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A remark on extrapolation of rearrangement operators on dyadic H^s, $0 < s \leq 1$

by

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Abstract. For an injective map τ acting on the dyadic subintervals of the unit interval $[0,1)$ we define the rearrangement operator T_s, $0 < s < 2$, to be the linear extension of the map

$$\frac{h_I}{|I|^{1/s}} \mapsto \frac{h_{\tau(I)}}{|\tau(I)|^{1/s}},$$

where h_I denotes the L^∞-normalized Haar function supported on the dyadic interval I. We prove the following extrapolation result: If there exists at least one $0 < s_0 < 2$ such that T_{s_0} is bounded on H^{s_0}, then for all $0 < s < 2$ the operator T_s is bounded on H^s.

1. Introduction. In this paper we prove extrapolation estimates for rearrangement operators of the Haar system, normalized in H^s, $0 < s < 2$. Here H^s denotes the dyadic Hardy space of sequences $(g(I))_{I\in\mathcal{D}}$ for which

$$\|(g(I))_{I\in\mathcal{D}}\|_{H^s}^s := \int_0^1 \Big(\sum_{I\in\mathcal{D}} g(I)^2 h_I^2(x)\Big)^{s/2}\,dx < \infty. \tag{1}$$

In (1) we let $\mathcal{D}$ denote the collection of all dyadic intervals $[a,b)$ in the unit interval $[0,1)$ and correspondingly $(h_I)_{I\in\mathcal{D}}$ denotes the L^∞-normalized Haar system. For an injective map $\tau : \mathcal{D} \to \mathcal{D}$ we define the rearrangement operator T_s to be the linear extension of

$$T_s : \frac{h_I}{|I|^{1/s}} \mapsto \frac{h_{\tau(I)}}{|\tau(I)|^{1/s}}.$$

We show that

$$\|T_s : H^s \to H^s\|^{1-\theta} \leq c\|T_p : H^p \to H^p\|, \quad 0 < s < p < 2, \tag{2}$$

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where $0 < \theta < 1$ is chosen such that
$$\frac{1}{p} = \frac{1-\theta}{s} + \frac{\theta}{2},$$
$c > 0$ depends at most on s and p, and
$$\|T_s : H^s \to H^s\| := \sup\{\|T_s g\|_{H^s} : \|g\|_{H^s} \le 1\}.$$

The novelty of (2) lies in the range of admissible values for s. In [4] the estimate (2) was obtained for the range $1 \le s < p < 2$. The proof in [4] is based on duality and therefore strictly limited to the case $s \ge 1$. An alternative proof of (2) for $1 \le s < p < 2$ has been given by exploiting the norm devised by G. Pisier in the context of general Banach lattices [6]. For example, for $g = (g(I))_{I \in \mathcal{D}} \in H^1$ Pisier's result reads in our setting as

$$(3) \qquad \frac{1}{d}\|g\|_{H^1}^{1-\theta} \le \sup\Big\{\Big\|\sum_{I \in \mathcal{D}} |g(I)|^{1-\theta}|w(I)|^{\theta} h_I\Big\|_{H^p} : \|w\|_{H^2} \le 1\Big\} \le \|g\|_{H^1}^{1-\theta}$$

with $0 < \theta < 1$ and
$$\frac{1}{p} = 1 - \frac{\theta}{2},$$
where $d \ge 1$ depends at most on p and θ. We do not know who should be credited for finding the proof of (2), $1 \le s < p < 2$, using (3). A proof of (3) follows by specializing the ideas of G. Pisier to the context of H^1. The work of M. Cwikel, P. G. Nilsson and G. Schechtman [1, Ch. 3] plays a crucial role in linking (3) to G. Pisier's original construction [6].

2. Extrapolation estimates. The aim of this paper is to present a proof of the following two theorems.

THEOREM 1. *Let $\tau : \mathcal{D} \to \mathcal{D}$ be an injection, and let $0 < s < p < 2$ and $0 < \theta < 1$ be such that*
$$\frac{1}{p} = \frac{1-\theta}{s} + \frac{\theta}{2}.$$
Then there exists a constant $c > 0$, depending at most on s and p, such that
$$\frac{1}{c}\|T_s : H^s \to H^s\|^{1-\theta} \le \|T_p : H^p \to H^p\| \le c\|T_s : H^s \to H^s\|^{1-\theta}.$$

The point of the above theorem is the left-hand inequality which corresponds to an *extrapolation.* The right-hand one is rather standard and follows by interpolation. The proof of the extrapolation inequality is based on

THEOREM 2. *For $0 < s < p < q \le 2$ and $0 < \theta < 1$ such that*
$$\frac{1}{p} = \frac{1-\theta}{s} + \frac{\theta}{q}$$

there exists a constant $c > 0$, *depending at most on* s, p, *and* q, *such that*

$$\frac{1}{c}\,\|g\|_{H^s}^{1-\theta} \le \sup\Big\{\Big\|\sum_{I\in\mathcal{D}} |g(I)|^{1-\theta}|w(I)|^{\theta}h_I\Big\|_{H^p} : \|w\|_{H^q} \le 1\Big\} \le \|g\|_{H^s}^{1-\theta} \tag{4}$$

for all $g \in H^s$.

The main estimate in Theorem 2 is the left-hand inequality for which we present two approaches. One is by reduction to the case of Banach lattices and duality. The second approach is via Theorem 5 which is the desired inequality for $q = 2$ and $p + s \ge 2$ and which is—despite the parameter restriction—sufficient to prove the extrapolation part of Theorem 1 as well. The proof of Theorem 5 circumvents the use of duality and is based instead on the atomic decomposition; it provides additional information by finding a particular w_0 that realizes the supremum in (4) up to a multiplicative constant.

Let us start with the proof of Theorem 2 by introducing the following Banach lattices of Triebel type.

Definition 3. For $1 \le \alpha < \infty$ we let

$$f_1^{\alpha} := \Big\{g = (g(I))_{I\in\mathcal{D}} : \|g\|_{f_1^{\alpha}} := \Big\|\Big(\sum_{I\in\mathcal{D}} |g(I)|^{\alpha}h_I^2\Big)^{1/\alpha}\Big\|_{L^1} < \infty\Big\}.$$

The lattice structure of the spaces f_1^{α} is defined through the canonical lattice structure of the *sequences* $(g(I))_{I\in\mathcal{D}}$. The Triebel spaces f_1^{α} form an interpolation scale compatible with the Calderón product: For

$$\frac{1}{\gamma} = \frac{1-\eta}{\alpha} + \frac{\eta}{\beta},$$

$0 < \eta < 1$, and $1 \le \alpha < \gamma < \beta < \infty$, M. Frazier and B. Jawerth [2, Theorem 8.2] ([1]) proved that

$$\|g\|_{f_1^{\gamma}} \le \|g\|_{(f_1^{\alpha})^{1-\eta}(f_1^{\beta})^{\eta}} \le c\|g\|_{f_1^{\gamma}} \tag{5}$$

with $c \ge 1$ depending at most on α, γ, and β, where the Calderón product is given by

$$\|g\|_{(f_1^{\alpha})^{1-\eta}(f_1^{\beta})^{\eta}} := \inf\{\|g_0\|_{f_1^{\alpha}}^{1-\eta}\|g_1\|_{f_1^{\beta}}^{\eta} : |g| = |g_0|^{1-\eta}|g_1|^{\eta}\}.$$

(The left-hand inequality of (5) follows by an appropriate application of Hölder's inequality.) Our main tool will be the extrapolation formula

$$\|g\|_{f_1^{\beta}}^{\eta} = \sup\big\{\big\||g|^{\eta}|w|^{1-\eta}\big\|_{(f_1^{\alpha})^{1-\eta}(f_1^{\beta})^{\eta}} : \|w\|_{f_1^{\alpha}} \le 1\big\} \tag{6}$$

with $1 \le \alpha < \beta < \infty$ and $0 < \eta < 1$ from M. Cwikel, P. G. Nilsson and G. Schechtman [1, Theorem 3.5].

([1]) The spaces we use are complemented subspaces of the spaces $\dot{f}_1^{-1/2,p}$ from [2, p. 38], complemented in a way that [2, Theorem 8.2] remains true.

Proof of Theorem 2. For $0 < t \le 2$ and $g = (g(I))_{I\in\mathcal{D}}$ we get

$$(7)\qquad \|g\|_{H^t}^t = \int_0^1 \Big(\sum_{I\in\mathcal{D}} g(I)^2 h_I^2(x)\Big)^{t/2} dx = \int_0^1 \Big(\sum_{I\in\mathcal{D}} (|g(I)|^t)^{2/t} h_I^2(x)\Big)^{t/2} dx = \big\||g|^t\big\|_{f_1^{2/t}}.$$

Consequently, rewriting (4) we need to prove that

$$\frac{1}{c}\,\big\||g|^s\big\|_{f_1^{2/s}}^{(1-\theta)/s} \le \sup\big\{\|(|g(I)|^{p(1-\theta)}|w(I)|^{p\theta})_{I\in\mathcal{D}}\|_{f_1^{2/p}}^{1/p} : \big\||w|^q\big\|_{f_1^{2/q}}^{1/q} \le 1\big\} \le \big\||g|^s\big\|_{f_1^{2/s}}^{(1-\theta)/s}.$$

Replacing in the above estimates g by $|g|^{1/s}$ and w by $|w|^{1/q}$ we obtain

$$(8)\qquad \frac{1}{c^p}\,\|g\|_{f_1^{2/s}}^{p(1-\theta)/s} \le \sup\{\|(|g(I)|^{p(1-\theta)/s}|w(I)|^{p\theta/q})_{I\in\mathcal{D}}\|_{f_1^{2/p}} : \|w\|_{f_1^{2/q}} \le 1\} \le \|g\|_{f_1^{2/s}}^{p(1-\theta)/s}.$$

With $\alpha := 2/q$, $\beta := 2/s$, $\gamma := 2/p$, and $\eta := (q-p)/(q-s) \in (0,1)$ so that

$$1 \le \alpha < \gamma < \beta, \qquad \frac{1}{\gamma} = \frac{1-\eta}{\alpha} + \frac{\eta}{\beta},$$

the estimates (8) are equivalent to

$$\frac{1}{c^p}\,\|g\|_{f_1^\beta}^\eta \le \sup\{\|(|g(I)|^\eta|w(I)|^{1-\eta})_{I\in\mathcal{D}}\|_{f_1^\gamma} : \|w\|_{f_1^\alpha} \le 1\} \le \|g\|_{f_1^\beta}^\eta,$$

which follows immediately from (5) and (6). ∎

Proof of Theorem 1. (a) First we prove the left-hand inequality. Assume that $\|T_p : H^p \to H^p\| < \infty$ (otherwise there is nothing to prove). Fix $g = (g(I))_{I\in\mathcal{D}} \in H^s$ and $w = (w(I))_{I\in\mathcal{D}} \in H^2$. Define

$$u := \sum_{I\in\mathcal{D}} |g(I)|^{1-\theta}|w(I)|^\theta h_I.$$

As $1/p = (1-\theta)/s + \theta/2$ we have

$$(T_p u)(J) = |(T_s g)(J)|^{1-\theta}|(T_2 w)(J)|^\theta$$

for the corresponding Haar coefficients. By Theorem 2 we get

$$\frac{1}{c}\,\|T_s g\|_{H^s}^{1-\theta} \le \sup\Big\{\Big\|\sum_{J\in\mathcal{D}} |(T_s g)(J)|^{1-\theta}|w(J)|^\theta h_J\Big\|_{H^p} : \|w\|_{H^2} \le 1\Big\}.$$

Since T_2 preserves the H^2-norm and the supremum in the above expression can be restricted to those w such that $w(J) = 0$ whenever $J \notin \tau(\mathcal{D})$, we can rewrite the above inequality as

$$\frac{1}{c}\,\|T_s g\|_{H^s}^{1-\theta} \le \sup\Big\{\Big\|\sum_{J\in\mathcal{D}} |(T_s g)(J)|^{1-\theta}|(T_2 w)(J)|^{\theta} h_J\Big\|_{H^p} : \|T_2 w\|_{H^2}\le 1\Big\}$$
$$= \sup\Big\{\Big\|T_p\Big(\sum_{I\in\mathcal{D}} |g(I)|^{1-\theta}|w(I)|^{\theta} h_I\Big)\Big\|_{H^p} : \|w\|_{H^2}\le 1\Big\}.$$

As T_p is bounded on H^p,

$$\sup\Big\{\Big\|T_p\Big(\sum_{I\in\mathcal{D}} |g(I)|^{1-\theta}|w(I)|^{\theta} h_I\Big)\Big\|_{H^p} : \|w\|_{H^2}\le 1\Big\}$$
$$\le \|T_p : H^p\to H^p\| \sup\Big\{\Big\|\sum_{I\in\mathcal{D}} |g(I)|^{1-\theta}|w(I)|^{\theta} h_I\Big\|_{H^p} : \|w\|_{H^2}\le 1\Big\}.$$

By Theorem 2 the supremum above is bounded by $\|g\|_{H^s}^{1-\theta}$ so that

$$\frac{1}{c}\,\|T_s g\|_{H^s}^{1-\theta} \le \|T_p : H^p\to H^p\|\,\|g\|_{H^s}^{1-\theta}$$

and the claim follows.

(b) Because $\|T_2 g\|_{H^2} = \|g\|_{H^2}$ the right-hand inequality follows from a general interpolation property of the operators T_p: for $0<s<p<q\le 2$ and $0<\theta'<1$ with $1/p = (1-\theta')/s + \theta'/q$ one has

$$\|T_p : H^p\to H^p\| \le c\|T_s : H^s\to H^s\|^{1-\theta'}\|T_q : H^q\to H^q\|^{\theta'} \tag{9}$$

where $c>0$ depends at most on s, p, and q. There are different ways to deduce (9). We reduce the family of operators $(T_p)_{0<p\le 2}$ to a *single* operator T and exploit the interpolation property of the Calderón product. The map T is given by $T((a(I))_{I\in\mathcal{D}}) := (g(J))_{J\in\mathcal{D}}$ with

$$g(J) := \begin{cases} a(\tau^{-1}(J))\,\dfrac{|\tau^{-1}(J)|}{|J|}, & J\in\tau(\mathcal{D}),\\ 0, & J\notin\tau(\mathcal{D}),\end{cases}$$

so that, for $0<t\le 2$,

$$\begin{aligned} ||T_t g||_{H^t}^t &= \int_0^1\Big(\sum_{I\in\mathcal{D}}\Big[g(I)\Big(\frac{|I|}{|\tau(I)|}\Big)^{1/t}\Big]^2 h_{\tau(I)}^2(x)\Big)^{t/2} dx\\ &= \int_0^1\Big(\sum_{I\in\mathcal{D}}\Big[|g(I)|^t\,\frac{|I|}{|\tau(I)|}\Big]^{2/t} h_{\tau(I)}^2(x)\Big)^{t/2} dx\\ &= \int_0^1\Big(\sum_{J\in\tau(\mathcal{D})}\Big[|g(\tau^{-1}(J))|^t\,\frac{|\tau^{-1}(J)|}{|J|}\Big]^{2/t} h_J^2(x)\Big)^{t/2} dx = \|T(|g|^t)\|_{f_1^{2/t}}.\end{aligned}$$

Together with (7) this implies

$$\|T_t : H^t\to H^t\|^t = \|T : f_1^{2/t}\to f_1^{2/t}\|. \tag{10}$$

Now, from (5), [2, Proposition 8.1], and the positivity of T we obtain

$$\|T : f_1^\gamma \to f_1^\gamma\| \le c\|T : f_1^\alpha \to f_1^\alpha\|^{1-\eta}\|T : f_1^\beta \to f_1^\beta\|^\eta$$

for $1 \le \alpha < \gamma < \beta < \infty$ and $0 < \eta < 1$ such that $1/\gamma = (1-\eta)/\alpha + \eta/\beta$, where $c > 0$ depends at most on α, β, and γ. Together with (10) we end up with (9) by letting $\alpha = 2/q$, $\beta = 2/s$, and $\gamma = 2/p$. ∎

3. A constructive aspect of Theorem 2. Given $g \in H^s$ it follows from Theorem 2 that there exists a $w_0 \in H^2$ with $\|w_0\|_{H^2} = 1$ such that

$$\|g\|_{H^s}^{1-\theta} \sim \Big\| \sum_{I \in \mathcal{D}} |g(I)|^{1-\theta}|w_0(I)|^\theta h_I \Big\|_{H^p}$$

whenever $0 < s < p < 2$, $0 < \theta < 1$, and $1/p = (1-\theta)/s + \theta/2$. The duality proof of Theorem 2 yields just the existence of such a $w_0 \in H^2$. In order to get an explicit formula for $w_0 \in H^2$ we exploit an atomic decomposition for $g \in H^s$ in this section. To simplify the notion we use the square function

$$S(g)(x) := \Big(\sum_{I \in \mathcal{D}} g(I)^2 h_I^2(x) \Big)^{1/2} \quad \text{for } g = (g(I))_{I \in \mathcal{D}} \in H^s.$$

The following lemma summarizes the properties of the stopping time decomposition originating with the work of S. Janson and P. W. Jones [3].

LEMMA 4. *Let $0 < s, p < \infty$ and $g = (g(I))_{I \in \mathcal{D}} \in H^s$. Then there exists a system $\mathcal{E} \subseteq \mathcal{D}$ of dyadic intervals and $\mathcal{T}(K) \subseteq \mathcal{D}$ for $K \in \mathcal{E}$ such that, for*

$$g_K := \sum_{I \in \mathcal{T}(K)} g(I) h_I,$$

one has the following:

(i) $(\mathcal{T}(K))_{K \in \mathcal{E}}$ *is a disjoint partition of* $\mathcal{D}$,

(ii) $\mathrm{supp}(S(g_K)) \subseteq K$,

(iii) *there is a constant $c > 0$, depending on s only, such that*

$$\sum_{K \in \mathcal{E}} \|S(g_K)\|_\infty^s |K| \le c\|g\|_{H^s}^s, \tag{11}$$

(iv) *there is an absolute constant $d \ge 1$ such that*

$$\sum_{K \in \mathcal{E}} |\alpha(K)|^p \|g_K\|_{H^p}^p \le d \Big\| \sum_{K \in \mathcal{E}} \alpha(K) g_K \Big\|_{H^p}^p \tag{12}$$

for any sequence of scalars $(\alpha_K)_{K \in \mathcal{E}}$ where the sides might be infinite.

The above decomposition is obtained by applying a stopping time procedure based on the size of the square function $S(g)$. This argument is due to S. Janson and P. W. Jones [3]; it is reproduced in many places, for instance in [5] (cf. Theorem 2.3.3 and Proposition 3.1.5). By renumbering we replace $(g_K, K)_{K\in\mathcal{E}}$ by $(g_i, I_i)_{i\in\mathcal{N}}$ with $\mathcal{N} \subseteq \{1, 2, \ldots\}$. The family (g_i, I_i) is called an *atomic decomposition* of g where we may assume without loss of generality that $\|g_i\|_{H^2} = \|S(g_i)\|_2 > 0$ for all i by leaving out those elements g_K with $\|g_K\|_{H^2} = 0$.

THEOREM 5. *Let $0 < s < p < 2$ and $p + s \geq 2$, and let $0 < \theta < 1$ be such that*

$$\frac{1}{p} = \frac{1-\theta}{s} + \frac{\theta}{2}.$$

For $g \in H^s$ with $\|g\|_{H^s} > 0$ and atomic decomposition (g_i, I_i) define

$$w_0 := \|g\|_{H^s}^{-s/2} \sum_i Y_i^{1/2} g_i \quad \textit{where} \quad Y_i := \frac{\|S(g_i)\|_\infty^s |I_i|}{\|S(g_i)\|_2^2}.$$

Then $w_0 \in H^2$ with $\|w_0\|_{H^2} \leq c$ with $c > 0$ depending on s only, and

$$\|g\|_{H^s}^{1-\theta} \leq d \Big\| \sum_{I\in\mathcal{D}} |g(I)|^{1-\theta} |w_0(I)|^\theta h_I \Big\|_{H^p}$$

where $d > 0$ is an absolute constant.

Proof. We may assume that $\|g\|_{H^s} = 1$ in the following. As the sequence (g_i) is disjointly supported over the Haar system, we have $S^2(w_0) = \sum_i Y_i S(g_i)^2$. Inserting the definition of Y_i and using the estimate (11) yields

$$\|w_0\|_{H^2} = \Big(\sum_i \|S(g_i)\|_\infty^s |I_i| \Big)^{1/2} \leq c^{1/2} \|g\|_{H^s}^{s/2} = c^{1/2} < \infty.$$

Let $(g_i(I))_{I\in\mathcal{D}}$ denote the Haar coefficients of g_i. Because

$$\sum_{I\in\mathcal{D}} |g(I)|^{1-\theta} |w_0(I)|^\theta h_I = \sum_i Y_i^{\theta/2} |g_i|,$$

from (12) we get

$$\sum_i Y_i^{\theta p/2} \|g_i\|_{H^p}^p \leq d \Big\| \sum_{I\in\mathcal{D}} |g(I)|^{1-\theta} |w_0(I)|^\theta h_I \Big\|_{H^p}^p$$

where the right-hand side is finite because $g \in H^s$, $w_0 \in H^2$, and by the right-hand inequality of (4) (we are interested in an alternative proof for the left-hand side). As $s \leq 2$ we have

$$S(g)^s = \Big(\sum_i S(g_i)^2 \Big)^{s/2} \leq \sum_i S(g_i)^s$$

so that $1 = \|g\|_{H^s}^s \leq \sum_i \|g_i\|_{H^s}^s$. Thus in order to prove

$$1 \leq d \Big\| \sum_{I\in\mathcal{D}} |g(I)|^{1-\theta} |w_0(I)|^\theta h_I \Big\|_{H^p}^p$$

it suffices to show
$$\sum_i \|g_i\|_{H^s}^s \le \sum_i Y_i^{\theta p/2} \|g_i\|_{H^p}^p .$$
Since
$$Y_i^{\theta p/2}\|g_i\|_{H^p}^p = \|S(g_i)\|_\infty^{\theta ps/2} |I_i|^{\theta p/2} \|S(g_i)\|_2^{-\theta p} \|g_i\|_{H^p}^p$$
we will prove that
$$\Big(\int_0^1 S(g_i)^s(x)\,dx\Big)\Big(\int_0^1 S(g_i)^2(x)\,dx\Big)^{\theta p/2} \le \|S(g_i)\|_\infty^{\theta ps/2} |I_i|^{\theta p/2} \Big(\int_0^1 S(g_i)^p(x)\,dx\Big).$$
Replacing dx by $dx/|I|$ and taking into the account that the support of $S(g_i)$ is contained in I_i we only need to prove for a non-negative random variable Z that
$$(EZ^s)(EZ^2)^{\theta p/2} \le \|Z\|_\infty^{\theta ps/2} EZ^p,$$
which follows from
$$\begin{aligned}(EZ^s)(EZ^2)^{\theta p/2} &\le (EZ^s)(EZ^{2-s})^{\theta p/2}\|Z\|_\infty^{\theta ps/2} \\ &\le (EZ^p)^{s/p}(EZ^p)^{(2-s)\theta p/(2p)}\|Z\|_\infty^{\theta ps/2}\end{aligned}$$
and
$$\frac{s}{p} + \frac{2-s}{p}\,\frac{\theta p}{2} = 1. \ \blacksquare$$

Second proof of the left-hand inequality of Theorem 1. For $0 < s < p < 2$ we find $p \le p' < 2$ such that $s + p' \ge 2$. Then we get
$$\begin{aligned}\|T_s : H^s \to H^s\| &\le c_1 \|T_{p'} : H^{p'} \to H^{p'}\|^{(1-\theta_1)^{-1}} \\ &\le c_2 \|T_p : H^p \to H^p\|^{(1-\theta_1)^{-1}(1-\theta_2)}\end{aligned}$$
where
$$\frac{1}{p'} = \frac{1-\theta_1}{s} + \frac{\theta_1}{2} \quad \text{and} \quad \frac{1}{p'} = \frac{1-\theta_2}{p} + \frac{\theta_2}{2}$$
with $c_1, c_2 > 0$ depending at most on s, p, and p' and where we used in the first step Theorem 5 together with the arguments of part (a) of the proof of Theorem 1, and in the second one formula (9) for $q = 2$ (note that T_2 preserves the H^2-norm). Because
$$(1-\theta_1)^{-1}(1-\theta_2) = (1-\theta)^{-1}$$
with θ defined in Theorem 1, we are done. $\blacksquare$

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