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Corrigendum to "Carleson measures associated with families of multilinear operators"

(Studia Math. 211 (2012), 71–94)

by

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Abstract. We provide a modification for part of the proof of Theorem 1.2 of our article, pages 85–89, under the multivariable $T(1)$ cancellation condition.

In this note we fix an erroneous derivation in [2]. We do not introduce any notation here but we adhere to the notation introduced in that article.

We reexamine the pointwise estimates for $L_{t,s_1,\ldots,s_m}$, defined in equation (4.21) of [2] as the kernel of the m-linear operator $\Theta_t(Q_{s_1}f_1,\ldots,Q_{s_m}f_m)$. We claimed in (4.25) that when $s_1,\ldots,s_m \geq t$ we have

$$|\Theta_t(Q_{s_1}f_1,\ldots,Q_{s_m}f_m)| \lesssim w(s_1,\ldots,s_m,t)\prod_{i=1}^{m} M(f_i)\,, \tag{0.1}$$

where

$$w(s_1,\ldots,s_m,t) = \prod_{i=1}^{m} \min\left(\frac{t}{s_i},\frac{s_i}{t}\right)^{\epsilon} \tag{0.2}$$

for some $\epsilon > 0$. Although (0.1) holds for some function $w(s_1,\ldots,s_m,t)$, it is not valid for the specific function in (0.2); in particular it is not the case that

$$\sup_t \int_0^\infty \cdots \int_0^\infty w(s_1,\ldots,s_m,t)\,\frac{ds_1}{s_1}\cdots\frac{ds_m}{s_m} < \infty\,,$$

which is required to complete the proof in [2].

In what follows, we fix this point providing an alternative argument, which resembles the approach in [3]. Basically, we need to prove the in-

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equality

$$|\Theta_t \Pi_{j,s}(f_1,\dots,f_m)(x)| \lesssim w(t,s) \prod_{i\neq j} M(f_i)(x) \sum_{k=1}^{n} M(Q_s^{2,k} f_j)(x).$$

We first state a proposition about the Calderón reproducing formulae for tensor products that will be useful in this revision.

Proposition 0.1. *Denote by*

$$(f_1 \otimes \dots \otimes f_m)(x_1,\dots,x_m) = f_1(x_1)\dots f_m(x_m)$$

the tensor product of m *functions and let* $P_s f = \varphi_s \star f$ *be a convolution operator with a nice function that satisfies* $P_s^2 f \to f$ *when* $s \to 0$ *and* $P_s^2 f \to 0$ *when* $s \to \infty$ *(convergence in* L^p *norm or in the sense of distributions). Then the following Calderón representation formulae hold (see* [1, p. 199] *for the case* $m = 1$*) for Schwartz functions* f_j*:*

$$\begin{aligned} f_1 \otimes \dots \otimes f_m &= \lim_{\epsilon\to 0} (P_\epsilon^2 f_1 \otimes \dots \otimes P_\epsilon^2 f_m - P_{1/\epsilon}^2 f_1 \otimes \dots \otimes P_{1/\epsilon}^2 f_m) \\ &= \lim_{\epsilon\to 0} \int_\epsilon^{1/\epsilon} s\frac{d}{ds}(P_s^2 f_1 \otimes \dots \otimes P_s^2 f_m)\,\frac{ds}{s} \\ &= \int_0^\infty s\frac{d}{ds}(P_s^2 f_1 \otimes \dots \otimes P_s^2 f_m)\,\frac{ds}{s} \\ &= \sum_{j=1}^{m} \int_0^\infty \Pi_{j,s}(f_1,\dots,f_m)\,\frac{ds}{s}, \end{aligned}$$

where

$$\Pi_{j,s}(f_1,\dots,f_m) = P_s^2 f_1 \otimes \dots \otimes \left(s\frac{d}{ds}P_s^2 f_j\right) \otimes \dots \otimes P_s^2 f_m$$

and where $s\frac{d}{ds}P_s^2$ *are operators of the type* Q_s^2 *introduced in* [2, p. 75]*, that is, squares of Littlewood–Paley projections.*

The following formula, which can be found in [1], gives an explicit expression for the derivatives of squares of Littlewood–Paley projections (where now we adopt the notation of [2], and write Q_s^2 instead of P_s^2 for the Littlewood–Paley projections):

$$s\frac{d}{ds}Q_s^2 = \sum_{k=1}^{n} Q_s^{k,1} Q_s^{k,2}.$$

In the preceding expression, $Q_s^{k,1}, Q_s^{k,2}$ are operators given by multiplication on the Fourier transform with bumps supported in balls and annuli, respectively, of size comparable to s^{-1}.

We can use all this information together to obtain the decomposition

$$\begin{aligned}\Theta_t(f_1,\dots,f_m) &= \widetilde{\Theta}_t(f_1\otimes\cdots\otimes f_m) = \widetilde{\Theta}_t\biggl(\int_0^\infty s\frac{d}{ds}(Q_s^2 f_1\otimes\cdots\otimes Q_s^2 f_m)\,\frac{ds}{s}\biggr)\\ &= \widetilde{\Theta}_t\biggl(\sum_{j=1}^m\int_0^\infty \Pi_{j,s}(f_1,\dots,f_m)\,\frac{ds}{s}\biggr)\\ &= \sum_{j=1}^m\int_0^\infty \Theta_t(\Pi_{j,s}(f_1,\dots,f_m))\,\frac{ds}{s}.\end{aligned}$$

Applying duality gives

$$\|S(f_1,\dots,f_m)\|_p = \sup_{\|h\|_{p',2}\le 1}\biggl|\int_{\mathbb{R}^n}\int_0^\infty \Theta_t(f_1,\dots,f_m)(x)h(x,t)\,\frac{dt}{t}\,dx\biggr|. \tag{0.3}$$

Using the above expression we obtain

$$\begin{aligned}&\biggl|\int_{\mathbb{R}^n}\int_0^\infty \Theta_t(f_1,\dots,f_m)(x)h(x,t)\,\frac{dt}{t}\,dx\biggr|\\ &\quad= \biggl|\int_{\mathbb{R}^n}\int_0^\infty\biggl(\sum_{j=1}^m\int_0^\infty \Theta_t\Pi_{j,s}(f_1,\dots,f_m)(x)\,\frac{ds}{s}\biggr)h(x,t)\,\frac{dt}{t}\,dx\biggr|\\ &\quad= \biggl|\sum_{j=1}^m\int_{\mathbb{R}^n}\int_0^\infty\int_0^\infty \Theta_t\Pi_{j,s}(f_1,\dots,f_m)(x)h(x,t)\,\frac{dt}{t}\,\frac{ds}{s}\,dx\biggr|\\ &\quad\le \sum_{j=1}^m\biggl|\int_{\mathbb{R}^n}\int_0^\infty\int_0^\infty \Theta_t\Pi_{j,s}(f_1,\dots,f_m)(x)h(x,t)\,\frac{dt}{t}\,\frac{ds}{s}\,dx\biggr|\\ &\quad\le \sum_{j=1}^m\biggl|\int_{\mathbb{R}^n}\biggl(\int_0^\infty\int_0^\infty |\Theta_t\Pi_{j,s}(f_1,\dots,f_m)(x)|^2 w(t,s)^{-1}\,\frac{ds}{s}\,\frac{dt}{t}\biggr)^{1/2}\\ &\qquad\times\biggl(\int_0^\infty\int_0^\infty |h(x,t)|^2 w(t,s)\,\frac{ds}{s}\,\frac{dt}{t}\biggr)^{1/2}dx\biggr|,\end{aligned} \tag{0.4}$$

where

$$w(t,s)=\min\biggl(\frac{t}{s},\frac{s}{t}\biggr)^{\epsilon}$$

for some $\epsilon>0$. An easy calculation allows us to deduce

$$\begin{aligned}&\biggl(\int_{\mathbb{R}^n}\biggl(\int_0^\infty\int_0^\infty |h(x,t)|^2 w(t,s)\,\frac{ds}{s}\,\frac{dt}{t}\biggr)^{p'/2}dx\biggr)^{1/p'}\\ &\qquad\lesssim \biggl(\int_{\mathbb{R}^n}\biggl(\int_0^\infty |h(x,t)|^2\,\frac{dt}{t}\biggr)^{p'/2}dx\biggr)^{1/p'} = \|h\|_{p',2}.\end{aligned}$$

Proceeding exactly as in the case of one-variable cancellation, we reduce the problem to showing that

$$|\Theta_t \Pi_{j,s}(f_1,\dots,f_m)(x)| \lesssim w(t,s) \prod_{i\neq j} M(f_i)(x) \sum_{k=1}^{n} M(Q_s^{2,k} f_j)(x). \tag{0.5}$$

This follows using the same idea as in the one-variable case.

References

[1] L. Grafakos, *Modern Fourier Analysis*, Grad. Texts in Math. 250, Springer, New York, 2008.

[2] L. Grafakos and L. Oliveira, *Carleson measures associated with families of multilinear operators*, Studia Math. 211 (2012), 71–94.

[3] J. Hart, *A new proof of the bilinear $T(1)$ theorem*, Proc. Amer. Math. Soc. (2013), to appear.

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