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The "Full Clarkson–Erdős–Schwartz Theorem" on the closure of non-dense Müntz spaces

by

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Abstract. Denote by $\mathrm{span}\{f_1, f_2, \dots\}$ the collection of all finite linear combinations of the functions $f_1, f_2, \dots$ over $\mathbb{R}$. The principal result of the paper is the following.

THEOREM (Full Clarkson–Erdős–Schwartz Theorem). *Suppose* $(\lambda_j)_{j=1}^{\infty}$ *is a sequence of distinct positive numbers. Then* $\mathrm{span}\{1, x^{\lambda_1}, x^{\lambda_2}, \dots\}$ *is dense in* $C[0,1]$ *if and only if*

$$\sum_{j=1}^{\infty} \frac{\lambda_j}{\lambda_j^2 + 1} = \infty.$$

Moreover, *if*

$$\sum_{j=1}^{\infty} \frac{\lambda_j}{\lambda_j^2 + 1} < \infty,$$

then every function from the $C[0,1]$ *closure of* $\mathrm{span}\{1, x^{\lambda_1}, x^{\lambda_2}, \dots\}$ *can be represented as an analytic function on* $\{z \in \mathbb{C} \setminus (-\infty, 0] : |z| < 1\}$ *restricted to* $(0,1)$.

This result improves an earlier result by P. Borwein and Erdélyi stating that if

$$\sum_{j=1}^{\infty} \frac{\lambda_j}{\lambda_j^2 + 1} < \infty,$$

then every function from the $C[0,1]$ closure of $\mathrm{span}\{1, x^{\lambda_1}, x^{\lambda_2}, \dots\}$ is in $C^{\infty}(0,1)$. Our result may also be viewed as an improvement, extension, or completion of earlier results by Müntz, Szász, Clarkson, Erdős, L. Schwartz, P. Borwein, Erdélyi, W. B. Johnson, and Operstein.

1. Introduction and notation.

Müntz's beautiful classical theorem characterizes sequences $(\lambda_j)_{j=0}^{\infty}$ with

$$0 = \lambda_0 < \lambda_1 < \lambda_2 < \dots$$

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for which the Müntz space $\mathrm{span}\{x^{\lambda_0}, x^{\lambda_1}, \ldots\}$ is dense in $C[0,1]$. Here, and in what follows, $\mathrm{span}\{x^{\lambda_0}, x^{\lambda_1}, \ldots\}$ denotes the collection of finite linear combinations of the functions $x^{\lambda_0}, x^{\lambda_1}, \ldots$ with real coefficients, and $C[a,b]$ is the space of all real-valued continuous functions on $[a,b] \subset \mathbb{R}$ equipped with the uniform norm. Müntz's Theorem [Bo-Er3, De-Lo, Go, Mü, Szá] states the following.

THEOREM 1.A (Müntz). *Suppose* $(\lambda_j)_{j=0}^{\infty}$ *is a sequence with* $0 = \lambda_0 < \lambda_1 < \ldots$ *Then* $\mathrm{span}\{x^{\lambda_0}, x^{\lambda_1}, \ldots\}$ *is dense in* $C[0,1]$ *if and only if* $\sum_{j=1}^{\infty} 1/\lambda_j = \infty$.

The original Müntz Theorem proved by Müntz [Mü] in 1914, by Szász [Szá] in 1916, and anticipated by Bernstein [Be] was only for sequences of exponents tending to infinity. The point 0 is special in the study of Müntz spaces. Even replacing $[0,1]$ by an interval $[a,b] \subset [0,\infty)$ in Müntz's Theorem is a non-trivial issue. This is, in large measure, due to Clarkson and Erdős [Cl-Er] and Schwartz [Sch] whose works include the result that if $\sum_{j=1}^{\infty} 1/\lambda_j < \infty$, then every function belonging to the uniform closure of $\mathrm{span}\{x^{\lambda_0}, x^{\lambda_1}, \ldots\}$ on $[a,b]$ can be extended analytically throughout the region $\{z \in \mathbb{C} \setminus (-\infty, 0] : |z| < b\}$.

There are many variations and generalizations of Müntz's Theorem [An, Be, Boa, Bo1, Bo2, Bo-Er1, Bo-Er2, Bo-Er3, Bo-Er4, Bo-Er5, Bo-Er6, Bo-Er7, B-E-Z, Ch, Cl-Er, De-Lo, Er-Jo, Go, Lu-Ko, Ma, Op, Sch, So]. There are also still many open problems. In [Bo-Er6] it is shown that the interval $[0,1]$ in Müntz's Theorem can be replaced by an arbitrary compact set $A \subset [0,\infty)$ of positive Lebesgue measure. That is, if $A \subset [0,\infty)$ is a compact set of positive Lebesgue measure, then $\mathrm{span}\{x^{\lambda_0}, x^{\lambda_1}, \ldots\}$ is dense in $C(A)$ if and only if $\sum_{j=1}^{\infty} 1/\lambda_j = \infty$. Here $C(A)$ denotes the space of all real-valued continuous functions on A equipped with the uniform norm. If A contains an interval then this follows from the already mentioned results of Clarkson, Erdős, and Schwartz. However, their results and methods cannot handle the case when, for example, $A \subset [0,1]$ is a Cantor-type set of positive measure.

In the case that $\sum_{j=1}^{\infty} 1/\lambda_j < \infty$, analyticity properties of the functions belonging to the uniform closure of $\mathrm{span}\{x^{\lambda_0}, x^{\lambda_1}, \ldots\}$ on A are also established in [Bo-Er6].

In [Bo-Er3, Section 4.2] and in [Bo-Er4] the following result is proved.

THEOREM 1.B (Full Müntz Theorem in $C[0,1]$). *Suppose* $(\lambda_j)_{j=1}^{\infty}$ *is a sequence of distinct positive real numbers. Then* $\mathrm{span}\{1, x^{\lambda_1}, x^{\lambda_2}, \ldots\}$ *is dense in* $C[0,1]$ *if and only if*

$$\sum_{j=1}^{\infty} \frac{\lambda_j}{\lambda_j^2 + 1} = \infty.$$

Moreover, *if*

$$\sum_{j=1}^{\infty} \frac{\lambda_j}{\lambda_j^2+1} < \infty,$$

then every function from the $C[0,1]$ *closure of* $\mathrm{span}\{1, x^{\lambda_1}, x^{\lambda_2}, \ldots\}$ *is infinitely many times differentiable on* $(0,1)$.

The new result of this paper is the following.

THEOREM 1.1 (Full Clarkson–Erdős–Schwartz Theorem). *Let* $(\lambda_j)_{j=1}^{\infty}$ *be a sequence of distinct positive numbers. Then* $\mathrm{span}\{1, x^{\lambda_1}, x^{\lambda_2}, \ldots\}$ *is dense in* $C[0,1]$ *if and only if*

$$\sum_{j=1}^{\infty} \frac{\lambda_j}{\lambda_j^2+1} = \infty.$$

Moreover, *if*

$$\sum_{j=1}^{\infty} \frac{\lambda_j}{\lambda_j^2+1} < \infty,$$

then every function from the $C[0,1]$ *closure of* $\mathrm{span}\{1, x^{\lambda_1}, x^{\lambda_2}, \ldots\}$ *can be represented as an analytic function on* $\{z \in \mathbb{C} \setminus (-\infty, 0] : |z| < 1\}$ *restricted to* $(0,1)$.

The notation

$$\|f\|_A := \sup_{x \in A} |f(x)|$$

is used throughout this paper for real-valued measurable functions f defined on a set $A \subset \mathbb{R}$.

2. Auxiliary results. The following result is the "bounded Remez-type inequality for non-dense Müntz spaces" due to P. Borwein and Erdélyi [Bo-Er6].

THEOREM 2.1. *Suppose* $(\gamma_j)_{j=1}^{\infty}$ *is a sequence of distinct positive numbers satisfying*

$$\sum_{j=1}^{\infty} 1/\gamma_j < \infty.$$

Let $s > 0$. *Then there exists a constant* $c_1(\Gamma, s)$ *depending only on* $\Gamma := (\gamma_j)_{j=1}^{\infty}$ *and* s (*and not on* ϱ, B, *or the "length" of* Q) *so that*

$$\|Q\|_{[0,\varrho]} \le c_1(\Gamma, s) \|Q\|_B$$

for every $Q \in \mathrm{span}\{1, x^{\gamma_1}, x^{\gamma_2}, \ldots\}$ *and for every set* $B \subset [\varrho, 1]$ *of Lebesgue measure at least* s.

Combining a result of Clarkson and Erdős [Cl-Er] and its extension given by Schwartz [Sch] we can state the following.

Theorem 2.2. *Suppose* $(\gamma_j)_{j=1}^{\infty}$ *is a sequence of distinct positive numbers satisfying* $\sum_{j=1}^{\infty} 1/\gamma_j < \infty$. *Then* $\operatorname{span}\{1, x^{\gamma_1}, x^{\gamma_2}, \ldots\}$ *is not dense in* $C[0,1]$. *In addition, if the gap condition*

$$\inf\{\gamma_{j+1} - \gamma_j : j = 1, 2, \ldots\} > 0 \tag{2.1}$$

holds, then every function $f \in C[0,1]$ *belonging to the* $C[0,1]$ *closure of* $\operatorname{span}\{1, x^{\gamma_1}, x^{\gamma_2}, \ldots\}$ *can be represented as*

$$f(x) = \sum_{j=0}^{\infty} a_j x^{\gamma_j}, \qquad x \in [0,1).$$

If the gap condition (2.1) *does not hold, then every function* $f \in C[0,1]$ *belonging to the* $C[0,1]$ *closure of* $\operatorname{span}\{1, x^{\gamma_1}, x^{\gamma_2}, \ldots\}$ *can still be represented as an analytic function on* $\{z \in \mathbb{C} \setminus (-\infty, 0] : |z| < 1\}$ *restricted to* $(0,1)$.

Now we offer a sufficient condition for $\operatorname{span}\{x^{\beta_1}, x^{\beta_2}, \ldots\}$ to be non-dense in $C[0,1]$ for a sequence $(\beta_j)_{j=1}^{\infty}$ of distinct positive numbers converging to 0.

Theorem 2.3. *Suppose that* $(\beta_j)_{j=1}^{\infty}$ *is a sequence of distinct real numbers greater than* 0 *satisfying*

$$\sum_{j=1}^{\infty} \beta_j =: \eta < \infty.$$

Then $\operatorname{span}\{x^{\beta_1}, x^{\beta_2}, \ldots\}$ *is not dense in* $C[0,1]$. *In addition, every function in the* $C[0,1]$ *closure of* $\operatorname{span}\{x^{\beta_1}, x^{\beta_2}, \ldots\}$ *can be represented as an analytic function on* $\mathbb{C} \setminus (-\infty, 0]$ *restricted to* $(0,1)$.

Proof. The theorem is a consequence of D. J. Newman's Markov-type inequality [Bo-Er3, Theorem 6.1.1 on page 276] (see also [Ne]). We state this as Theorem 2.4. Repeated application of Theorem 2.4 with the substitution $x = e^{-t}$ implies that

$$\|(Q(e^{-t}))^{(m)}\|_{[0,\infty)} \le (9\eta)^m \|Q(e^{-t})\|_{[0,\infty)}, \qquad m = 1, 2, \ldots,$$

in particular

$$|(Q(e^{-t}))^{(m)}(0)| \le (9\eta)^m \|Q(e^{-t})\|_{[0,\infty)}, \qquad m = 1, 2, \ldots,$$

for every $Q \in \operatorname{span}\{x^{\beta_1}, x^{\beta_2}, \ldots\}$. By using the Taylor series expansion of $Q(e^{-t})$ around 0, we obtain

$$|Q(z)| \le c_2(K, \eta) \|Q\|_{[0,1]}, \qquad z \in K, \tag{2.2}$$

for every $Q \in \operatorname{span}\{x^{\beta_1}, x^{\beta_2}, \dots\}$ and for every compact $K \subset \mathbb{C} \setminus \{0\}$, where

$$c_2(K, \eta) := \sum_{m=0}^{\infty} \frac{(9\eta)^m (\max_{z \in K} |\log z|)^m}{m!} = \exp(9\eta \max_{z \in K} |\log z|)$$

is a constant depending only on K and η. Now (2.2) shows that if

$$Q_n \in \operatorname{span}\{x^{\beta_1}, x^{\beta_2}, \dots\}$$

converges in $C[0,1]$, then it converges uniformly on every compact $K \subset \mathbb{C} \setminus \{0\}$. ∎

The following Markov-type inequality for Müntz polynomials is due to Newman [Bo-Er3, Theorem 6.1.1 on page 276] (see also [Ne]).

THEOREM 2.4 (Markov-type inequality for Müntz polynomials). *Suppose that $\beta_1, \dots, \beta_n$ are distinct non-negative numbers. Then*

$$\|xQ'(x)\|_{[0,1]} \le 9\Big(\sum_{j=1}^{n} \beta_j\Big) \|Q\|_{[0,1]}$$

for every $Q \in \operatorname{span}\{x^{\beta_1}, \dots, x^{\beta_n}\}$.

We will also need the bounded Bernstein-type inequality below (see [Bo-Er3, page 178]).

THEOREM 2.5 (Bernstein-type inequality for non-dense Müntz spaces). *Suppose $\varGamma := (\gamma_j)_{j=1}^{\infty}$ is a sequence of distinct positive numbers satisfying $\sum_{j=1}^{\infty} 1/\gamma_j < \infty$. Then*

$$\|Q'\|_{[0,x]} \le c_3(\varGamma, x) \|Q\|_{[0,1]}$$

for every $Q \in \operatorname{span}\{1, x^{\gamma_1}, x^{\gamma_2}, \dots\}$ and for every $x \in [0,1)$, where $c_3(\varGamma, x)$ depends only on x and $\varGamma$.

The following simple fact will also be needed.

LEMMA 2.6. *Let $U \subset C[0,1]$ be a cosed linear subspace and let $V \subset C[0,1]$ be a finite-dimensional (hence closed) linear subspace. Then $U + V$ is closed.*

3. Proof of Theorem 1.1. The first part of the theorem is contained in Theorem 1.B, so we need to prove only the second part. Suppose $(\lambda_j)_{j=1}^{\infty}$ is a sequence of distinct positive numbers satisfying

$$\sum_{j=1}^{\infty} \frac{\lambda_j}{\lambda_j^2 + 1} < \infty.$$

Then there are positive numbers η, β_j, γ_j, and δ_j such that

$$\{\lambda_j : j = 1, 2, \dots\} = \{\beta_j : j = 1, 2, \dots\} \cup \{\gamma_j : j = 1, 2, \dots\} \cup \{\delta_j : j = 1, \dots, k\},$$

where $\gamma_1 \ge 1$, $(\beta_j)_{j=1}^{\infty}$ is increasing, $(\gamma_j)_{j=1}^{\infty}$ is increasing,

$$\sum_{j=1}^{\infty} \beta_j \le \eta, \qquad \sum_{j=1}^{\infty} 1/\gamma_j < \infty,$$

and with $\Gamma := (\gamma_j)_{j=1}^{\infty}$ we have

$$c_1(\Gamma, 1/2) < \frac{1}{36\eta}$$

($c_1(\Gamma, 1/2)$ is defined in Theorem 2.1). Let

$$H_\beta := \operatorname{span}\{x^{\beta_1}, x^{\beta_2}, \dots\}, \qquad H_\gamma := \operatorname{span}\{1, x^{\gamma_1}, x^{\gamma_2}, \dots\},$$
$$H_\delta := \operatorname{span}\{x^{\delta_1}, \dots, x^{\delta_k}\}.$$

Every $Q \in H_\beta + H_\gamma$ can be written as $Q = Q_\beta + Q_\gamma$ with some $Q_\beta \in H_\beta$ and $Q_\gamma \in H_\gamma$. First we show that there are constans C_β and C_γ depending only on H_β and H_γ, respectively, so that

$$\|Q_\beta\|_{[0,1]} \le C_\beta \|Q\|_{[0,1]}, \tag{3.1}$$

$$\|Q_\gamma\|_{[0,1]} \le C_\gamma \|Q\|_{[0,1]}, \tag{3.2}$$

for all $Q \in H_\beta + H_\gamma$. Suppose to the contrary that, say, the first inequality fails. Then there are Müntz polynomials $Q_{\beta,n} \in H_\beta$ and $Q_{\gamma,n} \in H_\gamma$ so that

$$\|Q_{\beta,n}\|_{[0,1]} = 1, \qquad \lim_{n\to\infty} \|Q_{\gamma,n}\|_{[0,1]} = 1, \tag{3.3}$$

$$\lim_{n\to\infty} \|Q_{\beta,n} + Q_{\gamma,n}\|_{[0,1]} = 0. \tag{3.4}$$

Then by Theorem 2.4, $\{Q_{\beta,n} : n = 1, 2, \dots\}$ is a family of bounded, equicontinuous functions on $[1/3, 1]$, while by Theorem 2.5, $\{Q_{\gamma,n} : n = 1, 2, \dots\}$ is a family of bounded, equicontinuous functions on $[0, 2/3]$. So by the Arzelà–Ascoli Theorem there are a subsequence of $(Q_{\beta,n})$ (without loss of generality we may assume that this is $(Q_{\beta,n})$ itself) and a subsequence of $(Q_{\gamma,n})$ (we may assume that this is $(Q_{\gamma,n})$ itself) so that

$$\lim_{n\to\infty} \|Q_{\beta,n} - f\|_{[1/3,1]} = 0, \tag{3.5}$$

$$\lim_{n\to\infty} \|Q_{\gamma,n} - g\|_{[0,2/3]} = 0, \tag{3.6}$$

with some continuous functions f and g on $[1/3, 1]$ and $[0, 2/3]$, respectively. By (3.4)–(3.6) we have $f = -g$ on $[1/3, 2/3]$, so the function

$$h(x) := \begin{cases} f(x), & x \in [1/3, 1], \\ -g(x), & x \in [0, 2/3], \end{cases} \tag{3.7}$$

is well defined on $[0, 1]$. By (3.4)–(3.7) we can deduce that

$$\lim_{n\to\infty} \|Q_{\beta,n} - h\|_{[0,1]} = 0, \tag{3.8}$$

$$\lim_{n\to\infty} \|Q_{\gamma,n} - h\|_{[0,1]} = 0. \tag{3.9}$$

Using (3.3), (3.8), Theorem 2.4, and $\sum_{j=1}^{\infty} \beta_j \le \eta$, we can deduce that

$$h(x) - h(1) \le 18\eta, \qquad x \in [1/2, 1].$$

Note that (3.3), (3.5), and (3.7) imply that $\|h\|_{[0,1]} = 1$ and $h(0) = 0$. Now observe that the function $h - h(1)$ is in the uniform closure of

$$H_\gamma = \operatorname{span}\{1, x^{\gamma_1}, x^{\gamma_2}, \ldots\},$$

hence Theorem 2.1 implies

$$\|h - h(1)\|_{[0,1]} \le c_1(\Gamma, 1/2)\, \|h - h(1)\|_{[1/2,1]} \le c_1(\Gamma, 1/2) 18\eta < 1/2.$$

This contradicts the fact that $h(0) = 0$ and $\|h\|_{[0,1]} = 1$. Hence the proof of (3.1) is finished. The proof of (3.2) goes in the same way, so we omit it.

Let $\overline{H}$ denote the uniform closure of a subspace $H \subset C[0,1]$. We want to prove that $\overline{H_\beta + H_\gamma + H_\delta} \subset \mathcal{A}$, where $\mathcal{A} \subset C[0,1]$ denotes the collection of functions $f \in C[0,1]$ which can be represented as an analytic function on $\{z \in \mathbb{C} \setminus (-\infty, 0] : |z| < 1\}$ restricted to $(0,1)$. Since H_δ is finite-dimensional, Theorem 2.6 implies that

$$\overline{H_\beta + H_\gamma + H_\delta} \subset \overline{H_\beta + H_\gamma} + H_\delta,$$

so it is sufficient to prove that

$$\overline{H_\beta + H_\gamma} \subset \mathcal{A}. \tag{3.10}$$

However, (3.1) and (3.2) imply that

$$\overline{H_\beta + H_\gamma} \subset \overline{H}_\beta + \overline{H}_\gamma,$$

where $\overline{H}_\beta \subset \mathcal{A}$ by Theorem 2.3 and $\overline{H}_\gamma \subset \mathcal{A}$ by Theorem 2.2. Hence (3.10) holds indeed. ∎

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