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The bounded approximation property for the predual of the space of bounded holomorphic mappings

by

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Abstract. When U is the open unit ball of a separable Banach space E, we show that $G^\infty(U)$, the predual of the space of bounded holomorphic mappings on U, has the bounded approximation property if and only if E has the bounded approximation property.

1. Introduction. Let E and F be complex Banach spaces, and let $L(E;F)$ be the Banach space of all continuous linear operators. Let $1 \leq \lambda < \infty$. A Banach space E is said to have:

- the *approximation property* (AP for short) if given a compact set $K \subset E$ and $\varepsilon > 0$, there is a finite rank operator $T \in L(E;E)$ such that $\|Tx - x\| < \varepsilon$ for every $x \in K$;
- the λ-*bounded approximation property* (λ-BAP for short) if given a compact set $K \subset E$ and $\varepsilon > 0$, there is a finite rank operator $T \in L(E;E)$ so that $\|T\| \leq \lambda$ and $\|Tx - x\| < \varepsilon$ for every $x \in K$;
- the *bounded approximation property* (BAP for short) if it has the λ-BAP for some λ;
- the *metric approximation property* (MAP for short) if it has the 1-BAP.

Clearly the BAP implies the AP. But the converse is in general false. In [5] Figiel and Johnson gave an example of a separable Banach space with the AP which fails to have the BAP. (See also Casazza [2].)

Let U be an open subset of a Banach space E, and let $G^\infty(U)$ denote the predual of the space $\mathcal{H}^\infty(U)$ of all bounded holomorphic mappings constructed by Mujica in [10]. If U is a bounded balanced open subset of E then Mujica [10] proved that E has the AP (resp. MAP) if and only if $G^\infty(U)$ has the AP (resp. MAP). In [1] the author proved that E has the compact AP if and only if $G^\infty(U)$ has the compact AP.

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In the present paper, we establish our notation and terminology in Section 2, and in Section 3 we give two basic results, one of which characterizes the weakly compactly generated Banach spaces with the BAP. In Section 4 we show that a Banach space E has the λ-BAP if and only if each m-homogeneous continuous Banach-valued polynomial on E of norm ≤ 1 can be uniformly approximated on compact sets by m-homogeneous finite rank polynomials of norm $\leq \lambda^m$, for some $1 \leq \lambda < \infty$. In our main result we show that a separable Banach space E has the BAP if and only if $G^\infty(U)$ has the BAP, where U is the open unit ball of E. This gives a partial solution for a problem posed by Mujica in [10].

I wish to thank Professor Jorge Mujica for many useful suggestions on the exposition.

2. Notation and terminology. The symbol $\mathbb{C}$ represents the field of all complex numbers, $\mathbb{N}$ the set of all positive integers, and $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. Unless stated otherwise E and F denote complex Banach spaces. The letter U denotes a nonvoid open subset of E, and U_E is the open unit ball of E.

The symbol B_E^λ, $1 \leq \lambda < \infty$, represents the subset of E consisting of the elements of norm $\leq \lambda$. We write B_E instead of B_E^1, the closed unit ball of E.

The symbol τ_{c} denotes the compact-open topology.

Let $L(E;F)$ denote the Banach space of all continuous linear operators from E into F. When $F = \mathbb{C}$ we write E' instead of $L(E;\mathbb{C})$.

We denote by $E \otimes F$ the tensor product of E and F.

Let $\mathcal{P}(E;F)$ denote the vector space of all continuous polynomials from E into F, and $\mathcal{P}(^mE;F)$ the subspace of all m-homogeneous members of $\mathcal{P}(E;F)$. If $F = \mathbb{C}$ then we denote $\mathcal{P}(^mE;\mathbb{C})$ by $\mathcal{P}(^mE)$.

A polynomial $P \in \mathcal{P}(^mE;F)$ is said to be *of finite type* if it is a linear combination of functions $\phi^m \otimes y$ ($\phi \in E'$, $y \in F$), where $\phi^m \otimes y(x) = \phi^m(x)y$ for all $x \in E$. Let $\mathcal{P}_{\mathrm{f}}(^mE;F)$ denote the subspace of all members of $\mathcal{P}(^mE;F)$ which are of finite type, for every $m \in \mathbb{N}_0$.

We denote by $\mathcal{H}^\infty(U;F)$ the Banach space of all bounded holomorphic mappings from U into F, with the supremum norm. When $F = \mathbb{C}$ we write $\mathcal{H}^\infty(U)$ instead of $\mathcal{H}^\infty(U;\mathbb{C})$.

We refer to [3] or [9] for the properties of polynomials and holomorphic mappings on infinite-dimensional spaces, and to [7] for the theory of Banach spaces.

3. The bounded approximation property in Banach spaces. An operator T in $L(E;F)$ is said to have a *finite rank* if $T(E)$ is finite-dimensional. Observe that the subspace of all finite rank operators $T \in L(E;F)$ can be identified with the space $E' \otimes F$.

In [7, Theorem 1.e.4] Lindenstrauss and Tzafriri give a list of equivalent formulations of the AP for Banach spaces. The following analogous proposition characterizing spaces with the BAP can be proved easily (see also [2]).

PROPOSITION 1. *Let E be a Banach space and $1 \le \lambda < \infty$. The following statements are equivalent*:

(a) *E has the λ-BAP.*
(b) $B_{L(E;E)} = \overline{B^{\lambda}_{E'\otimes E}}^{\tau_c}$.
(c) *For every Banach space F*, $B_{L(F;E)} = \overline{B^{\lambda}_{F'\otimes E}}^{\tau_c}$.
(d) *For every Banach space F*, $B_{L(E;F)} = \overline{B^{\lambda}_{E'\otimes F}}^{\tau_c}$.
(e) *For every $c > 0$, every $(x_n)_{n=1}^{\infty} \subset E$, and $(x'_n)_{n=1}^{\infty} \subset E'$ with*

$$\sum_{n=1}^{\infty} \|x_n\| \cdot \|x'_n\| < \infty \quad \textit{and} \quad \Big|\sum_{n=1}^{\infty} x'_n(Tx_n)\Big| \le c \quad \textit{for all } T \in B^{\lambda}_{E'\otimes E},$$

we have $|\sum_{n=1}^{\infty} x'_n(x_n)| \le c$.

A common generalization of the notions of a reflexive and a separable Banach space is given in the following definition.

DEFINITION 1. A Banach space E is said to be *weakly compactly generated* (WCG for short) if there is a weakly compact subset K of E such that $E = \overline{\bigcup_{n=1}^{\infty} nK}$.

For examples of WCG Banach spaces other than separable and reflexive spaces see [6].

One can easily show that if E has the λ-BAP, $1 \le \lambda < \infty$, then every complemented subspace of E with the projection P has the $\lambda\|P\|$-BAP. Recall that a complemented subspace of E is said to be 1-*complemented* if the corresponding projection has norm 1. Using Proposition 1 and [6, Theorem 2.1] we easily get the following characterization of the BAP for WCG Banach spaces, which we have not found in the literature.

COROLLARY 1. *Let E be a WCG Banach space*, *and $1 \le \lambda < \infty$. The following are equivalent*:

(a) *E has the λ-BAP.*
(b) *Every 1-complemented subspace of E has the λ-BAP.*
(c) *Every 1-complemented separable subspace of E has the λ-BAP.*

One can also obtain results similar to Proposition 1 and Corollary 1 for the bounded compact approximation property (for the definition see [2]).

4. The bounded approximation property for the predual of the space of bounded holomorphic mappings. In [13] Ryan constructed a Banach space $Q(^mE)$, $m \in \mathbb{N}$, and a mapping $\delta_m \in \mathcal{P}(^mE; Q(^mE))$

with the following universal property: For each Banach space F and each $P \in \mathcal{P}(^mE;F)$, there is a unique operator $T_P \in L(Q(^mE);F)$ such that $T_P \circ \delta_m = P$. The space $Q(^mE)$ is defined as the closed subspace of all linear functionals $v \in \mathcal{P}(^mE)'$ such that $v|_{B_{\mathcal{P}(^mE)}}$ is τ_c-continuous; it is called the predual of $\mathcal{P}(^mE)$. (See also [11], or [10, Theorems 2.4 and 4.1].)

The next result asserts that a Banach space E has the BAP if and only if, for each $m \in \mathbb{N}$, $Q(^mE)$ has the BAP.

Proposition 2. *Let E be a Banach space and let $1 \leq \lambda < \infty$. The following statements are equivalent*:

(a) *E has the λ-BAP.*
(b) *For each Banach space F and each $m \in \mathbb{N}$, $B_{\mathcal{P}(^mE;F)} = \overline{B^{\lambda^m}_{\mathcal{P}_f(^mE;F)}}^{\tau_c}$.*
(c) *For each $m \in \mathbb{N}$, $Q(^mE)$ has the λ^m-BAP.*

Proof. The proof of (a)⇒(b) is standard while (b)⇒(c) is a consequence of [10, Theorem 2.4 and Proposition 3.1]. Finally, (c)⇒(a) follows from the fact that $Q(^1E) = E$ (see [10, p. 871]). ∎

We note that the implication (a)⇒(c) was stated in [10, p. 885] without proof.

The space $\mathcal{H}^\infty(U)$ also has a predual. In [10] Mujica constructed a Banach space $G^\infty(U)$ and a mapping $\delta_U \in \mathcal{H}^\infty(U;G^\infty(U))$ with the following universal property: For each Banach space F and each mapping $f \in \mathcal{H}^\infty(U;F)$, there is a unique operator $T_f \in L(G^\infty(U);F)$ such that $T_f \circ \delta_U = f$. The space $G^\infty(U)$ is defined as the closed subspace of all linear functionals $u \in \mathcal{H}^\infty(U)'$ such that $u|_{B_{\mathcal{H}^\infty(U)}}$ is τ_c-continuous; it is called the predual of $\mathcal{H}^\infty(U)$.

Now it is natural to ask whether there is a result similar to Proposition 2 for $G^\infty(U)$ concerning the BAP. In fact, in [10] Mujica posed the following problem (see [10, 5.9 Problem]): Does $G^\infty(U_E)$ have the BAP whenever E does? We have no answer to this question in general, but we give a positive answer if E is separable. First we give a general relation between E and $G^\infty(U)$ in connection with the BAP.

If U is a bounded open subset of a Banach space E, then by [10, Proposition 2.3], E is topologically isomorphic to a complemented subspace of $G^\infty(U)$. In the following proposition $P \in L(G^\infty(U);E)$ will indicate the relevant projection.

Proposition 3. *Let U be an open subset of a Banach space E and let $1 \leq \lambda < \infty$. Consider the following statements*:

(a) *For each Banach space F, $B_{\mathcal{H}^\infty(U;F)} = \overline{B^{\lambda}_{\mathcal{H}^\infty(U)\otimes F}}^{\tau_c}$.*
(b) *$G^\infty(U)$ has the λ-BAP.*
(c) *E has the $\lambda\|P\|$-BAP.*

(d) *For each Banach space* F *and for each open subset* $V \subset F$,

$$B_{\mathcal{H}^\infty(V;E)} = \overline{B^{\lambda\|P\|}_{\mathcal{H}^\infty(V)\otimes E}}^{\tau_c}.$$

(e) $I_U \in \overline{B^{\lambda\|P\|}_{\mathcal{H}^\infty(U)\otimes E}}^{\tau_c}$.

Then we always have the implications (a)⇔(b) *and* (c)⇒(d). *If, in addition,* U *is bounded then* (b)⇒(c) *and* (d)⇒(c). *If* U *is the open unit ball then also* (d)⇒(e).

Proof. The equivalence (a)⇔(b) follows easily from Proposition 1 and [10, Theorem 2.1, Proposition 3.1 and Proposition 4.9], and (c)⇒(d) follows from Proposition 1 and [10, Theorem 2.1, Corollary 4.10 and Proposition 3.1].

If U is bounded, we get (b)⇒(c) from [10, Proposition 2.3]. Now by modifying an idea of Mujica [10, p. 883] we shall show that (d) implies (c). From (d), by using, [10, Theorem 2.1, Proposition 3.1 and Corollary 4.10], we deduce that

$$B_{L(G^\infty(U);E)} = \overline{B^{\lambda\|P\|}_{G^\infty(U)'\otimes E}}^{\tau_c}$$

for every Banach space F. Let $A \in B_{L(F;E)}$ be given. By [10, Proposition 2.3] there are operators $S \in B_{L(F;G^\infty(U_F))}$ and $S \in B_{L(G^\infty(U_F);F)}$ such that $T \circ S(y) = y$ for every $y \in F$. Then $A \circ T \in B_{L(G^\infty(U_F);E)}$ and therefore there is a net $(B_\alpha)_{\alpha\in\Lambda} \subset B^{\lambda\|P\|}_{G^\infty(U_F)'\otimes E}$ which converges to $A\circ T$ for τ_c. Hence $(B_\alpha \circ S)_{\alpha\in\Lambda} \subset B^{\lambda\|P\|}_{F'\otimes E}$ and $B_\alpha S \to A \circ T \circ S = A$ for the topology τ_c. Thus, by Proposition 1, E has the $\lambda\|P\|$-BAP.

Finally, if U is the open unit ball of E then (d)⇒(e) follows from the fact that $I_U \in B_{\mathcal{H}^\infty(U;E)}$. ∎

Note that, if U is a bounded open subset of a separable Banach space E which has the AP, but does not have the BAP (see [5]), then $G^\infty(U)$ has the AP [10, Theorem 5.4], but does not have the BAP by Proposition 3. We do not know, in the preceding proposition, if the implication (c)⇒(b) is true for any Banach space E, i.e., whether or not $G^\infty(U)$ has the BAP whenever E does. As stated above, we have an affirmative answer for E separable. Before proving this result we need some preparation.

If E has a Schauder basis $(e_n)_{n=1}^\infty$ then $E_n := \langle e_1, \ldots, e_n\rangle$ denotes the subspace generated by $e_1, \ldots, e_n$, and $T_n : E \to E_n$ the canonical projection, for each $n \in \mathbb{N}$. If U is an open subset of E then we set $U_n := U \cap E_n$ for each $n \in \mathbb{N}$.

The following lemma, due to Mujica [8], plays an important role in the proof of Theorem 1 (see also [4, Lemma 3.1]).

LEMMA 1 ([8], [4]). *Let* E *be a Banach space with a Schauder basis. Let* U *be a balanced convex open subset of* E. *Then there are three increas-*

ing sequences $(A_n)_{n=1}^{\infty}$, $(B_n)_{n=1}^{\infty}$ *and* $(C_n)_{n=1}^{\infty}$ *of open subsets of* U *with the following properties*:

(a) $C_n \subset B_n \subset A_n$ *for every* n, *and* $\bigcup_{n=1}^{\infty} C_n = U$.
(b) $T_n(A_n) \subset U_n \subset A_n$ *for every* n.
(c) $B_j \cap U_n$ *is a compact subset of* $A_j \cap U_n$ *for every* j *and* n.
(d) $T_n(C_j) \subset B_j \cap U_n$ *whenever* $n \geq j$.
(e) *For each compact subset* K *of* U *and balanced convex open neighborhood of zero* V *in* E *such that* $K + V \subset U$, *there exists* $n_0 \in \mathbb{N}$ *such that* $K \subset C_n$ *and* $T_n(x) \in x + V$ *whenever* $x \in K$ *and* $n \geq n_0$.

The proof of the following result was suggested to us by Professor Jorge Mujica. This greatly simplified our original proof.

THEOREM 1. *Let* E *be a Banach space with a monotone Schauder basis. Then* $\mathcal{H}^{\infty}(U_E) \otimes F$ *is sequentially dense in* $(\mathcal{H}^{\infty}(U_E; F), \tau_{\mathrm{c}})$ *for every Banach space* F.

Proof. Let $f \in \mathcal{H}^{\infty}(U_E; F)$ be given. We may assume, without loss of generality, that $\|f\| \leq 1$. Since, by hypothesis, E has the MAP (see, for example, [7, p. 38]), by [10, Proposition 5.7] we have $B_{\mathcal{H}^{\infty}(U_E;F)} = \overline{B_{\mathcal{H}^{\infty}(U_E) \otimes F}}^{\tau_{\mathrm{c}}}$. Hence, there exists a sequence $(h_n)_{n=1}^{\infty} \subset B_{\mathcal{H}^{\infty}(U_E) \otimes F}$ such that

$$(*) \qquad \|h_n(y) - f(y)\| < 1/n \quad \text{for all } y \in B_n \cap U_n.$$

Since $\|h_n\| \leq 1$ for every n, the sequence $(h_n)_{n=1}^{\infty}$ is equicontinuous (see [9, Proposition 9.15]). It follows from $(*)$ that $(h_n(y))_{n=1}^{\infty}$ converges to $f(y)$ for every $y \in D = \bigcup_{n=1}^{\infty}(B_n \cap U_n)$. As E has the MAP, it is easy to see that D is dense in U_E by Lemma 1. As $(h_n)_{n=1}^{\infty}$ is equicontinuous, it follows that $(h_n(y))_{n=1}^{\infty}$ is a Cauchy sequence in F for every $y \in U_E$ (see the proof of [14, Theorem 3]).

Define $h(y) = \lim h_n(y)$ for every $y \in U_E$. Since $(h_n)_{n=1}^{\infty}$ is equicontinuous, h is continuous (see [9, Lemma 9.10]). Since $h(y) = f(y)$ for every $y \in D$, and D is dense in U_E, it follows that $h(y) = f(y)$ for every $y \in U_E$. Now, from equicontinuity of $(h_n)_{n=1}^{\infty}$, it follows that $(h_n)_{n=1}^{\infty}$ converges to f uniformly on compact subsets of U_E (see [9, Proposition 9.11]). This completes the proof. ∎

Using a result of Pełczyński we obtain the following stronger version of the previous theorem.

COROLLARY 2. *Let* E *be a separable Banach space with the BAP. Then* $\mathcal{H}^{\infty}(U_E) \otimes F$ *is sequentially dense in* $(\mathcal{H}^{\infty}(U_E; F), \tau_{\mathrm{c}})$ *for every Banach space* F.

Proof. By a result of Pełczyński [12] there exists a Banach space M with a Schauder basis, and a Banach space N, such that $M = E \oplus N$. We may assume that M has a monotone Schauder basis. Now, $U_E \oplus N$ is an open subset of $E \oplus N = M$. Let $\sigma : x \in U_E \mapsto x + 0 \in U_E \oplus N$ and let $\pi : x + y \in U_E \oplus N \mapsto x \in U_E$. If $f \in \mathcal{H}^\infty(U_E; F)$, then $f \circ \pi \in \mathcal{H}^\infty(U_E \oplus N; F)$. By Theorem 1 there exists a sequence $(h_n)_{n=1}^\infty \subset \mathcal{H}^\infty(U_E \oplus N) \oplus F$ which converges to $f \circ \pi$ in $(\mathcal{H}^\infty(U_E \oplus N; F), \tau_c)$. Hence the sequence $(h_n \circ \sigma)_{n=1}^\infty$ lies in $\mathcal{H}^\infty(U_E) \oplus F$, and converges to $f \circ \pi \circ \sigma = f$ in $(\mathcal{H}^\infty(U_E; F), \tau_c)$. ∎

To attain our main result we need the following proposition which shows that sequential convergence on $\mathcal{H}^\infty(U; F)$ for the topology τ_c implies sequential convergence on $L(G^\infty(U); F)$ for the topology of pointwise convergence.

PROPOSITION 4. *Let U be an open subset of a Banach space E and let F be a Banach space. If $\mathcal{H}^\infty(U) \otimes F$ is sequentially dense in $(\mathcal{H}^\infty(U; F), \tau_c)$, then for each $T \in L(G^\infty(U); F)$ there exists a sequence $(T_n)_{n=1}^\infty \subset G^\infty(U)' \otimes F$ such that $T_n(u) \to T(u)$ for every $u \in G^\infty(U)$.*

Proof. Let $T \in L(G^\infty(U); F)$ be given. Then by [10, Theorem 2.1] there exists a corresponding mapping $f \in \mathcal{H}^\infty(U; F)$ and hence, by hypothesis, there is a sequence $(f_n)_{n=1}^\infty \subset \mathcal{H}^\infty(U) \otimes F$ such that $f_n \to f$ for the compact-open topology τ_c. By [10, Theorem 2.1 and Proposition 3.1] there is a corresponding sequence $(T_{f_n})_{n=1}^\infty \subset G^\infty(U)' \otimes F$.

We claim that $T_{f_n}(u) \to T(u)$ for every $u \in G^\infty(U)$. Let $u \in G^\infty(U)$ and $\varepsilon > 0$ be given. By [10, Corollary 4.12] there exist $(\alpha_i)_{i=1}^\infty \subset l_1$ and $(x_i)_{i=1}^\infty \subset U$ such that $u = \sum_{i=1}^\infty \alpha_i \delta_U(x_i)$, where δ_U is the bounded holomorphic mapping given in [10, Theorem 2.1] for which $T_{f_n} \circ \delta_U = f_n$ for every $n \in \mathbb{N}$, and $T \circ \delta_U = f$. Let m be any positive integer and consider $u_m := \sum_{i=1}^m \alpha_i \delta_U(x_i)$. Since f_n converges to f for the topology τ_c, and $(x_i)_{i=1}^m$ is a compact subset of U, there is an $n_0 \in \mathbb{N}$ such that

$$\sup_{1 \le i \le m} \|f_n(x_i) - f(x_i)\| \le \frac{\varepsilon}{m \sup_{1 \le i \le m} |\alpha_i|}$$

for all $n \ge n_0$. Hence, for each $m \in \mathbb{N}$ we get

$$\begin{aligned}
\|T_{f_n}(u_m) - T(u_m)\| &= \Big\| \sum_{i=1}^m \alpha_i (T_{f_n}(\delta_U(x_i)) - T(\delta_U(x_i))) \Big\| \\
&\le \sum_{i=1}^m |\alpha_i| \, \|(T_{f_n} \circ \delta_U(x_i) - T \circ \delta_U(x_i)\| \\
&\le \sup_{1 \le i \le m} |\alpha_i| \sum_{i=1}^m \|f_n(x_i) - f(x_i)\| \\
&\le m \sup_{1 \le i \le m} |\alpha_i| \frac{\varepsilon}{m \sup_{1 \le i \le m} |\alpha_i|} = \varepsilon,
\end{aligned}$$

for all $n \ge n_0$. Therefore

$$\|T_{f_n}(u) - T(u)\| = \lim_{m \to \infty} \|T_{f_n}(u_m) - T(u_m)\| \le \varepsilon$$

for all $n \ge n_0$, which completes the proof. ∎

Now from Corollary 2 and Proposition 4 we obtain our main result.

COROLLARY 3. *A separable Banach space E has the BAP if and only if $G^\infty(U_E)$ has the BAP.*

Proof. Let E be a separable Banach space and suppose that E has the BAP. Let I be the identity mapping on $G^\infty(U_E)$. Then by Corollary 2 and Proposition 4 there exists a sequence $(T_n)_{n=1}^\infty \subset G^\infty(U_E)' \otimes G^\infty(U_E)$ such that $T_n(u) \to I(u)$ for every $u \in G^\infty(U_E)$. Since $G^\infty(U_E)$ is also a separable Banach space (see [10, Remark 2.2]), by a characterization of the BAP for separable Banach spaces (see, for example, [9, Theorem 27.4]), we conclude that $G^\infty(U_E)$ has the BAP. Since the converse implication follows from Proposition 3, the proof is complete. ■

Hence, the above result provides a positive answer to the problem posed by Mujica in [10] in the case of separable Banach spaces.

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