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# Erratum/addendum to the paper: "Quasi $^*$-algebras and generalized inductive limits of $C^*$-algebras"

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by

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In Example 5.3 of the paper cited in the title, we tried to construct a family $\{W_A \in \mathfrak{B}(\mathcal{H}_A) : A \in \mathcal{L}^\dagger(\mathcal{D})\}$ so that the partial multiplication, defined in $\mathfrak{L}_{\mathsf{B}}(\mathcal{D}, \mathcal{D}^\times)$ by the method discussed *ibidem* [Section 3.6], would reproduce the well-known quasi $^*$-algebra structure of $(\mathfrak{L}_{\mathsf{B}}(\mathcal{D}, \mathcal{D}^\times), \mathcal{L}^\dagger(\mathcal{D}))$. Unfortunately, some misplaced inverses and a more relevant mistake made there produced an incorrect conclusion of that discussion. The argument in Example 5.3 shows, in fact, that it is not possible to find a family $\{W_A \in \mathfrak{B}(\mathcal{H}_A) : A \in \mathcal{L}^\dagger(\mathcal{D})\}$ satisfying the required condition. We will prove the last statement in detail, referring the reader, of course, for notations and definitions to the above mentioned article.

If $A \in \mathcal{L}^\dagger(\mathcal{D})$, then $(I + A^*\overline{A})^{-1} \in \mathfrak{B}(\mathcal{H}_A)$, and, for its norm in $\mathfrak{B}(\mathcal{H}_A)$ one has $\|(I + A^*\overline{A})^{-1}\|_{A,A} \le 1$. Moreover, for every $\xi, \eta \in \mathcal{D}$,

$$\begin{aligned}\langle (I + A^*\overline{A})^{-1}\xi \,|\, \eta\rangle_A &= \langle (I + A^*\overline{A})^{1/2}(I + A^*\overline{A})^{-1}\xi \,|\, (I + A^*\overline{A})^{1/2}\eta\rangle \\ &= \langle \xi \,|\, \eta\rangle.\end{aligned}$$

This means that the identity operator $I$ of $\mathcal{D}$ is represented in every space $\mathfrak{B}(\mathcal{H}_A)$ by the operator $(I + A^*\overline{A})^{-1}$. If $Y \in \mathcal{L}^\dagger(\mathcal{D})$, then $Y \in \mathfrak{L}^A_{\mathsf{B}}(\mathcal{D}, \mathcal{D}^\times)$ for some $A \in \mathcal{L}^\dagger(\mathcal{D})$, and the operator $Y_A \in \mathfrak{B}(\mathcal{H}_A)$, corresponding to $Y$, satisfies $Y_A \restriction \mathcal{D} = (I + A^\dagger A)^{-1}Y$. If $X \in \mathfrak{L}_{\mathsf{B}}(\mathcal{D}, \mathcal{D}^\times)$, then $X \in \mathfrak{L}^S_{\mathsf{B}}(\mathcal{D}, \mathcal{D}^\times)$ for a sufficiently large $S \in \mathcal{L}^\dagger(\mathcal{D})$. If a family $\{W_A \in \mathfrak{B}(\mathcal{H}_A) : A \in \mathcal{L}^\dagger(\mathcal{D})\}$ satisfying the required conditions were to exist, one should have, for every $\xi \in \mathcal{D}$,

$$X_T W_T Y_T \xi = X_T W_T (I + T^*\overline{T})^{-1} Y\xi = X_T Y\xi, \qquad T \succeq S,$$

with $X_T = \Phi_T^{-1}(X)$. This is possible only if $W_T = I + T^*\overline{T}$. But this operator *does not belong* to $\mathfrak{B}(\mathcal{H}_T)$, unless $T$ is bounded.

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This proves that the method developed in the paper cannot be applied to this space of operators. In conclusion, $\mathfrak{L}_{\mathsf{B}}(\mathcal{D}, \mathcal{D}^{\times})$ has the structure of a $C^*$-inductive locally convex space, but it is not possible to make it a $C^*$-inductive locally convex quasi $^*$-algebra via the method of Section 3.6.

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