfrak{b}}(N) \leq f_{\mathfrak{a}}^{\mathfrak{b}}(M,N)$ for any R-module M. This yields a new version of the Local-Global Principle for annihilation of local cohomology modules. Moreover, we obtain a generalization of the Faltings Lemma.

1. Introduction. Let R be a commutative Noetherian ring, let N be a finitely generated R-module, let $\mathfrak{a}$ and $\mathfrak{b}$ be ideals of R and let s be a positive integer. We use $\mathbb{N}$ and $\mathbb{N}_0$ to denote the sets of positive and non-negative integers respectively.

Consider the following statement:

$$(\dagger) \qquad f_{\mathfrak{a}R_{\mathfrak{p}}}^{\mathfrak{b}R_{\mathfrak{p}}}(N_{\mathfrak{p}}) > s \text{ for all } \mathfrak{p} \in \operatorname{Spec}(R) \;\Leftrightarrow\; f_{\mathfrak{a}}^{\mathfrak{b}}(N) > s,$$

where $f_{\mathfrak{a}}^{\mathfrak{b}}(N)$ denotes the $\mathfrak{b}$-finiteness dimension of N relative to $\mathfrak{a}$. We say that the *Local-Global Principle* (for annihilation of local cohomology modules) holds at level s for R-module N if statement $(\dagger)$ holds for any ideals $\mathfrak{a}$ and $\mathfrak{b}$ of R.

This Local-Global Principle has been investigated by Faltings in [7, Satz 1], in the particular case where the ideals $\mathfrak{a}$ and $\mathfrak{b}$ coincide, and in this case he has proved that the Principle holds at all levels $s \in \mathbb{N}$. In [14], Raghavan showed that the Local-Global Principle holds at level 1; he deduced from the Faltings Annihilator Theorem [6] that if R is a homomorphic image of a regular (commutative Noetherian) ring, then the Principle holds at all levels $s \in \mathbb{N}$. Furthermore, Brodmann, Rotthaus and Sharp [4] established

2000 *Mathematics Subject Classification*: 13D45, 13E5.

Key words and phrases: local cohomology module, generalized local cohomology module, filter regular sequence.

The first author was partially supported by a grant from the Institute for Studies in Theoretical Physics and Mathematics (IPM), Iran (No. 82130030).

the Principle over an arbitrary commutative ring at level 2. They have also established it at all levels over an arbitrary commutative Noetherian ring of dimension not exceeding 4. They explored interrelations between the Local-Global Principle and the Annihilator Theorem for local cohomology, and showed that if R is universally catenary and all formal fibers of all localizations of R satisfy a certain Serre condition, then the Annihilator Theorem for local cohomology holds at level s over R if and only if the Local-Global Principle for annihilation of local cohomology modules holds at level s over R.

To specify our main results, let us begin with a definition which is a natural generalization of the $\mathfrak{b}$-finiteness dimension $f_{\mathfrak{a}}^{\mathfrak{b}}(N)$ of N relative to $\mathfrak{a}$ in the context of generalized local cohomology modules.

Suppose that $\mathfrak{a}$ and $\mathfrak{b}$ are ideals of R and that $M,\ N$ are R-modules. Then we define

$$\begin{aligned} f_{\mathfrak{a}}^{\mathfrak{b}}(M,N) &:= \inf\{i \in \mathbb{N}_0 \mid \mathfrak{b} \nsubseteq \sqrt{(0 :_R H_{\mathfrak{a}}^i(M,N))}\} \\ &= \inf\{i \in \mathbb{N}_0 \mid \mathfrak{b}^n H_{\mathfrak{a}}^i(M,N) \neq 0 \text{ for all } n \in \mathbb{N}\}. \end{aligned}$$

We will show that $f_{\mathfrak{a}}^{\mathfrak{b}}(N) \leq f_{\mathfrak{a}}^{\mathfrak{b}}(M,N)$ for every R-module M. This leads to a new version of the Local-Global Principle as follows:

$$(\ddagger) \qquad f_{\mathfrak{a}R_{\mathfrak{p}}}^{\mathfrak{b}R_{\mathfrak{p}}}(N_{\mathfrak{p}}) > s \text{ for all } \mathfrak{p} \in \operatorname{Spec}(R) \quad \Leftrightarrow \quad f_{\mathfrak{a}}^{\mathfrak{b}}(M,N) > s \text{ for any } R\text{-module } M.$$

Hence, statement (†) implies (‡) and vice versa. Also, statement (‡) enables one to extend the Local-Global Principle (see for example 2.5 and 2.8). Our results yield a generalization of the Faltings Lemma (see 2.9). Finally, we obtain a generalization of the main result of [3] and [11] (see 2.10).

Throughout this note, let R be a Noetherian ring, let $\mathfrak{a}$ and $\mathfrak{b}$ be ideals of R, let $M,\ N$ be R-modules, and suppose N is finitely generated. If $i \in \mathbb{N}_0$ we write $H_{\mathfrak{a}}^i(N)$ for the ith local cohomology module of N with respect to $\mathfrak{a}$. For convenience, we write $H_{\mathfrak{a}}^j(N) = 0$ whenever j is a negative integer. For unexplained terminology we refer to [5].

2. Finiteness properties of generalized local cohomology modules. First of all, we mention a generalization of the concept of regular sequence which is needed in this paper.

A sequence $x_1, \ldots, x_n$ of elements of an ideal $\mathfrak{a}$ is said to be an $\mathfrak{a}$-*filter regular sequence* on N if

$$\operatorname{Supp}_R\left(\frac{(x_1, \ldots, x_{i-1})N :_N x_i}{(x_1, \ldots, x_{i-1})N}\right) \subseteq V(\mathfrak{a})$$

for all $i = 1, \ldots, n$, where $V(\mathfrak{a})$ denotes the set of all prime ideals of R containing $\mathfrak{a}$. This concept is a generalization of the one of a filter regular sequence which has been studied in [15], [16] and [8], and has led to some interesting results. Note that both concepts coincide if $\mathfrak{a}$ is the maximal ideal of a local ring. Also note that $x_1, \ldots, x_n$ is a weak sequence on N if and only if it is an R-filter regular sequence on N. It is easy to see that the analogue of [16, Appendix 2(ii)] holds for the ideal $\mathfrak{a}$ whenever R is Noetherian and N is finitely generated; so that, if $x_1, \ldots, x_n$ is an $\mathfrak{a}$-filter regular sequence on N, then there is an element $y \in \mathfrak{a}$ such that $x_1, \ldots, x_n, y$ is an $\mathfrak{a}$-filter regular sequence on N. Thus, for a positive integer n, there exists an $\mathfrak{a}$-filter regular sequence on N of length n.

PROPOSITION 2.1 (see [12, 3.4] and [10, 1.2]). *Suppose that $n \geq 1$ and that $x_1, \ldots, x_n$ is an $\mathfrak{a}$-filter regular sequence on N. Then*

(i) $H^i_{\mathfrak{a}}(N) \cong H^i_{(x_1,\ldots,x_n)}(N)$ *whenever* $0 \leq i < n$.
(ii) $H^i_{\mathfrak{a}}(N) \cong H^{i-n}_{\mathfrak{a}}(H^n_{(x_1,\ldots,x_n)}(N))$ *whenever* $i \geq n$.

The definition of the $\mathfrak{b}$-finiteness dimension $f^{\mathfrak{b}}_{\mathfrak{a}}(N)$ of N relative to $\mathfrak{a}$ (cf. [5, 9.1.5]) provides some motivation for the following definition. Here, we adopt the convention that the infimum of the empty set of integers is ∞.

DEFINITION 2.2. Given two ideals $\mathfrak{a}$ and $\mathfrak{b}$ of R and two R-modules M and N, we set

$$\begin{aligned} f^{\mathfrak{b}}_{\mathfrak{a}}(M,N) &:= \inf\{i \in \mathbb{N}_0 \mid \mathfrak{b} \nsubseteq \sqrt{(0 :_R H^i_{\mathfrak{a}}(M,N))}\} \\ &= \inf\{i \in \mathbb{N}_0 \mid \mathfrak{b}^n H^i_{\mathfrak{a}}(M,N) \neq 0 \text{ for all } n \in \mathbb{N}\}. \end{aligned}$$

LEMMA 2.3. *Suppose that $\mathfrak{a}$ and $\mathfrak{b}$ are ideals of R, and N is a finitely generated R-module. Then, for every R-module M, $f^{\mathfrak{b}}_{\mathfrak{a}}(N) \leq f^{\mathfrak{b}}_{\mathfrak{a}}(M,N)$. Moreover, if $f^{\mathfrak{b}}_{\mathfrak{a}}(N)$ is infinite, then so also is $f^{\mathfrak{b}}_{\mathfrak{a}}(M,N)$.*

Proof. Let M be an R-module and $t \in \mathbb{N}_0$. It is enough to show that $f^{\mathfrak{b}}_{\mathfrak{a}}(M,N) \geq t$ whenever $f^{\mathfrak{b}}_{\mathfrak{a}}(N) \geq t$. We proceed by induction on t.

To begin, note that the case $t = 0$ is clear. Now suppose, inductively, that $t > 0$ and the result is proved for non-negative integers less than t. So, there exists $u \in \mathbb{N}$ such that $\mathfrak{b}^u H^i_{\mathfrak{a}}(N) = 0$ for all $i = 0, \ldots, t-1$. Our inductive hypothesis ensures that there exists $v \in \mathbb{N}$ such that $\mathfrak{b}^v H^i_{\mathfrak{a}}(M,N) = 0$ for all $i = 0, \ldots, t-2$. Thus it remains to show that $H^{t-1}_{\mathfrak{a}}(M,N)$ is annihilated by some power of $\mathfrak{b}$.

Let $x_1, \ldots, x_t$ be an $\mathfrak{a}$-filter regular sequence on N (the existence of such sequences is shown at the beginning of this section). Put $S_0 := N$ and $S_i := H^i_{(x_1,\ldots,x_i)}(N)$ for all $i = 1, \ldots, t$. By [5, 2.2.17], for all $i = 0, \ldots, t-1$, we obtain the exact sequence

$$0 \to H^0_{(x_{i+1})}(S_i) \to S_i \to (S_i)_{x_{i+1}} \to H^1_{(x_{i+1})}(S_i) \to 0.$$

Now, using [5, 2.1.9], in the light of 2.1, we have the isomorphisms

$$H^0_{(x_{i+1})}(S_i) \cong H^0_{(x_1,\dots,x_{i+1})}(S_i) \cong H^i_{(x_1,\dots,x_{i+1})}(N) \cong H^i_{\mathfrak{a}}(N),$$
$$H^1_{(x_{i+1})}(S_i) \cong H^{i+1}_{(x_1,\dots,x_{i+1})}(N) = S_{i+1}.$$

Summing up, we deduce the exact sequence

$$0 \to H^i_{\mathfrak{a}}(N) \to S_i \xrightarrow{f_i} (S_i)_{x_{i+1}} \to S_{i+1} \to 0,$$

and hence the exact sequences

$$(*) \qquad 0 \to H^i_{\mathfrak{a}}(N) \to S_i \to \operatorname{Im} f_i \to 0,$$
$$(**) \qquad 0 \to \operatorname{Im} f_i \to (S_i)_{x_{i+1}} \to S_{i+1} \to 0,$$

for all $i = 0, \dots, t-1$. We use the facts that the R-module $H^j_{\mathfrak{a}}(M, (S_i)_{x_{i+1}})$ is $\mathfrak{a}$-torsion for $j \in \mathbb{N}_0$, and multiplication by x_{i+1} provides an automorphism of $(S_i)_{x_{i+1}}$. Applying the functor $H^j_{\mathfrak{a}}(M, \cdot)$ to $(**)$ in conjunction with the above properties ensures that $H^j_{\mathfrak{a}}(M, S_{i+1}) \cong H^{j+1}_{\mathfrak{a}}(M, \operatorname{Im} f_i)$ for all $j \in \mathbb{N}_0$. Therefore, by applying again $H^j_{\mathfrak{a}}(M, \cdot)$ to $(*)$, we get the exact sequence

$$H^j_{\mathfrak{a}}(M, H^i_{\mathfrak{a}}(N)) \to H^j_{\mathfrak{a}}(M, S_i) \to H^{j-1}_{\mathfrak{a}}(M, S_{i+1})$$

for all $j \in \mathbb{N}$. Now, by [5, 9.1.1] and the telescoping method for the above exact sequence, it is enough to show that $H^0_{\mathfrak{a}}(M, S_{t-1})$ is annihilated by some power of $\mathfrak{b}$. To this end, apply the functor $H^0_{\mathfrak{a}}(M, \cdot)$ to the exact sequence

$$0 \to H^{t-1}_{\mathfrak{a}}(N) \to S_{t-1} \xrightarrow{f_{t-1}} (S_{t-1})_{x_t}$$

to obtain an isomorphism $H^0_{\mathfrak{a}}(M, S_{t-1}) \cong H^0_{\mathfrak{a}}(M, H^{t-1}_{\mathfrak{a}}(N))$. Also it is well known that $H^0_{\mathfrak{a}}(M, H^{t-1}_{\mathfrak{a}}(N)) \cong H^0_{\mathfrak{a}}(\operatorname{Hom}_R(M, H^{t-1}_{\mathfrak{a}}(N)))$. Hence, the result follows since $H^{t-1}_{\mathfrak{a}}(N)$ is annihilated by some power of $\mathfrak{b}$.

We are now in a position to present one of the main theorems which is an immediate consequence of Lemma 2.3.

THEOREM 2.4. *Assume that R is a Noetherian ring, $\mathfrak{a}$ and $\mathfrak{b}$ ideals of R, N a finitely generated R-module, and s a positive integer. The following statements are equivalent*:

(†) $f^{\mathfrak{b}R_{\mathfrak{p}}}_{\mathfrak{a}R_{\mathfrak{p}}}(N_{\mathfrak{p}}) > s$ *for all* $\mathfrak{p} \in \operatorname{Spec}(R)$ *if and only if* $f^{\mathfrak{b}}_{\mathfrak{a}}(N) > s$;

(‡) $f^{\mathfrak{b}R_{\mathfrak{p}}}_{\mathfrak{a}R_{\mathfrak{p}}}(N_{\mathfrak{p}}) > s$ *for all* $\mathfrak{p} \in \operatorname{Spec}(R)$ *if and only if* $f^{\mathfrak{b}}_{\mathfrak{a}}(M, N) > s$ *for any* R-module M.

Theorem 2.4 shows that in studying statement (‡), it suffices to examine the Local-Global Principle for annihilation of local cohomology modules. For instance, we have the following consequences (see [4]).

COROLLARY 2.5. *Statement* (‡) *holds in the following cases*:

(i) $\dim R$ *is finite and* $s \geq \dim R - 1$.
(ii) $\dim R \leq 4$.
(iii) $s = 1$ or $s = 2$.

REMARK 2.6. Suppose that $k, n \in \mathbb{N}$ are such that $\mathfrak{b}^k H^i_{\mathfrak{a}}(N) = 0$ for all $i = 0, \ldots, n-1$. By a little work on the proof of 2.3, one can conclude that $\mathfrak{b}^{kn} H^i_{\mathfrak{a}}(M, N) = 0$ for all $i = 0, \ldots, n-1$. This gives an analogue to what Raghavan [13] calls "uniform annihilation of local cohomology"; we are going to present it in Corollary 2.8 below.

Now we recall the definition of the $\mathfrak{b}$-minimum $\mathfrak{a}$-adjusted depth $\lambda^{\mathfrak{b}}_{\mathfrak{a}}(N)$ of N and we introduce some notations (see [13, 2.1]).

DEFINITION 2.7. Let $\mathfrak{a}$ and $\mathfrak{b}$ be ideals of R, and N be a finitely generated R-module. Let $D(\mathfrak{b})$ denote the set of all prime ideals of R which do not contain $\mathfrak{b}$. Define

$$\lambda^{\mathfrak{b}}_{\mathfrak{a}}(N) = \min_{\mathfrak{p} \in D(\mathfrak{b})} \{\operatorname{depth}(N_{\mathfrak{p}}) + \operatorname{ht}(\mathfrak{a} + \mathfrak{p}/\mathfrak{p})\}.$$

The depth of the zero module and the height of the unit ideal are ∞ by convention. If $D(\mathfrak{b})$ is empty, then define $\lambda^{\mathfrak{b}}_{\mathfrak{a}}(N) = \infty$.

COROLLARY 2.8 (Uniform annihilation of generalized local cohomology). *Let R be a homomorphic image of a biequidimensional regular ring of finite Krull dimension, and N be a finitely generated R-module. Then there exists an integer k (depending only on N) such that, given any two ideals $\mathfrak{a}$ and $\mathfrak{b}$ of R, an R-module M, and an integer j less than $\lambda^{\mathfrak{b}}_{\mathfrak{a}}(N)$, we have $\mathfrak{b}^k H^j_{\mathfrak{a}}(M, N) = 0$.*

Proof. Apply [13, 3.1] and Remark 2.6.

One of the interesting results in the theory of local cohomology is the Faltings Lemma [6, Lemma 3]. The following theorem provides a generalization of the Faltings Lemma in the context of generalized local cohomology modules.

THEOREM 2.9. *Let R be a Noetherian ring and $\mathfrak{a}$ be an ideal of R. Let M, N be finitely generated R-modules, and $t \in \mathbb{N}_0$. Then the following statements are equivalent:*

(a) *$H^i_{\mathfrak{a}}(M, N)$ is finitely generated for all $i \leq t$;*
(b) *$H^i_{\mathfrak{a}R_{\mathfrak{p}}}(M_{\mathfrak{p}}, N_{\mathfrak{p}})$ is finitely generated for all $i \leq t$ and all $\mathfrak{p} \in \operatorname{Spec}(R)$;*
(c) *there exists $k \in \mathbb{N}$ such that $\mathfrak{a}^k H^i_{\mathfrak{a}}(M, N) = 0$ for all $i \leq t$.*

Proof. The implication (a)⇒(b) is clear.
(c)⇒(a). Consider the exact sequence

$$0 \to \Gamma_{\mathfrak{a}}(N) \to N \to N/\Gamma_{\mathfrak{a}}(N) \to 0$$

yielding the exact sequence

$$H^i_{\mathfrak{a}}(M, \Gamma_{\mathfrak{a}}(N)) \to H^i_{\mathfrak{a}}(M, N) \to H^i_{\mathfrak{a}}(M, N/\Gamma_{\mathfrak{a}}(N)) \to H^{i+1}_{\mathfrak{a}}(M, \Gamma_{\mathfrak{a}}(N))$$

for each $i \in \mathbb{N}_0$. First, since $\Gamma_{\mathfrak{a}}(N)$ is finitely generated, by [1, 1.1], so is $H^i_{\mathfrak{a}}(M, \Gamma_{\mathfrak{a}}(N))$, and hence, it is annihilated by some power of $\mathfrak{a}$ for all $i \in \mathbb{N}_0$.

Thus, in view of [5, 9.1.1], we can replace N by $N/\Gamma_{\mathfrak{a}}(N)$ and assume that there exists $x \in \mathfrak{a}$ which is a non-zerodivisor on N. Therefore, if we consider the long exact sequence of generalized local cohomology modules induced by the exact sequence $0 \to N \xrightarrow{x^k} N \to N/x^kN \to 0$, we see that there exists $u \in \mathbb{N}$ such that $\mathfrak{a}^u H^i_{\mathfrak{a}}(M, N/x^kN) = 0$ for all $i < t$. Another use of that short exact sequence shows that $H^{i+1}_{\mathfrak{a}}(M,N)$ is a homomorphic image of $H^i_{\mathfrak{a}}(M, N/x^kN)$ for all $i < t$, and so it is finitely generated.

(b)$\Rightarrow$(c). We argue by induction on t. When $t = 0$, the result is clear. We therefore suppose, inductively, that $t > 0$ and that the result has been proved for smaller values of t. By inductive hypothesis, there exists $k \in \mathbb{N}$ such that $\mathfrak{a}^k H^i_{\mathfrak{a}}(M,N) = 0$ for all $i < t$. Hence, by the implication (c)$\Rightarrow$(a), $H^i_{\mathfrak{a}}(M,N)$ is finitely generated for all $i < t$. So, by [1, 1.3], $\operatorname{Ass}_R H^t_{\mathfrak{a}}(M,N)$ is finite. The result now immediately follows from [4, 2.1].

The following corollary is a generalization of the main results of [3] and [11].

Corollary 2.10. *Let M and N be finitely generated R-modules and $\mathfrak{a}$ be an ideal of R. Let n be a positive integer such that $H^i_{\mathfrak{a}}(N)$ is finitely generated for all $i = 0, 1, \dots, n-1$. Then* $\operatorname{Hom}_R(R/\mathfrak{a}, H^n_{\mathfrak{a}}(M,N))$ *is finitely generated and the set of associated prime ideals of $H^n_{\mathfrak{a}}(M,N)$ is finite.*

Proof. Apply [5, 9.1.2], Lemma 2.3, Theorem 2.9 and [1, 1.2 and 1.3].

We also note that Theorem 2.9 provides some motivation for definition of the finiteness dimension of generalized local cohomology modules.

Definition 2.11. Let $\mathfrak{a}$ be an ideal of R, and M, N be finitely generated R-modules. Define the *finiteness dimension* $f_{\mathfrak{a}}(M,N)$ of (M,N) *relative to* $\mathfrak{a}$ by

$$\begin{aligned} f_{\mathfrak{a}}(M,N) &:= \inf\{i \in \mathbb{N} \mid H^i_{\mathfrak{a}}(M,N) \text{ is not finitely generated}\} \\ &= \inf\{i \in \mathbb{N} \mid \mathfrak{a} \nsubseteq \sqrt{(0 :_R H^i_{\mathfrak{a}}(M,N))}\} \\ &= \inf\{i \in \mathbb{N} \mid \mathfrak{a}^n H^i_{\mathfrak{a}}(M,N) \neq 0 \text{ for all } n \in \mathbb{N}\}. \end{aligned}$$

Note that $f_{\mathfrak{a}}(M,N) = f^{\mathfrak{a}}_{\mathfrak{a}}(M,N)$ and that

$$f_{\mathfrak{a}}(N) = \inf\{i \in \mathbb{N} \mid H^i_{\mathfrak{a}}(N) \text{ is not finitely generated}\} = f_{\mathfrak{a}}(R,N).$$

Finally, we obtain the following proposition.

Proposition 2.12. *Suppose that R is a Noetherian ring, $\mathfrak{a}$ is an ideal of R, and M, N are finitely generated R-modules. Then $f_{\mathfrak{a}}(N) \le f_{\mathfrak{a}}(M,N)$.*

Acknowledgments. We are especially grateful to the referee for his/her comments regarding this paper and to Professor Daniel Simson for sending some worthwhile papers.

REFERENCES

[1] J. Asadollahi, K. Khashyarmanesh and Sh. Salarian, *On the finiteness properties of generalized local cohomology modules*, Comm. Algebra 30 (2002), 859–867.

[2] M. H. Bijan-Zadeh, *A common generalization of local cohomology theories*, Glasgow Math. J. 21 (1980), 173–181.

[3] M. Brodmann and A. Lashgari Faghani, *A finiteness result for associated primes of local cohomology modules*, Proc. Amer. Math. Soc. 128 (2000), 2851–2853.

[4] M. Brodmann, Ch. Rotthaus and R. Y. Sharp, *On annihilators and associated primes of local cohomology modules*, J. Pure Appl. Algebra 153 (2000), 197–227.

[5] M. Brodmann and R. Y. Sharp, *Local Cohomology—An Algebraic Introduction with Geometric Applications*, Cambridge Stud. Adv. Math. 60, Cambridge Univ. Press, 1998.

[6] G. Faltings, *Über die Annulatoren lokaler Kohomologiegruppen*, Arch. Math. (Basel) 30 (1978), 473–476.

[7] —, *Der Endlichkeitssatz in der lokalen Kohomologie*, Math. Ann. 255 (1981), 45–56.

[8] S. Goto and K.Yamagishi, *The theory of unconditioned strong d-sequences and modules of finite local cohomology*, preprint.

[9] J. Herzog, *Komplexe, Auflösungen und dualität in der lokalen Algebra*, preprint, Univ. Regensburg, 1974.

[10] K. Khashyarmanesh and Sh. Salarian, *Filter regular sequences and the finiteness of local cohomology modules*, Comm. Algebra 26 (1998), 2483–2490.

[11] —, —, *On the associated primes of local cohomology modules*, ibid. 27 (1999), 6191–6198.

[12] U. Nagel and P. Schenzel, *Cohomological annihilators and Castelnuovo–Mumford regularity*, in: Commutative Algebra: Syzygies, Multiplicities, and Birational Algebra (South Hadley, MA, 1992), Contemp. Math. 159, Amer. Math. Soc., Providence, RI, 1994, 307–328.

[13] K. Raghavan, *Uniform annihilation of local cohomology and of Koszul homology*, Math. Proc. Cambridge Philos. Soc. 112 (1992), 487–494.

[14] —, *Local-Gobal Principle of local cohomology*, in: Commutative Algebra: Syzygies, Multiplicities, and Birational Algebra (South Hadley, MA, 1992), Contemp. Math. 159, Amer. Math. Soc., 1994, 329–331.

[15] P. Schenzel, N. V. Trung und N. T. Cuong, *Verallgemeinerte Cohen–Macaulay-Moduln*, Math. Nachr. 85 (1978), 57–73.

[16] J. Stückrad and W. Vogel, *Buchsbaum Rings and Applications*, Springer, Berlin, 1986.

[17] N. Suzuki, *On the generalized local cohomology and its duality*, J. Math. Kyoto Univ. 18 (1978), 71–85.

Institute for Studies in
Theoretical Physics and Mathematics
P.O. Box 19395-5746
Tehran, Iran
E-mail: khashyar@ipm.ir

Department of Mathematics
Mashhad University
P.O. Box 1159-91775
Mashhad, Iran
E-mail: aabbasi@guilan.ac.ir

Received 8 January 2003;
revised 2 June 2004 (4303)