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F_σ-additive covers of Čech complete and scattered-K-analytic spaces

by

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Abstract. We prove that an F_σ-additive cover of a Čech complete, or more generally scattered-K-analytic space, has a σ-scattered refinement. This generalizes results of G. Koumoullis and R. W. Hansell.

1. Introduction. The main goal of our paper is an extension of results of A. G. El'kin, G. Koumoullis and R. W. Hansell (see [1], [5, Theorem 2.1] and [9, Theorem 2]) to nonmetrizable topological spaces.

A. G. El'kin [1] showed that an absolute Suslin metric space is either discrete or contains a perfect compact subset. (We recall that a metric space is *absolute Suslin* if it is homeomorphic to a Suslin subset of a complete metric space.) G. Koumoullis improved this result in the following way: a disjoint cover $\mathcal{A}$ of an absolute Suslin metric space Y consisting of F_σ-sets is either σ-discretely decomposable or there exists a compact set $K \subset Y$ which meets uncountably many sets of $\mathcal{A}$. As a corollary, any disjoint F_σ-additive cover of an absolute Suslin metric space has a σ-discrete refinement (see [5, Section 3]).

Later on, R. W. Hansell [5, Theorem 2.1] generalized this result for point-countable families of F_σ-sets by proving that if $\mathcal{A}$ is a point-countable cover of an absolute Suslin metric space Y consisting of F_σ-sets, then either $\mathcal{A}$ has a σ-discrete refinement or there exists a compact set $K \subset Y$ which is not covered by any countable subfamily of $\mathcal{A}$. It follows that a point-countable F_σ-additive cover of an absolute Suslin metric space has a σ-discrete refinement (see [5, Theorem 3.3]).

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In Theorem 3.4 we are able to get rid of the assumption of metrizability. Namely, we prove that an F_σ-additive cover of a Čech complete space has a σ-scattered refinement.

This result is further generalized in Theorem 4.3, where the same is proved for an F_σ-additive cover of a scattered-K-analytic space.

Nevertheless, these results for topological spaces are not completely satisfactory since the notion of F_σ-sets is much more special than within metric spaces. A natural generalization of F_σ-sets are $(F \wedge G)_\sigma$-*sets*, i.e., sets of the form $\bigcup_n (F_n \cap G_n)$ where each F_n is closed and G_n open. Unfortunately, the method of proof of Theorem 3.4 does not seem to work for this class of sets.

Partial results under additional set-theoretical assumptions were obtained by R. Pol (see [10, Theorem 1]) and P. Holický (see [6, Theorem 1]).

2. Preliminaries. By a *space* we mean a completely regular Hausdorff topological space.

Let $\mathcal{F}$ be a family of sets in a topological space X. A family $\mathcal{R}$ is a *refinement* of $\mathcal{F}$ if $\bigcup \mathcal{R} = \bigcup \mathcal{F}$ and for every $R \in \mathcal{R}$ there exists $F \in \mathcal{F}$ with $R \subset F$.

A family $\mathcal{F}$ is called *point-countable* if each $x \in X$ lies in at most countably many sets from $\mathcal{F}$.

If $\mathcal{S}$ is a system of sets in X, we say that $\mathcal{F}$ is $\mathcal{S}$-*additive* if $\bigcup \mathcal{F}_0$ is in $\mathcal{S}$ for every subfamily $\mathcal{F}_0$ of $\mathcal{F}$.

A family $\mathcal{D}$ in a topological space X is *scattered* if it is disjoint and for every nonempty subfamily $\mathcal{D}_0$ of $\mathcal{D}$, $\mathcal{D}$ contains an element that is relatively open in $\bigcup \mathcal{D}_0$ (see [4, Definition 6.1]). If $\mathcal{F}$ is a scattered family of sets, then there is a well-ordering $\le$ of $\mathcal{F}$ and open sets $U(F)$, $F \in \mathcal{F}$, such that

$$U(F) \cap \bigcup \mathcal{F} = \bigcup \{E \in \mathcal{F} : E \le F\}.$$

We call the family $\{U(F) : F \in \mathcal{F}\}$ the *associated open sets for* $\mathcal{F}$.

An indexed family $\mathcal{F} = \{F_i : i \in I\}$ is called σ-*scattered resolvable* if each E_i is the union of a family $\{F_i(n,l) : n \in \mathbb{N},\, l \in J(n,i)\}$ such that $\{F_i(n,l) : i \in I,\, l \in J(n,i)\}$ is scattered for each $n \in \mathbb{N}$. We may suppose that the index sets $J(n,i)$ are all equal (see [8, p. 3]). We remark that the notions of σ-scattered resolvable family and of σ-scattered-decomposable family defined in [4, Definition 6.6] are equivalent.

A set-valued mapping $f : X \to Y$ between topological spaces is said to be *index-σ-scattered* if $\{f(F_i) : i \in I\}$ is σ-scattered resolvable in Y whenever $\{F_i : i \in I\}$ is σ-scattered resolvable in X.

A topological space X is called *Čech complete* if X is a G_δ-subset of its Stone–Čech compactification βX (see [2, Theorem 3.9.1]).

A topological space X is *scattered-K-analytic* if X is the image of a complete metric space M under an usco index-σ-scattered map $f : M \to X$

(see [3, p. 11], [4, Definition 6.7] and [7, Definition 1]). (We recall that a set-valued map $f : X \to Y$ between topological spaces is an *usco map* if f has nonempty compact values and

$$f^{-1}(F) = \{x \in X : f(x) \cap F \neq \emptyset\}$$

is closed in X for every closed set $F \subset Y$.)

We say that a family $\mathcal{R}$ is *σ-scattered* if $\mathcal{R} = \bigcup_n \mathcal{R}_n$ and each family $\mathcal{R}_n$ is scattered. We remark that a σ-scattered family in a metrizable space is σ-discretely decomposable due to the existence of a σ-discrete basis.

We denote by $\{0,1\}^{<\mathbb{N}}$ the space of finite sequences of 0's and 1's. Let $|s|$ be the length of s. We denote by $\emptyset$ the empty sequence, of length 0 by convention. For $s \in \{0,1\}^{<\mathbb{N}}$ and $i \in \{0,1\}$ we write $s^\wedge i$ for the sequence $(s_1, \dots, s_{|s|}, i)$.

For a sequence σ in the Cantor set $\{0,1\}^{\mathbb{N}}$ and $n \in \mathbb{N}$ we write $\sigma\restriction n$ for the finite sequence $(\sigma_1, \dots, \sigma_n)$. We adopt the convention that $\sigma\restriction 0 = \emptyset$.

If $\mathcal{F}$ is a family of sets in a space X and $A \subset X$, we denote by $\mathcal{F}\restriction_A$ the family $\{F \cap A : F \in \mathcal{F}\}$.

3. Čech complete spaces. We start with the following easy result whose proof is based upon [3, Lemma 2.2]. As the proof is rather standard, we omit it.

LEMMA 3.1. *Let $\mathcal{F} = \{F_i : i \in I\}$ be a σ-scattered family of sets in a space X. For each $i \in I$, let $\mathcal{F}_i = \{F_{i,j} : j \in J_i\}$ be a σ-scattered family contained in F_i. Then $\{F_{i,j} : i \in I,\ j \in J_i\}$ is σ-scattered.*

LEMMA 3.2. *Let $\mathcal{F}$ be a cover of a space X such that every $x \in X$ has an open neighbourhood U such that $\mathcal{F}\restriction_U$ has a σ-scattered refinement. Then $\mathcal{F}$ has a σ-scattered refinement.*

Proof. We will find by transfinite induction an ordinal κ and an increasing sequence $\{U_\alpha : \alpha \in [0,\kappa]\}$ of open sets in X such that $\mathcal{F}\restriction_{U_{\alpha+1}\setminus U_\alpha}$ has a σ-scattered refinement and $U_\kappa = X$.

We set $U_0 := \emptyset$ and find an open set U_1 such that $\mathcal{F}\restriction_{U_1}$ has a σ-scattered refinement. Let α be an ordinal and suppose that the construction has been completed for every ordinal $\beta < \alpha$. If α is a limit ordinal, set $U_\alpha := \bigcup_{\beta<\alpha} U_\beta$.

If $\alpha = \eta + 1$ and $U_\eta = X$, we stop the construction. Otherwise we use the assumption on $\mathcal{F}$ and find a nonempty open set U such that $U \cap (X \setminus U_\eta) \neq \emptyset$ and $\mathcal{F}\restriction_U$ has a σ-scattered refinement. Then we set $U_\alpha := U_\eta \cup U$. This finishes the inductive step.

Since U_α's are strictly increasing, there exists an ordinal κ such that $U_\kappa = X$. Then $\{U_{\alpha+1} \setminus U_\alpha : \alpha \in [0,\kappa)\}$ is a scattered family with associated open sets $\{U_\alpha : \alpha \in [0,\kappa]\}$ such that $\mathcal{F}\restriction_{U_{\alpha+1}\setminus U_\alpha}$ has a σ-scattered refinement for each $\alpha \in [0,\kappa)$. By Lemma 3.1, $\mathcal{F}$ has a σ-scattered refinement. ∎

Lemma 3.3. *Let $\mathcal{F}$ be a point-countable F_σ-additive cover of a Čech complete space X. Then there exists $F \in \mathcal{F}$ with nonempty interior.*

Proof. Let $\mathcal{F} = \{F_i : i \in I\}$. Suppose that each F_i has empty interior. Since X is Čech complete, it is a G_δ-set in every compactification of X. We select some compactification K of X and find open sets V_n, $n \in \mathbb{N}$, in K such that $X = \bigcap V_n$. Set $V_0 := K$.

If $A \subset I$, then $X \setminus \bigcup_{i \in A} F_i$ is a G_δ-set in X and consequently in K. Let $\{G(A,n)\}$ be a decreasing sequence of open sets in K such that

$$\bigcap_{n=1}^{\infty} G(A,n) = X \setminus \bigcup_{i \in A} F_i.$$

Notice that if $A \subset I$ is countable and $U \subset K$ is nonempty and open, then $(U \cap X) \setminus \bigcup_{i \in A} F_i \neq \emptyset$. Indeed, the union of a countable subfamily of $\mathcal{F}$ is of the first category by our assumption, and thus cannot cover a nonempty open subset of a Čech complete space (see [2, Theorem 3.9.3]).

We set $U_\emptyset := K$ and pick $x_\emptyset \in U_\emptyset \cap X$. We also set $A_\emptyset := \{i \in I : x_\emptyset \in F_i\}$ and $B_\emptyset := I \setminus A_\emptyset$. For each $s \in \{0,1\}^{<\mathbb{N}}$ we will find points $x_s \in X$, nonempty open sets $U_s \subset K$ and sets $A_s, B_s \subset I$ such that

(i) $\overline{U}_{s^\wedge 0} \cup \overline{U}_{s^\wedge 1} \subset U_s \subset V_{|s|}$, $\overline{U}_{s^\wedge 0} \cap \overline{U}_{s^\wedge 1} = \emptyset$;

(ii) $x_s \in U_s$, $x_{s^\wedge 0} = x_s$, and

$$x_{s^\wedge 1} \notin \bigcup_{k=0}^{|s|} \bigcup_{|t|=k} \{F_i : i \in A_t\};$$

(iii) $A_{s^\wedge 1} = \{i \in I : x_{s^\wedge 1} \in F_i\}$, $A_{s^\wedge 0} = A_s$ and $B_s = I \setminus A_s$;

(iv) $U_{s^\wedge 0} \subset G(B_{s^\wedge 0}, |s^\wedge 0|)$ and

$$U_{s^\wedge 1} \subset \bigcap_{k=1}^{|s|} \bigcap_{|t|=k} G(A_t, |s^\wedge 1|).$$

To start the construction, we set $x_0 := x_\emptyset$, $A_0 := A_\emptyset$ and $B_0 := B_\emptyset$. We choose nonempty open sets U_0, U_1 in K such that $x_0 \in U_0$ and $\emptyset = \overline{U}_0 \cap \overline{U}_1 \subset \overline{U}_0 \cup \overline{U}_1 \subset V_1$ and $U_0 \subset G(B_0, 1)$. We pick a point $x_1 \in (U_1 \cap X) \setminus \bigcup_{i \in A_0} F_i$ and set $A_1 := \{i \in I : x_1 \in F_i\}$, $B_1 := I \setminus A_1$. This finishes the first step of the construction.

Let $n \in \mathbb{N}$ and suppose that the construction has been completed for each $s \in \{0,1\}^{<\mathbb{N}}$ with $|s| \leq n$. Let now $s \in \{0,1\}^{<\mathbb{N}}$ have length n. We set $x_{s^\wedge 0} := x_s$, $A_{s^\wedge 0} := A_s$ and $B_{s^\wedge 0} := B_s$. We choose nonempty open sets $U_{s^\wedge 0}$, $U_{s^\wedge 1}$ in K such that $x_{s^\wedge 0} \in U_{s^\wedge 0}$ and conditions (i) and (iv) are satisfied. (Notice that $\bigcap_{k=1}^{|s|} \bigcap_{|t|=k} G(A_t, |s^\wedge 1|)$ is a dense open set in K.)

Further we pick a point $x_{s^\wedge 1} \in U_{s^\wedge 1} \cap X$ such that

$$x_{s^\wedge 1} \notin \bigcup_{k=1}^{|s|} \bigcup_{|t|=k} \{F_i : i \in A_t\}.$$

To finish the inductive step of the construction it is enough to define $A_{s^\wedge 1}$ and $B_{s^\wedge 1}$ according to condition (iii).

Set

$$C := \bigcap_{n=1}^{\infty} \bigcup_{|s|=n} \overline{U}_s$$

and define a mapping $\varphi : C \to \{0,1\}^{\mathbb{N}}$ by the formula

$$\varphi(x) = \sigma \in \{0,1\}^{\mathbb{N}} \quad \text{if and only if} \quad x \in \bigcap_{n=1}^{\infty} \overline{U}_{\sigma\restriction n}.$$

We have $C \subset X$ as $U_s \subset V_{|s|}$ for each $s \in \{0,1\}^{<\mathbb{N}}$. Moreover, φ is a continuous mapping of C onto $\{0,1\}^{\mathbb{N}}$. Let

$$A := \{\sigma \in \{0,1\}^{\mathbb{N}} : \sigma = (s_1, \dots, s_{|s|}, 0, 0, \dots) \text{ for some } \sigma \in \{0,1\}^{\mathbb{N}}\}.$$

Set $I_A := \bigcup\{A_s : s \in \{0,1\}^{<\mathbb{N}}\}$, $I_B := I \setminus I_A$ and define

$$\widehat{A} := \bigcup\{F_i : i \in I_A\} \quad \text{and} \quad \widehat{B} := \bigcup\{F_i : i \in I_B\}.$$

We need the following claim.

Claim. *We have* $\varphi^{-1}(A) = C \cap \widehat{A} = C \setminus \widehat{B}$ *and* $\varphi^{-1}(\{0,1\}^{\mathbb{N}} \setminus A) = C \cap \widehat{B} = C \setminus \widehat{A}$.

Proof of Claim. We start by showing

(1) $$\varphi^{-1}(A) \subset \widehat{A} \quad \text{and} \quad \varphi^{-1}(A) \cap \widehat{B} = \emptyset.$$

Let $\sigma \in A$, i.e., $\sigma = (s_1, \dots, s_{|s|}, 0, 0, \dots)$ for some $s \in \{0,1\}^{<\mathbb{N}}$. If $n > |s|$, it follows from (iv) that

$$U_{\sigma\restriction n} = U_{(s_1, \dots, s_{|s|}, 0, \dots, 0)} \subset G(B_s, n).$$

Thus

$$\varphi^{-1}(\sigma) = \bigcap_{n=|s|+1}^{\infty} \overline{U}_{\sigma\restriction n} = \bigcap_{n=|s|+1}^{\infty} U_{\sigma\restriction n} \subset \bigcap_{n=|s|+1}^{\infty} G(B_s, n) = X \setminus \bigcup_{i \in B_s} F_i.$$

As $I_B \subset B_s$, we have $\varphi^{-1}(\sigma) \cap \widehat{B} = \emptyset$. Also $\varphi^{-1}(\sigma) \subset \widehat{A}$ because $\mathcal{F}$ is a cover of X. Since $\sigma \in A$ is arbitrary, (1) follows.

Further, we show that

(2) $$\varphi^{-1}(\{0,1\}^{\mathbb{N}} \setminus A) \subset \widehat{B} \quad \text{and} \quad \varphi^{-1}(\{0,1\}^{\mathbb{N}} \setminus A) \cap \widehat{A} = \emptyset.$$

Let $\sigma \in \{0,1\}^{\mathbb{N}} \setminus A$, i.e., σ contains digit 1 infinitely often. Let $\{n_k\}$ be an increasing sequence of natural numbers such that $\sigma_{n_k} = 1$ for all $k \in \mathbb{N}$.

For a fixed sequence $t \in \{0,1\}^{<\mathbb{N}}$ we choose $k_0 \in \mathbb{N}$ such that $n_{k_0} - 1 \geq |t|$. It follows from (iv) that

$$U_{(\sigma_1,\dots,\sigma_{n_k-1},1)} \subset G(A_t, n_k)$$

for each integer $k \geq k_0$. Hence

$$\varphi^{-1}(\sigma) = \bigcap_{n=1}^{\infty} U_{\sigma\restriction n} = \bigcap_{k=k_0}^{\infty} U_{\sigma\restriction n_k} \subset \bigcap_{k=k_0}^{\infty} G(A_t, n_k) = X \setminus \bigcup_{i\in A_t} F_i.$$

Since this inclusion holds for each $t \in \{0,1\}^{<\mathbb{N}}$, we get

$$\varphi^{-1}(\sigma) \cap \bigcup\{F_i : i \in I_A\} = \varphi^{-1}(\sigma) \cap \widehat{A} = \emptyset.$$

Hence $\varphi^{-1}(\{0,1\}^{\mathbb{N}} \setminus A) \cap \widehat{A} = \emptyset$. Again we use the fact that $\mathcal{F}$ is a cover to deduce that $\varphi^{-1}(\{0,1\}^{\mathbb{N}} \setminus A) \subset \widehat{B}$. This concludes the proof of(2). By combining (1) and (2) we finish the proof of the claim. ■

Now we are ready to finish the proof of the lemma. Since $\mathcal{F}$ is F_σ-additive, the Claim shows that both $\varphi^{-1}(A)$ and $\varphi^{-1}(\{0,1\}^{\mathbb{N}} \setminus A)$ are F_σ. Since C is compact and φ is continuous, both A and $\{0,1\}^{\mathbb{N}} \setminus A$ are also F_σ. But this is impossible as they are both dense in the Baire space $\{0,1\}^{\mathbb{N}}$. This contradiction finishes the proof. ■

THEOREM 3.4. *Let $\mathcal{F}$ be a point-countable F_σ-additive cover of a Čech complete space X. Then $\mathcal{F}$ has a σ-scattered refinement.*

Proof. Set

$$G := \bigcup\{U : U \text{ open and } \mathcal{F}\restriction_U \text{ has a } \sigma\text{-scattered refinement}\}.$$

We claim that $G = X$.

Indeed, assuming the contrary, we set $H := X \setminus G$ and consider the restriction $\mathcal{F}\restriction_H$. According to Lemma 3.2, $\mathcal{F}\restriction_G$ has a σ-scattered refinement. Since H is a Čech complete (see [2, Theorem 3.9.6]), Lemma 3.3 yields an $F \in \mathcal{F}$ such that $F \cap H$ has nonempty interior in H. Let $U \subset X$ be an open set such that $U \cap H \neq \emptyset$ and $U \cap H \subset F \cap H$. Then $\mathcal{F}\restriction_{U\cap H}$ has a σ-scattered refinement. Since $\mathcal{F}\restriction_{U\setminus H}$ has a σ-scattered refinement as well, $\mathcal{F}\restriction_U$ has a σ-scattered refinement. Thus $U \subset G$, contrary to $U \cap H \neq \emptyset$.

Thus $G = X$ and $\mathcal{F}$ has a σ-scattered refinement by Lemma 3.2. ■

4. Scattered-K-analytic spaces. This section is devoted to a generalization of Theorem 3.4 to scattered-K-analytic spaces. We start with the following lemma proved in [8] as Lemma 2.5.

LEMMA 4.1. *Let Y and X be spaces and suppose Y has a σ-scattered network $\mathcal{N}$. Let $p : Y \times X \to X$ be the projection. For every scattered family*

$\mathcal{T}$ of sets in $Y \times X$ there are open sets $U_T^N \subset X$, $N \in \mathcal{N}$, $T \in \mathcal{T}$, so that the sets $T^N = T \cap (N \times U_T^N)$, $N \in \mathcal{N}$, satisfy

- *$T = \bigcup_{N\in\mathcal{N}} T^N$ for each $T \in \mathcal{T}$;*
- *$\{p(T^N) : T \in \mathcal{T}\}$ is a scattered family in X with associated open sets $U_T^N = U(p(T^N))$, $T \in \mathcal{T}$, for every $N \in \mathcal{N}$.*

The proof of the following proposition closely follows the proof of [8, Lemma 2.6].

PROPOSITION 4.2. *Let X be a scattered-K-analytic space. Then there exists a (single-valued) continuous mapping p from a Čech complete space Z onto X such that p maps scattered families in Z to families admitting a σ-scattered refinement.*

Proof. Let $f : Y \to X$ be an index-σ-scattered usco mapping of a complete metric space Y onto X. Let Z be the graph of f, i.e.,

$$Z = \{(y, x) \in Y \times X : x \in f(y)\}.$$

Then Z is Čech complete by [2, Theorem 3.9.10], since the projection $Z \to Y$ is a perfect map.

To finish the proof it is enough to show that the projection $p : Z \to X$ maps scattered families to families admitting a σ-scattered refinement. Let $\mathcal{T}$ be a scattered family in Z with associated open sets $\{U(T) : T \in \mathcal{T}\}$. Let $\mathcal{N}$ be a σ-scattered network for Y. (Since Y is a metric space, we can take $\mathcal{N}$ to be a σ-discrete basis.) Given $N \in \mathcal{N}$, Lemma 4.1 provides open sets U_T^N, $T \in \mathcal{T}$, in X such that

$$T = \bigcup_{N\in\mathcal{N}} T \cap (N \times U_T^N), \qquad T \in \mathcal{T}.$$

Moreover, if we set $T^N := T \cap (N \times U_T^N)$, then $\{p(T^N) : T \in \mathcal{T}\}$ is a scattered family in X with associated open sets U_T^N, $T \in \mathcal{T}$.

As

$$p(T^N) = p(T \cap (N \times U_T^N)) \subset f(N)$$

for each $N \in \mathcal{N}$ and $\{f(N) : N \in \mathcal{N}\}$ is even σ-scattered resolvable, the family $\{p(T^N) : N \in \mathcal{N}\}$ is σ-scattered resolvable for each $T \in \mathcal{T}$. Since $\{p(T^N) : T \in \mathcal{T}\}$ is scattered for each $N \in \mathcal{N}$, Lemma 3.1 implies that $\{p(T^N) : T \in \mathcal{T}, N \in \mathcal{N}\}$ has a σ-scattered refinement. Thus the family $\{p(T) : T \in \mathcal{T}\}$ has a σ-scattered refinement as well. This concludes the proof. ∎

THEOREM 4.3. *Let $\mathcal{F}$ be a point-countable F_σ-additive cover of a scattered-K-analytic space. Then $\mathcal{F}$ has a σ-scattered refinement.*

Proof. Using Proposition 4.2 we find a continuous mapping f of a Čech complete space Y onto X such that f maps scattered families to families admitting a σ-scattered refinement. Then $\widehat{\mathcal{F}} := \{f^{-1}(F) : F \in \mathcal{F}\}$ is an F_σ-additive cover of Y. According to Theorem 3.4, $\widehat{\mathcal{F}}$ has a σ-scattered refinement. Thus $\mathcal{F}$ itself has a σ-scattered refinement and we are done. ∎

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