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On many-sorted ω-categorical theories

by

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Abstract. We prove that every many-sorted ω-categorical theory is completely interpretable in a one-sorted ω-categorical theory. As an application, we give a short proof of the existence of non-G-compact ω-categorical theories.

1. Introduction. A many-sorted structure can be easily transformed into a one-sorted one by adding new unary predicates for the different sorts. However ω-categoricity is not preserved. In this article we present a general method for producing ω-categorical one-sorted structures from ω-categorical many-sorted structures. This is stated in Corollary 3.2, the main theorem in this paper. Our initial motivation was to understand Alexander Ivanov's example (in [4]) of an ω-categorical non-G-compact theory. In Corollary 3.3 we apply our results to offer a short proof of the existence of such theories.

Our method is based on the use of a particular theory T_E of equivalence relations E_n on n-tuples. The quotient by E_n is an imaginary sort containing a predicate P_n which can be used to copy the nth sort of the given many-sorted theory. Since the complexity of T_E is part of the complexity of the ω-categorical one-sorted theory obtained by our method, it is important to classify T_E from the point of view of stability, simplicity and related properties. It turns out that T_E is non-simple but it does not have SOP_2. A similar example of a theory with such properties has been presented by Shelah and Usvyatsov in [7]. Their proof, as ours, relies on Claim 2.11 of [3], which is known to have some gaps. A revised version of [3] has been posted on arxiv.org. In the meantime Kim and Kim have obtained a new proof of the same result: Proposition 2.3 in [5].

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The one-sorted theory T_E is interdefinable with some many-sorted theory T^* which is presented and discussed in Section 2. In order to describe T^* we need a version of Fraïssé's amalgamation method that can be applied to the many-sorted case (see Lemma 2.1). In Section 3 some results of the third author [8] on stable embeddedness are extended and used to prove Corollary 3.2. Section 4 is devoted to classifying T_E from the stability point of view.

A previous version of these results appeared in the second author's Ph.D. dissertation [6]. They have been corrected in some points and in general they have been elaborated and made more compact.

2. T^* and Fraïssé's amalgamation. Let L be a countable many-sorted language with sorts S_i $(i \in I)$, and let $\mathcal{K}$ be a class of finitely generated L-structures ([1]). We call an L-structure M a *Fraïssé limit* of $\mathcal{K}$ if the following hold:

1. $\mathcal{K} = \mathrm{Age}(M)$, where $\mathrm{Age}(M)$ is the class of all finitely generated L-structures which are embeddable in M.
2. M is at most countable.
3. M is *ultra-homogeneous*, i.e., any isomorphism between finitely generated substructures extends to an automorphism of M.

By a well-known argument, $\mathcal{K}$ can only have one Fraïssé limit, up to isomorphism.

LEMMA 2.1. *Let $\mathcal{K}$ be as above. Then the following are equivalent:*

(a) *The Fraïssé limit of $\mathcal{K}$ exists and is ω-categorical.*
(b) *$\mathcal{K}$ has the amalgamation property* AP*, the joint embedding property* JEP*, the hereditary property* HP (*i.e., finitely generated L-structures which are embeddable in elements of $\mathcal{K}$ belong themselves to $\mathcal{K}$*) *and satisfies*

    (∗) *for all $i_1, \ldots, i_n \in I$ there are only finitely many quantifier-free types of tuples $(a_1, \ldots, a_n)$ where the a_j are elements of sort S_{i_j} in some structure $A \in \mathcal{K}$.*

If the Fraïssé limit of $\mathcal{K}$ exists, it has quantifier elimination.

Proof. (a)⇒(b). It is well known that the age of an ultra-homogeneous structure has AP, JEP and HP. All quantifier-free types which occur in elements of $\mathcal{K}$ are quantifier-free types of tuples of the Fraïssé limit. So property (∗) follows from the Ryll-Nardzewski theorem.

([1]) We allow empty sorts if L has no constant symbols of that sort.

(b)⇒(a). The quantifier-free type $\mathrm{qftp}(\bar{a})$ determines the isomorphism type of the structure generated by $\bar{a}$. Hence $(*)$ implies that $\mathcal{K}$ contains at most countably many isomorphism types. The existence of the Fraïssé limit M follows now from AP, JEP and HP.

If two sequences $\bar{a}$ and $\bar{b}$ have the same quantifier-free type in M, there is an automorphism of M which maps $\bar{a}$ to $\bar{b}$ and so it follows that $\bar{a}$ and $\bar{b}$ have the same type in M. Consider a formula $\varphi(\bar{x})$ and the set $P_{\varphi(x)} = \{\mathrm{qftp}(\bar{a}) : M \models \varphi(\bar{a})\}$. Then

$$M \models \varphi(\bar{a}) \;\Leftrightarrow\; \mathrm{qftp}(\bar{a}) \in P_\varphi \;\Leftrightarrow\; M \models \bigvee_{p \in P_\varphi} p(\bar{a}).$$

Now, $(*)$ implies that P_φ is finite and that in M all $p = \mathrm{qftp}(\bar{a})$ are finitely axiomatizable, that is, $p = \langle \chi_p \rangle$ for some quantifier-free $\chi_p(x)$. Then $M \models \varphi(\bar{a}) \Leftrightarrow \bigvee_{p \in P_\varphi} \chi_p(\bar{a})$. So M has quantifier elimination and it is ω-categorical since there are only finitely many possibilities for the χ_p, depending only on the number and the sorts of the free variables of φ. ■

It is easy to see that the theory of the Fraïssé limit is the model-completion of the universal theory of $\mathcal{K}$.

Definition 2.2. Let L^* be the language with countably many sorts $S, S_1, \ldots$, function symbols $f_i : S^i \to S_i$, and constants $c_i \in S_i$, and let T^0 be the theory of all L^*-structures A with

$$f_i(\bar{a}) = c_i \;\Leftrightarrow\; \bar{a} \text{ has some repetition}$$

for all $\bar{a} \in S^i(A)$. Furthermore let $\mathcal{K}^*$ be the class of all finitely generated models of T^0.

Lemma 2.3. *$\mathcal{K}^*$ satisfies the conditions of Lemma* 2.1.

Proof. The class of all models of T^0 has AP and JEP and therefore also $\mathcal{K}^*$. Property $(*)$ follows easily from the fact that $f_i(a_{m_1} \ldots, a_{m_i}) = c_i$ for all $i > k$ and $\{a_{m_1} \ldots, a_{m_i}\} \subset \{a_1, \ldots, a_k\}$, ■

We define M^* to be the Fraïssé limit of $\mathcal{K}^*$ and T^* to be the complete theory of M^*. Then T^* is the model-completion of T^0.

Recall the following definition from [2]:

Definition 2.4. Let T be a complete theory and P a 0-definable predicate. P is called *stably embedded* if every definable relation on P is definable with parameters from P.

Remarks.

1. For many-sorted structures with sorts $(S_i)_{i \in I}$ this generalizes to the notion of a sequence $(P_i)_{i \in I}$ of 0-definable $P_i \subset S_i$ being stably embedded.

2. While the definition is meant in the monster model, an easy compactness argument shows that, if $P(M)$ is stably embedded in M for some weakly saturated [2] model M, then this is true for all models.
3. If M is saturated then P is stably embedded if and only if every automorphism (i.e. elementary permutation) of $P(M)$ extends to an automorphism of M. This was claimed in [2] only for the case that $|M| > |T|$. But the proof can easily be modified to work for the general case. One has to use the fact that if A has smaller size than M, then any type over a subset of $\mathrm{dcl}^{\mathrm{eq}}(A)$ can be realized in M.
4. If M is ω-categorical, it can be proved that for every finite tuple $a \in M$ there is a finite tuple $b \in P$ such that every relation on P which is definable over a can be defined using the parameter b.

LEMMA 2.5. *In T^* the sequence of sorts $(S_1, S_2, \ldots)$ is stably embedded.*

Proof. Clear since $\mathrm{tp}(\bar{a}/S_1, \ldots) = \mathrm{tp}(\bar{a}/f_1(\bar{a}), \ldots)$. See also the discussion in [2]. ∎

For a complete theory T and a 0-definable predicate P the *induced structure* on P consists of all 0-definable relations on P. Note that the automorphisms of P with its induced structure are exactly the elementary permutations of P in the sense of T.

LEMMA 2.6. *In T^* the induced structure on $(S_1, S_2, \ldots)$ equals its $L^*_{>0}$-structure, where $L^*_{>0}$ is the sublanguage of L^* which has only the sorts $S_1, S_2, \ldots$ and the constants $c_1, c_2, \ldots$.*

Proof. Quantifier elimination. ∎

Let $T^*_{>0}$ denote the theory of all $L^*_{>0}$-structures, where all sorts S_i are infinite. Clearly $T^*_{>0}$ is the restriction of T^* to $L^*_{>0}$.

LEMMA 2.7. *Every model of $T^*_{>0}$ can be expanded to a model of T^*.*

Proof. It is easy to see that the following amalgamation property is true:

> Let N be a model of T^0 with infinite sorts $S_i(N)$. Let A be a finitely generated substructure of N, and $B \in \mathcal{K}^*$ an extension of A. Then B can be embedded over A in an extension N' of N which is a model of T^0 and such that $S_i(N') = S_i(N)$ for all i.

If a model of $T^*_{>0}$ is given, we expand it arbitrarily to a model N of T^0 and apply the above amalgamation property repeatedly in such a way that the union of the resulting chain is a model of T^* which has the same sorts S_i as N. ∎

[2] M is *weakly saturated* if every type over the empty set is realized in M.

Corollary 2.8. *There is an ω-categorical one-sorted theory T_E with a series of* 0*-definable infinite predicates $P_1, P_2, \ldots$ in T_E^{eq} such that*

1. $(P_1, P_2, \ldots)$ *is stably embedded.*
2. *The many-sorted structure induced on $(P_1, P_2, \ldots)$ is trivial.*
3. *For every sequence $\kappa_1, \kappa_2, \ldots$ of infinite cardinals there is a model N of T_E such that $|P_i(N)| = \kappa_i$.*

Proof. The language L_E of T_E will contain, for each i, a symbol E_i for an equivalence relation between i-tuples. Let $M = (S, S_1, S_2, \ldots)$ be a model of T^*. For $a, b \in S^i$ define $E_i(a, b) \Leftrightarrow f_i(a) = f_i(b)$. Then T_E is the theory of $M_E = (S, E_1, E_2, \ldots)$. The S_i live in M_E^{eq} as M_E^i/E_i and the c_i are 0-definable in M_E^{eq}. We set $P_i = S_i \setminus \{c_i\}$. ■

It is easy to see that T_E as constructed in the proof is the model-completion of the theory of all structures $(M, E_1, E_2, \ldots)$ where E_n is an equivalence relation on M^n with one equivalence class consisting of all n-tuples which contain a repetition. That T_E has quantifier elimination can be proved as follows: Every formula $\varphi(\bar{x})$ of L_E is equivalent to a quantifier-free L^*-formula $\varphi'(\bar{x})$. Further, $\varphi'(\bar{x})$ is a boolean combination of formulas of the form $f_i(\bar{x}') \doteq f_i(\bar{x}'')$ and $f_i(\bar{x}') \doteq c_i$, which are equivalent to quantifier-free L_E-formulas: $f_i(\bar{x}') \doteq f_i(\bar{x}'')$ is equivalent to $E_i(\bar{x}', \bar{x}'')$, and $f_i(x'_1, \ldots, x'_i) \doteq c_i$ is equivalent to $\bigvee_{1 \le k < l \le i} x'_k \doteq x'_l$.

3. Expansions of stably embedded predicates. Let T be complete theory with two sorts S_0 and S_1. We consider S_1 as a structure of its own carrying the structure induced from T, and denote by $T{\restriction}S_1$ the theory of S_1.

Lemma 3.1. *Let T be complete theory with two sorts S_0 and S_1. Let $\widetilde{T}_1$ be a complete expansion of $T{\restriction}S_1$. Assume that S_1 is stably embedded. Then*

1. $\widetilde{T} = T \cup \widetilde{T}_1$ *is complete* (3) ([8, Lemma 3.1]).
2. S_1 *is stably embedded in* $\widetilde{T}$ *and* $\widetilde{T}{\restriction}S_1 = \widetilde{T}_1$.
3. *If T and $\widetilde{T}_1$ are ω-categorical, then $\widetilde{T}$ is also ω-categorical.*

Proof. 1. Let $\widetilde{M} = (M_0, \widetilde{M}_1)$ and $\widetilde{M}' = (M'_0, \widetilde{M}'_1)$ be saturated models of $\widetilde{T}$ of the same cardinality, and $M = (M_0, M_1)$ and $M' = (M'_0, M'_1)$ their restrictions to the language of T. Since T and $\widetilde{T}_1$ are complete, there are isomorphisms $f : M \to M'$ and $g : \widetilde{M}_1 \to \widetilde{M}'_1$. Then gf^{-1} is an auto-

(3) Actually we have: S_1 is stably embedded if and only if $\widetilde{T}$ is complete for all complete expansions $\widetilde{T}_1$.

morphism of M_1'. Since M_1' is stably embedded in M', gf^{-1} extends to an automorphism h of M'. Now hf is an isomorphism from M to M' which extends g.

2. We use the same notation as in the proof of 1. Let $\widetilde{M}$ be a saturated model of $\widetilde{T}$. We have to show that every automorphism f of $\widetilde{M_1}$ extends to an automorphism of $\widetilde{M}$. But f extends to an automorphism of M, which is automatically an automorphism of $\widetilde{M}$.

3. Start with two countable models $\widetilde{M}$ and $\widetilde{M'}$ and proceed as in the proof of 1. ∎

Corollary 3.2. *Every many-sorted ω-categorical theory T is completely interpretable in a one-sorted ω-categorical theory $\widetilde{T}$. This means that T is induced by $\widetilde{T}$ via the interpretation.*

Proof. Let T be a complete theory with countably many sorts $P_1, P_2, \ldots$. We consider T as an expansion of $T_E \restriction (P_1, P_2, \ldots)$ and set $\widetilde{T} = T_E \cup T$. Then $\widetilde{T}$ is a one-sorted complete theory. We have $\widetilde{T} \restriction (P_1, P_2, \ldots) = T$. If T is ω-categorical, $\widetilde{T}$ is also ω-categorical. ∎

Corollary 3.3 (Ivanov). *There is a one-sorted ω-categorical theory which is not G-compact.*

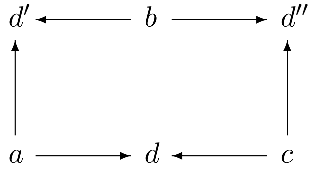
Proof. By [1] there is a many-sorted ω-categorical theory T which is not G-compact. Interpret T in a one-sorted ω-categorical theory $\widetilde{T}$ as in Corollary 3.2. Then T is not G-compact either. For this, one has to check that if $\widetilde{T}$ is G-compact, then every 0-definable subset with its induced structure is also G-compact. This follows from the following description of G-compactness: a, b of length ω are in the relation nc^ω if a and b are the first two elements of an infinite sequence of indiscernibles. A complete theory is G-compact if the transitive closure of nc^ω is type-definable. (Note that (a, b) is in the transitive closure of nc^ω if and only if a and b have the same Lascar-strong type.) ∎

4. Classification of T_E

Proposition 4.1. *T_E has TP_2, the tree property of the second kind, and therefore it is not simple.*

Proof. We show that $\varphi(x; y, u, v) = E_2(xy, uv)$ has TP_2. Let $(b_i : i < \omega)$, $(c_i : i < \omega)$, and $(d_i : i < \omega)$ be pairwise disjoint sequences of different elements such that $\neg E_2(c_i d_i, c_j d_j)$ for $i \neq j$. For $i, j \in \omega$, let $\bar{a}^i_j = b_i c_j d_j$. By compactness we can see that for any $\eta \in \omega^\omega$, the set $\{\varphi(x; \bar{a}^i_{\eta(i)}) : i < \omega\}$ is consistent, and since the $c_i d_i$'s are in different E_2-classes for each $i < \omega$, the set $\{\varphi(x; \bar{a}^i_j) : j < \omega\}$ is 2-inconsistent. ∎

Lemma 4.2 (Independence lemma). *Let a, b, c, d', d'' be tuples in the monster model of T_E, and F a finite subset. Assume that a and c have only elements from F in common. If $d'a \equiv_F d'b \equiv_F d''b \equiv_F d''c$, then there exists some d such that $d'a \equiv_F da \equiv_F dc \equiv_F d''c$.*



Proof. Let A, B, C, D' and D'' denote the set of elements of the tuples a, b, c, d' and d'', respectively. We note first that we can assume that F is contained in A, B and C, since otherwise we can increase a, b and c by elements from F. Then we note that if A and D' intersect in a subtuple f, this tuple also belongs to B and C and therefore to F. So we conclude that $A \cap D'$ is contained in F and similarly that $C \cap D''$ is contained in F.

It suffices to find an L_E-structure M extending AC and containing a new tuple d with the same quantifier-free type as d' over A and as d'' over C. Take as d a new tuple of the right length which intersects A and B in the subtuple f. We then have $d'a \equiv_F^{\mathrm{eq}} da \equiv_F^{\mathrm{eq}} dc \equiv_F^{\mathrm{eq}} d''c$, where $g \equiv_F^{\mathrm{eq}} h$ means that g and h satisfy the same equality formulas over F, i.e. $g_i = g_j$ iff $h_i = h_j$ and $g_i = f_j$ iff $h_i = f_j$. If D denotes the elements of d, it follows that the intersection of any two of A, C and D belongs to F.

It remains to define the relations E_n on ACD. Let E_n^0 denote the part of E_n which is already defined on AC. Let E_n' be the relation E_n transported from AD' to AD via the identification $d' \mapsto d$, and E_n'' the relation E_n transported from CD'' to CD via the identification $d'' \mapsto d$. Note that $d' \equiv_F d''$ implies that E_n' and E_n'' agree on DF. We define E_n on ACD as the transitive closure of

$$E_n^0 \cup E_n' \cup E_n'' \cup E_n^{\mathrm{rep}} \cup \Delta,$$

where E_n^{rep} is the set of all pairs of n-tuples from ACD which contain repetitions, and Δ is the identity on $(ACD)^n$.

We have to show that the new structure defined on AC agrees with the original structure. Also we must check that the structure on AD (and CD) agrees with the structure on AD' (and CD') via $d \mapsto d'$ (and $d \mapsto d''$). Using the fact that an n-tuple which e.g. belongs to AC and AD belongs already to A, it is easy to see that we have to show the following: For all n-tuples

$x \in A$, $y \in C$ and $z \in DF$,

1. $E'_n(x,z) \wedge E''_n(z,y) \Rightarrow E^0_n(y,x)$,
2. $E''_n(z,y) \wedge E^0_n(y,x) \Rightarrow E'_n(x,z)$,
3. $E^0_n(y,x) \wedge E'_n(x,z) \Rightarrow E''_n(z,y)$.

Let z' and z'' be the subtuples of $D'F$ and $D''F$ which correspond to z.

Proof of 1: Assume $E'_n(x,z)$ and $E''_n(z,y)$. We then have $E_n(x,z')$ and $E_n(z'',y)$. Further, $d'a \equiv_F d'b$ implies $z'a \equiv_F z'b$, which implies that there is a tuple x' in B such that $z'x \equiv_F z'x'$. So we see $E_n(z',x')$. In turn $d'b \equiv_F d''b$ implies $z'x' \equiv_F z''x'$ and hence $E_n(z'',x')$. Now we can connect y and x as follows: $y\ E_n\ z''\ E_n\ x'\ E_n\ z'\ E_n\ x$.

Proof of 2: Assume $E''_n(z,y)$ and $E^0_n(y,x)$. We then have $E_n(z'',y)$. As above we find a tuple $y' \in B$ such that $E_n(z'',y')$ and $E_n(z',y')$. The chain $x\ E_n\ y\ E_n\ z''\ E_n\ y'\ E_n\ z'$ shows that $E'_n(x,z)$.

Proof of 3: Symmetrical to the proof of 2. ∎

In order to state [5, Proposition 2.3] we need the following terminology:

(1) A tuple $\bar{\eta} = (\eta_0, \ldots, \eta_{d-1})$ of elements of $2^{<\omega}$ is $\cap$-*closed* if the set $\{\eta_0, \ldots, \eta_{d-1}\}$ is closed under intersection.

(2) Two $\cap$-closed tuples $\bar{\eta}$ and $\bar{\nu}$ are *isomorphic* if they have the same length and

  (i) $\eta_i \trianglelefteq \eta_j$ iff $\nu_i \trianglelefteq \nu_j$,

  (ii) $\eta_i{}^\frown t \trianglelefteq \eta_j$ iff $\nu_i{}^\frown t \trianglelefteq \nu_j$ for $t = 0, 1$.

(3) A tree $(a_\eta \colon \eta \in 2^{<\omega})$ of tuples of the same length is *modeled* by $(b_\eta \colon \eta \in 2^{<\omega})$ if for every formula $\phi(\bar{x})$ and every $\cap$-closed $\bar{\eta}$ there is a $\cap$-closed $\bar{\nu}$ isomorphic to $\bar{\eta}$ such that $\models \phi(b_{\bar{\eta}}) \Leftrightarrow\ \models \phi(a_{\bar{\nu}})$.

(4) $(b_\eta \colon \eta \in 2^{<\omega})$ is *indiscernible* if $\models \phi(b_{\bar{\eta}}) \Leftrightarrow\ \models \phi(b_{\bar{\nu}})$ for all isomorphic $\cap$-closed $\bar{\eta}, \bar{\nu}$.

Lemma 4.3 ([5, Proposition 2.3]; see also [3]). *Let T be a complete theory. Then any tree of tuples can be modeled by an indiscernible tree.*

Definition 4.4. The formula $\varphi(x,y)$ has SOP_2 in T if there is a binary tree $(a_\eta : \eta \in 2^{<\omega})$ such that for every $\eta \in 2^\omega$, $\{\varphi(x, a_{\eta\restriction n}) : n < \omega\}$ is consistent and for every incomparable $\eta, \nu \in 2^{<\omega}$, $\varphi(x,a_\eta) \wedge \varphi(x,a_\nu)$ is inconsistent. The theory T has SOP_2 if some formula $\varphi(x,y) \in L$ has SOP_2 in T.

Remark 4.5 (H. Adler). *The formula $\varphi(x,y)$ has* SOP_2 *in T if and only if $\varphi(x,y)$ has* TP_1 *the tree property of the first kind: there is a tree $(a_\eta : \eta \in \omega^{<\omega})$ such that for every $\eta \in \omega^\omega$, $\{\varphi(x,a_{\eta\restriction n}) : n < \omega\}$ is consistent and for every incomparable $\eta, \nu \in \omega^{<\omega}$, $\varphi(x,a_\eta) \wedge \varphi(x,a_\nu)$ is inconsistent.*

Proof. By compactness. ∎

Proposition 4.6. *T_E does not have* SOP_2.

Proof. We follow ideas from a similar proof in [7]. Assume $\varphi(x,y)$ has SOP_2 in T_E and the tree $(a_\eta : \eta \in 2^{<\omega})$ witnesses it. Choose for every η a tuple d_η such that $\models \phi(d_\eta, a_\nu)$ for all $\nu \subsetneq \eta$.

By Lemma 4.3 we can assume that the tree $(d_\eta a_\eta : \eta \in 2^{<\omega})$ is indiscernible. Let us now look at the elements $a_{00}, a_{\langle\rangle}, a_{01}, d_{000}, d_{010}$. We have by indiscernibility

$$d_{000}a_{00} \equiv d_{000}a_{\langle\rangle} \equiv d_{010}a_{\langle\rangle} \equiv d_{010}a_{01}.$$

If the tuples a_{00} and a_{01} are disjoint, we can apply the Independence Lemma to $a = a_{00}$, $b = a_{\langle\rangle}$, $c = a_{01}$, $d' = d_{000}$, $d'' = d_{010}$ to get a tuple d such that

$$d_{000}a_{00} \equiv da_{00} \equiv da_{01} \equiv d_{010}a_{01}.$$

It follows that $\models \varphi(d, a_{00}) \wedge \varphi(d, a_{01})$, which contradicts the SOP_2 of the tree.

If a_{00} and a_{01} are not disjoint, we argue as follows: Assume that a_{00} and a_{01} have an element f in common, say $f = a_{00,i} = a_{01,j}$. Then $a_{00}a_{01} \equiv a_{000}a_{01}$ implies $a_{000,i} = a_{01,j}$. So we have $a_{000,i} = a_{00,i}$ and it follows from indiscernibility that $f = a_{00,i} = a_{\langle\rangle,i} = a_{01,i}$. Let F be the set of elements which occur in both a_{00} and a_{01}. We have seen that the elements of F occur in a_{00}, $a_{\langle\rangle}$ and a_{01} at the same places. Therefore

$$d_{000}a_{00} \equiv_F d_{000}a_{\langle\rangle} \equiv_F d_{010}a_{\langle\rangle} \equiv_F d_{010}a_{01}$$

and we can again apply the Independence Lemma. ■

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