STUDIA MATHEMATICA 207 (1) (2011)

Automorphisms of central extensions of type I von Neumann algebras

by

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Abstract. Given a von Neumann algebra M we consider its central extension $E(M)$. For type I von Neumann algebras, $E(M)$ coincides with the algebra $LS(M)$ of all locally measurable operators affiliated with M. In this case we show that an arbitrary automorphism T of $E(M)$ can be decomposed as $T = T_a \circ T_\phi$, where $T_a(x) = axa^{-1}$ is an inner automorphism implemented by an element $a \in E(M)$, and T_ϕ is a special automorphism generated by an automorphism ϕ of the center of $E(M)$. In particular if M is of type I_∞ then every band preserving automorphism of $E(M)$ is inner.

1. Introduction. In a series of papers [1]–[3] we have considered derivations on the algebra $LS(M)$ of locally measurable operators affiliated with a von Neumann algebra M, and on various subalgebras of $LS(M)$. A complete description of derivations has been obtained in the case of von Neumann algebras of type I and III.

A comprehensive survey of recent results concerning derivations on various algebras of unbounded operators affiliated with von Neumann algebras is presented in [4].

It is well-known that properties of derivations on algebras are strongly correlated with properties of automorphisms of the algebras (see e.g. [6]). Algebraic automorphisms of C^*-algebras and von Neumann algebras were considered in the paper of R. Kadison and J. Ringrose [7], which is devoted to automatic continuity and innerness of automorphisms. In the present paper we initiate the study of automorphisms of the algebra $LS(M)$ and its various subalgebras. In the commutative case a similar problem has been considered by A. G. Kusraev [10] who proved by means of Booolean-valued analysis the existence of a nontrivial band preserving automorphism on al-

2010 *Mathematics Subject Classification*: Primary 46L40; Secondary 46L51, 46L52.
Key words and phrases: von Neumann algebras, central extensions, automorphism, inner automorphism.

DOI: 10.4064/sm207-1-1

gebras of the form $L^0(\Omega, \Sigma, \mu)$. The algebra $LS(M)$ and its subalgebras are noncommutative counterparts of $L^0(\Omega, \Sigma, \mu)$. In the present paper we establish a general form of automorphisms of the algebra $LS(M)$ for type I von Neumann algebras M.

Let $\mathcal{A}$ be an algebra. A one-to-one linear operator $T : \mathcal{A} \to \mathcal{A}$ is called an *automorphism* if $T(xy) = T(x)T(y)$ for all $x, y \in \mathcal{A}$. Given an invertible element $a \in A$ one can define an automorphism T_a of $\mathcal{A}$ by $T_a(x) = axa^{-1}$, $x \in \mathcal{A}$. Such automorphisms are called *inner*. It is clear that for a commutative (abelian) algebra $\mathcal{A}$ all inner automorphisms are trivial, i.e. act as the identity operator. In the general case inner automorphisms are identical on the center of $\mathcal{A}$. Essentially different classes of automorphisms are those which are generated by automorphisms of the center $Z(\mathcal{A})$ of $\mathcal{A}$. In some cases such automorphisms ϕ of $Z(\mathcal{A})$ can be extended to automorphisms T_ϕ of the whole algebra $\mathcal{A}$ (see e.g. Kaplansky [8, Theorem 1]). The main result of the present paper shows that for a type I von Neumann algebra M every automorphism T of the algebra $LS(M)$ can be uniquely decomposed as the composition $T = T_a \circ T_\phi$ of an inner automorphism T_a and an automorphism T_ϕ generated by an automorphism ϕ of the center of $LS(M)$.

In Section 2 we recall the notions of the algebras $S(M)$ of measurable operators and $LS(M)$ of locally measurable operators affiliated with a von Neumann algebra M. We also introduce the so-called *central extension* $E(M)$ of the von Neumann algebra M. In the general case $E(M)$ is a $*$-subalgebra of $LS(M)$, which coincides with $LS(M)$ if and only if M does not have direct summands of type II. We also introduce two generalizations of the topology of convergence locally in measure on $LS(M)$ and prove that for the type I case they coincide.

In Section 3 we consider automorphisms of the algebra $E(M)$, the central extension of a von Neumann algebra M. We prove (Theorem 3.10) that if M is of type I then each automorphism T of $E(M)$ which acts identically on $Z(E(M))$ is inner. We also show that for homogeneous type I von Neumann algebras M every automorphism ϕ of $Z(E(M))$ can be extended to an automorphism T_ϕ of the whole $E(M)$. Finally we prove the main result of the present paper which states that each automorphism T of $E(M)$ for a type I von Neumann algebra M can be uniquely represented as $T = T_a \circ T_\phi$, where T_a is the inner automorphism implemented by an element $a \in E(M)$, and T_ϕ is an automorphism generated by an automorphism ϕ of the center of $E(M)$. In particular we deduce that each band preserving automorphism of $E(M)$ is inner if M is of type I_∞.

2. Central extensions of von Neumann algebras. In this section we give some necessary definitions and preliminary information on algebras of measurable and locally measurable operators affiliated with a von Neumann algebra. We also introduce the notion of the central extension of a von Neumann algebra.

Let H be a complex Hilbert space and let $B(H)$ be the algebra of all bounded linear operators on H. Consider a von Neumann algebra M in $B(H)$ with the operator norm $\|\cdot\|_M$. Denote by $P(M)$ the lattice of projections in M.

A linear subspace $\mathcal{D}$ of H is said to be *affiliated* with M (written $\mathcal{D}\eta M$) if $u(\mathcal{D}) \subset \mathcal{D}$ for every unitary u from the commutant of M,

$$M' = \{y \in B(H) : xy = yx, \forall x \in M\},$$

A linear operator x on H with domain $\mathcal{D}(x)$ is said to be *affiliated* with M (written $x\eta M$) if $\mathcal{D}(x)\eta M$ and $u(x(\xi)) = x(u(\xi))$ for all $\xi \in \mathcal{D}(x)$.

A linear subspace $\mathcal{D}$ in H is said to be *strongly dense* in H with respect to the von Neumann algebra M if

- $\mathcal{D}\eta M$;
- there exists a sequence $\{p_n\}_{n=1}^{\infty}$ of projections in $P(M)$ such that $p_n \uparrow \mathbf{1}$, $p_n(H) \subset \mathcal{D}$ and $p_n^{\perp} = \mathbf{1} - p_n$ is finite in M for all $n \in \mathbb{N}$, where $\mathbf{1}$ is the identity in M.

A closed linear operator x acting in the Hilbert space H is said to be *measurable* with respect to the von Neumann algebra M if $x\eta M$ and $\mathcal{D}(x)$ is strongly dense in H.

Denote by $S(M)$ the set of all linear operators on H, measurable with respect to the von Neumann algebra M. If $x \in S(M)$ and $\lambda \in \mathbb{C}$, then $\lambda x \in S(M)$ and the operator x^*, adjoint to x, is also measurable with respect to M (see [15]). Moreover, if $x, y \in S(M)$, then the operators $x + y$ and xy are defined on dense subspaces and admit closures that are called, respectively, the *strong sum* and the *strong product* of x and y, and are denoted by $x \dotplus y$ and $x * y$. It was shown in [15] that $x \dotplus y$ and $x * y$ belong to $S(M)$ and these algebraic operations make $S(M)$ a $*$-algebra with the identity $\mathbf{1}$ over the field $\mathbb{C}$. Moreover, M is a $*$-subalgebra of $S(M)$. In what follows, the strong sum and the strong product of x and y will be denoted in the same way as the usual operations, by $x + y$ and xy.

A closed linear operator x in H is said to be *locally measurable* with respect to the von Neumann algebra M if $x\eta M$ and there exists a sequence $\{z_n\}_{n=1}^{\infty}$ of central projections in M such that $z_n \uparrow \mathbf{1}$ and $z_n x \in S(M)$ for all $n \in \mathbb{N}$ (see [16]).

Denote by $LS(M)$ the set of all linear operators that are locally measurable with respect to M. It was proved in [16] that $LS(M)$ is a $*$-algebra over the field $\mathbb{C}$ with identity $\mathbf{1}$ and with the operations of strong addition, strong multiplication, and taking the adjoint. Thus $S(M)$ is a $*$-subalgebra in $LS(M)$. In the case where M is a finite von Neumann algebra or a factor, the algebras $S(M)$ and $LS(M)$ coincide. This is not true in the general case. In [12] the class of von Neumann algebras M has been described for which the algebras $LS(M)$ and $S(M)$ coincide.

We say that a measure μ on a measure space (Ω, Σ, μ) has the *direct sum property* if there is a family $\{\Omega_i\}_{i\in J} \subset \Sigma$, $0 < \mu(\Omega_i) < \infty$, $i \in J$, such that for any $A \in \Sigma$ with $\mu(A) < \infty$, there exist a countable subset $J_0 \subset J$ and a set B with zero measure such that $A = \bigcup_{i\in J_0}(A \cap \Omega_i) \cup B$.

It is well-known (see e.g. [15]) that for each commutative von Neumann algebra M there exists a measure space (Ω, Σ, μ) with μ having the direct sum property such that M is $*$-isomorphic to the algebra $L^\infty(\Omega, \Sigma, \mu)$ of all (equivalence classes of) complex essentially bounded measurable functions on (Ω, Σ, μ). In this case $LS(M) = S(M) \cong L^0(\Omega, \Sigma, \mu)$, where $L^0(\Omega, \Sigma, \mu)$ is the algebra of all (equivalence classes of) complex measurable functions on (Ω, Σ, μ).

Further we consider the algebra $S(Z(M))$ of operators which are measurable with respect to the center $Z(M)$ of the von Neumann algebra M. Since $Z(M)$ is an abelian von Neumann algebra, it is $*$-isomorphic to $L^\infty(\Omega, \Sigma, \mu)$ for some measure space (Ω, Σ, μ). Therefore the algebra $S(Z(M))$ coincides with $Z(LS(M))$ and can be identified with $L^0(\Omega, \Sigma, \mu)$.

The basis of neighborhoods of zero in the topology of convergence locally in measure on $L^0(\Omega, \Sigma, \mu)$ consists of the sets

$$W(A, \varepsilon, \delta) = \{f \in L^0(\Omega, \Sigma, \mu) : \exists B \in \Sigma,\ B \subseteq A,\ \mu(A \setminus B) \le \delta,\\ f \cdot \chi_B \in L^\infty(\Omega, \Sigma, \mu),\ \|f \cdot \chi_B\|_{L^\infty(\Omega,\Sigma,\mu)} \le \varepsilon\},$$

where $\varepsilon, \delta > 0$, $A \in \Sigma$, $\mu(A) < \infty$, and χ_B is the characteristic function of the set $B \in \Sigma$.

Recall the definition of the dimension function on the lattice $P(M)$ of projections from M (see [11], [15]).

By L_+ we denote the set of all measurable functions $f : (\Omega, \Sigma, \mu) \to [0, \infty]$ (modulo functions equal to zero μ-almost everywhere).

Let M be an arbitrary von Neumann algebra with center $Z(M) \equiv L^\infty(\Omega, \Sigma, \mu)$. Then there exists a map $d : P(M) \to L_+$ with the following properties:

(i) $d(e)$ is a finite function if and only if the projection e is finite;
(ii) $d(e + q) = d(e) + d(q)$ for $e, q \in P(M)$ with $eq = 0$;
(iii) $d(uu^*) = d(u^*u)$ for every partial isometry $u \in M$;
(iv) $d(ze) = zd(e)$ for all $z \in P(Z(M))$ and $e \in P(M)$;
(v) if $\{e_\alpha\}_{\alpha\in J}$, $e \in P(M)$ and $e_\alpha \uparrow e$, then $d(e) = \sup_{\alpha\in J} d(e_\alpha)$.

The map $d : P(M) \to L_+$ is called the *dimension function* on $P(M)$.

Remark 2.1. Recall that for $x \in M$ the projection defined as

$$c(x) = \inf\{z \in P(Z(M)) : zx = x\}$$

is called the *central cover* of x.

Let M be a type I von Neumann algebra. If $p, q \in P(M)$ are abelian projections with $c(p) = c(q) = \mathbf{1}$, then the property (iii) implies that $0 < d(p)(\omega) = d(q)(\omega) < \infty$ for μ-almost every $\omega \in \Omega$. Therefore replacing d by $d(p)^{-1}d$ we can assume that $d(p) = c(p)$ for every abelian projection $p \in P(M)$. Thus for all $e \in P(M)$ we have $d(e) \geq c(e)$.

The basis of neighborhoods of zero in the topology $t(M)$ of *convergence locally in measure* on $LS(M)$ consists (in the above notation) of the sets

$$V(A, \varepsilon, \delta) = \{x \in LS(M) : \exists p \in P(M),\, \exists z \in P(Z(M)),\, xp \in M,$$
$$\|xp\|_M \leq \varepsilon,\, z^\perp \in W(A, \varepsilon, \delta),\, d(zp^\perp) \leq \varepsilon z\},$$

where $\varepsilon, \delta > 0$, $A \in \Sigma$, $\mu(A) < \infty$ (see [16]).

The topology $t(M)$ is metrizable if and only if the center $Z(M)$ is σ-finite (see [11]).

Given an arbitrary family $\{z_i\}_{i \in I}$ of mutually orthogonal central projections in M with $\bigvee_{i \in I} z_i = \mathbf{1}$ and a family of elements $\{x_i\}_{i \in I}$ in $LS(M)$ there exists a unique element $x \in LS(M)$ such that $z_i x = z_i x_i$ for all $i \in I$. This element is denoted by $x = \sum_{i \in I} z_i x_i$.

We denote by $E(M)$ the set of all $x \in LS(M)$ for which there exists a sequence $\{z_i\}_{i \in I}$ of mutually orthogonal central projections in M with $\bigvee_{i \in I} z_i = \mathbf{1}$ such that $z_i x \in M$ for all $i \in I$, i.e.

$$E(M) = \Big\{x \in LS(M) : \exists z_i \in P(Z(M)),\, z_i z_j = 0,\, i \neq j,$$
$$\bigvee_{i \in I} z_i = \mathbf{1},\, z_i x \in M,\, i \in I\Big\}.$$

It is known [3] that $E(M)$ is a $*$-subalgebra in $LS(M)$ with center $S(Z(M))$, the algebra of all measurable operators with respect to $Z(M)$; moreover, $LS(M) = E(M)$ if and only if M does not have direct summands of type II.

Example 2.2. There exists a type I von Neumann algebra such that

$$LS(M) \neq S(M) \quad \text{and} \quad S(M) \neq E(M).$$

Indeed, let M be a type I_∞ von Neumann algebra with infinite-dimensional center $Z(M)$. For example M is a C^*-product of a countable number of von Neumann algebras $B(H)$, where H is an infinite-dimensional Hilbert space, i.e.

$$M \equiv \bigoplus_{n \in \mathbb{N}} B(H).$$

Then there exists a sequence $\{p_n\}_{n=1}^{\infty}$ of nonzero mutually orthogonal projections in $Z(M)$. Put

$$x = \sum_{n=1}^{\infty} np_n.$$

Then $0 \leq x \in LS(M)$ and $e_n(x) = \sum_{k=1}^{n} p_k$, where $e_n(x)$ is the spectral projection of x corresponding to the interval $[0, n]$. Since M is a type I_∞ algebra, $e_n(x)^\perp = \sum_{k=n+1}^{\infty} p_k$ is an infinite projection for all $n \in \mathbb{N}$. This means that $x \notin S(M)$, i.e. $LS(M) \neq S(M)$.

Since M is of type I, from [3, Proposition 1.1] it follows that

$$LS(M) = E(M),$$

and therefore

$$S(M) \neq E(M).$$

In general, if a von Neumann algebra M is a direct product of an infinite number of von Neumann algebras that are not finite, then $LS(M) \neq S(M)$ (see [12, Proposition 4]).

The algebra $E(M)$ is called the *central extension* of M. A similar notion (of the algebra $E(\mathcal{A})$) for arbitrary $*$-subalgebras $\mathcal{A} \subset LS(M)$ was independently introduced recently by M. A. Muratov and V. I. Chilin [13].

It is known ([3], [13]) that an element $x \in LS(M)$ belongs to $E(M)$ if and only if there exists $f \in S(Z(M))$ such that $|x| \leq f$. Therefore for each $x \in E(M)$ one can define the following vector-valued norm:

$$\|x\| = \inf\{f \in S(Z(M)) : |x| \leq f\}. \tag{2.1}$$

This norm satisfies the following conditions:

- $\|x\| \geq 0; \|x\| = 0 \Leftrightarrow x = 0;$
- $\|fx\| = |f|\,\|x\|;$
- $\|x + y\| \leq \|x\| + \|y\|;$
- $\|xy\| \leq \|x\|\,\|y\|;$
- $\|xx^*\| = \|x\|^2$

for all $x, y \in E(M), f \in S(Z(M))$.

Let us equip $E(M)$ with the topology $t_c(M)$ defined by the following system of zero neighborhoods:

$$O(A, \varepsilon, \delta) = \{x \in E(M) : ||x|| \in W(A, \varepsilon, \delta)\},$$

where $\varepsilon, \delta > 0$, $A \in \Sigma$, $\mu(A) < \infty$.

LEMMA 2.3. *The topology $t_c(M)$ is stronger than the topology $t(M)$ of convergence locally in measure.*

Proof. It is sufficient to show that

$$O(A, \varepsilon, \delta) \subset V(A, \varepsilon, \delta). \tag{2.2}$$

Let $x \in O(A, \varepsilon, \delta)$, i.e. $\|x\| \in W(A, \varepsilon, \delta)$. Then there exists $B \in \Sigma$ such that

$$B \subseteq A, \quad \mu(A \setminus B) \leq \delta,$$

and

$$\|x\|\chi_B \in L^\infty(\Omega, \Sigma, \mu), \quad \big\|\|x\|\chi_B\big\|_M \leq \varepsilon.$$

Put $z = p = \chi_B$. Then $\|xp\| = \|x\chi_B\| = \|x\|\chi_B \in L^\infty(\Omega, \Sigma, \mu)$, i.e. $xp \in M$, and moreover $\|xp\|_M \leq \varepsilon$. Since $\mu(A \setminus B) \leq \delta$ and $z^\perp \chi_B = \chi_B^\perp \chi_B = 0$, one has $z^\perp \in W(A, \varepsilon, \delta)$. Therefore

$$\|xp\|_M \leq \varepsilon, \quad z^\perp \in W(A, \varepsilon, \delta), \quad zp^\perp = \chi_B \chi_B^\perp = 0,$$

and hence $x \in V(A, \varepsilon, \delta)$. ■

Lemma 2.4. *If M is a type I von Neumann algebra and $0 < \varepsilon < 1$, then*

$$O(A, \varepsilon, \delta) = V(A, \varepsilon, \delta). \tag{2.3}$$

Proof. By (2.2) it is sufficient to show that $V(A, \varepsilon, \delta) \subset O(A, \varepsilon, \delta)$.

Let $x \in V(A, \varepsilon, \delta)$. Then there exist $p \in P(M)$ and $z \in P(Z(M))$ such that

$$xp \in M, \quad \|xp\|_M \leq \varepsilon, \quad z^\perp \in W(A, \varepsilon, \delta), \quad d(zp^\perp) \leq \varepsilon z.$$

Since M is of type I, Remark 2.1 implies that $d(zp^\perp) \geq c(zp^\perp)$. Now from $d(zp^\perp) \leq \varepsilon z$ it follows that $c(zp^\perp) \leq \varepsilon z$. As $0 < \varepsilon < 1$ we find that $zp^\perp = 0$. Thus $z = zp$. Then $z = \chi_E$ for some $E \in \Sigma$. Since $z^\perp \in W(A, \varepsilon, \delta)$ one has $\chi_{\Omega \setminus E} \in W(A, \varepsilon, \delta)$. Thus there exists $B \in \Sigma$ such that $B \subseteq A$, $\mu(A \setminus B) \leq \delta$, $|\chi_{\Omega \setminus E}\chi_B| \leq \varepsilon < 1$. Hence $\chi_B \leq \chi_E$. So we obtain

$$\|x\|\chi_B \leq \|x\|\chi_E = \|x\|z = \|xz\| = \|xzp\| = \|xp\| \leq \varepsilon.$$

This means that $x \in O(A, \varepsilon, \delta)$. ■

Lemma 2.4 implies the following

Theorem 2.5. *If M is a type I von Neumann algebra then the topologies $t(M)$ and $t_c(M)$ coincide.*

Remark 2.6. The equality (2.3) implies that for type I von Neumann algebras the definition of $V(A, \varepsilon, \delta)$ can be written in a simpler way without using the dimension function:

$$V(A, \varepsilon, \delta) = \{x \in LS(M) : \exists z \in P(Z(M)),\ xz \in M,$$
$$\|xz\|_M \leq \varepsilon,\ z^\perp \in W(A, \varepsilon, \delta)\}.$$

It should be noted that the topology $t_c(M)$ on general Banach–Kantorovich spaces over a ring K of measurable functions was considered in [17]. An important property of this topology, which will be used in the next section (Theorem 3.1), is the following: the continuity of a K-linear operator on a Banach–Kantorovich space in this topology is equivalent to its K-boundedness [17, Theorem 3.1].

LEMMA 2.7. *Let M be a type I von Neumann algebra and let $x \in LS(M)$, $x \geq 0$. If $pxp = 0$ for all abelian projections $p \in M$ then $x = 0$.*

Proof. Since $x \geq 0$ we have $x = yy^*$ for some $y \in LS(M)$. Then

$$0 = pxp = pyy^*p = py(py)^*$$

and hence $py = 0$. Therefore $y^*py = 0$ for all abelian projections $p \in M$. But since M has type I there exists a family $\{p_i\}_{i\in J}$ of mutually orthogonal abelian projections such that $\sum_{i\in J} p_i = \mathbf{1}$. For any finite subset $F \subseteq J$ put $p_F = \sum_{i\in F} p_i$. Since $p_F \uparrow \mathbf{1}$, from $yp_Fy^* = 0$ we deduce that $yy^* = 0$, i.e. $x = yy^* = 0$. ∎

3. Automorphisms of central extensions for type I von Neumann algebras. Let $\mathcal{A}$ be an arbitrary algebra with center $Z(\mathcal{A})$ and let $T : \mathcal{A} \to \mathcal{A}$ be an automorphism. It is clear that T maps $Z(\mathcal{A})$ onto itself: indeed, for all $a \in Z(\mathcal{A})$ and $x \in \mathcal{A}$ one has

$$T(a)T(x) = T(ax) = T(xa) = T(x)T(a),$$

which means that $T(a) \in Z(\mathcal{A})$.

An operator $T : \mathcal{A} \to \mathcal{A}$ is said to be $Z(\mathcal{A})$-*linear* if $T(ax) = aT(x)$ for all $a \in Z(\mathcal{A})$ and $x \in \mathcal{A}$. It is easy to see that an automorphism $T : \mathcal{A} \to \mathcal{A}$ of a unital algebra $\mathcal{A}$ is $Z(\mathcal{A})$-linear if and only if it is identical on $Z(\mathcal{A})$.

THEOREM 3.1. *Let M be a von Neumann algebra of type I and let $E(M)$ be its central extension. Then each $Z(E(M))$-linear automorphism T of the algebra $E(M)$ is inner.*

Proof. Let us show that T is $t(M)$-continuous. First suppose that the center $Z(M)$ is σ-finite. Then the topology $t(M)$ is metrizable and hence it is sufficient to prove that T is $t(M)$-closed.

Consider a sequence $\{x_n\} \subset E(M)$ such that $x_n \xrightarrow{t(M)} 0$, $T(x_n) \xrightarrow{t(M)} y$. Take $x \in E(M)$ such that $T(x) = y$ and let us show that $x = 0$. Since

$$x^*x_n \xrightarrow{t(M)} 0$$

and

$$T(x^*x_n) = T(x^*)T(x_n) \xrightarrow{t(M)} T(x^*)y = T(x^*)T(x) = T(x^*x),$$

we may suppose (by replacing $\{x_n\}$ by $\{x^*x_n\}$) that $x \geq 0$.

Let $p \in M$ be an arbitrary abelian projection with $c(p) = \mathbf{1}$. Then $px_np = a_np$ for some $a_n \in S(Z(M))$ and all $n \in \mathbb{N}$. Since $x_n \xrightarrow{t(M)} 0$ and

$c(p) = \mathbf{1}$ it follows that $a_n \xrightarrow{t(M)} 0$. Therefore

$$T(p)T(x_n)T(p) = T(px_np) = T(a_np) = a_nT(p) \xrightarrow{t(M)} 0.$$

On the other hand

$$T(p)T(x_n)T(p) \xrightarrow{t(M)} T(p)yT(p),$$

thus $T(p)yT(p) = 0$ and hence

$$pxp = T^{-1}(T(p)yT(p)) = T(0) = 0,$$

i.e. $pxp = 0$ for all abelian projections with $c(p) = \mathbf{1}$. Therefore Lemma 2.7 implies that $x = 0$, i.e. T is $t(M)$-continuous.

Now consider the general case, i.e. when the center $Z(M)$ is arbitrary. Take a family $\{z_i\}_{i\in I}$ of mutually orthogonal central projections in M with $\bigvee_i z_i = \mathbf{1}$ such that $z_iZ(M)$ is σ-finite for all $i \in I$. By the above, z_iT is $t(z_iM)$-continuous on $z_iE(M)$ for all $i \in I$, where $(z_iT)(x) = T(z_ix) = z_iT(x)$ is the restriction of T to $z_iE(M)$, which is well-defined in view of the $Z(E(M))$-linearity of T. Therefore T is $t(M)$-continuous on the whole $E(M) = \prod_{i\in I} z_iE(M)$.

Further by Theorem 2.5 the topologies $t(M)$ and $t_c(M)$ coincide and hence T is also $t_c(M)$-continuous and according to [17, Theorem 3.1] there exists $c \in S(Z(M))$ such that $\|T(x)\| \le c\|x\|$ for all $x \in E(M)$.

Take a sequence $\{z_n\}_{n\in\mathbb{N}}$ of mutually orthogonal central projections in M with $\bigvee_n z_n = \mathbf{1}$ such that $z_nc \in Z(M)$ for all $n \in \mathbb{N}$. This means that the automorphism z_nT maps bounded elements from $z_nE(M)$ to bounded elements, i.e. $z_nT(z_nM) \subseteq z_nM$. Then given any $n \in \mathbb{N}$ the automorphism $z_nT|_{z_nM}$ is identical on the center of z_nM. By a theorem of Kaplansky [9, Theorem 10] there exist $a_n \in z_nM$ invertible in z_nM and such that $z_nT(x) = a_nxa_n^{-1}$ for all $x \in z_nM$. Put $a = \sum_{n\ge1} z_na_n$. It is clear that $a \in E(M)$ and

$$T(x) = \sum_{n\ge1} z_nT(x) = \sum_{n\ge1} z_nT(z_nx) = \sum_{n\ge1} a_n(z_nx)a_n = axa^{-1}$$

for all $x \in E(M)$. ∎

Let M be a von Neumann algebra of type I_n for some $n \in \mathbb{N}$. Then M is $*$-isomorphic to the algebra $M_n(Z(M))$ of all $n \times n$ matrices over $Z(M)$ (cf. [14, Theorem 2.3.3]). Moreover the algebra $S(M) = E(M)$ is $*$-isomorphic to $M_n(Z(S(M)))$, where $Z(S(M)) = S(Z(M))$ (see [2, Proposition 1.5]). If e_{ij}, $i, j = \overline{1, n}$, are matrix units in $M_n(S(Z(M)))$ then each $x \in M_n(S(Z(M)))$ has the form

$$x = \sum_{i,j=1}^{n} a_{ij}e_{ij}, \quad a_{ij} \in S(Z(M)),\ i, j = \overline{1, n}.$$

Let $\phi : S(Z(M)) \to S(Z(M))$ be an automorphism. Setting

$$T_\phi\Big(\sum_{i,j=1}^{n} a_{ij}e_{ij}\Big) = \sum_{i,j=1}^{n} \phi(a_{ij})e_{ij} \tag{3.1}$$

we obtain a linear operator T_ϕ on $M_n(S(Z(M)))$, which is in fact an algebra automorphism. Indeed, for

$$x = \sum_{i,j=1}^{n} a_{ij}e_{ij}, \quad y = \sum_{i,j=1}^{n} b_{ij}e_{ij}, \quad a_{ij}, b_{ij} \in S(Z(M)),\ i,j = \overline{1,n},$$

we have

$$\begin{aligned} T_\phi(xy) &= T_\phi\Big(\sum_{i,j=1}^{n} a_{ij}e_{ij}\sum_{k,s=1}^{n} b_{ks}e_{ks}\Big) = T_\phi\Big(\sum_{i,j,s=1}^{n} a_{ij}b_{js}e_{is}\Big) \\ &= \sum_{i,j,s=1}^{n} \phi(a_{ij}b_{js})e_{is} = \sum_{i,j,s=1}^{n} \phi(a_{ij})\phi(b_{js})e_{is} \\ &= \sum_{i,j=1}^{n} \phi(a_{ij})e_{ij}\sum_{k,s=1}^{n} \phi(b_{ks})e_{ks} = T_\phi(x)T_\phi(y), \end{aligned}$$

i.e. $T_\phi(xy) = T_\phi(x)T_\phi(y)$.

The following property immediately follows from the definition of T_ϕ: if φ and ϕ are two automorphisms of $S(Z(M))$ then $T_\phi \circ T_\varphi = T_{\phi\circ\varphi}$, in particular $T_\phi^{-1} = T_{\phi^{-1}}$.

Remark 3.2. (i) If the automorphism ϕ on $S(Z(M))$ is nontrivial (i.e. not identical) then it is clear that T_ϕ cannot be an inner automorphism on $M_n(S(Z(M)))$.

(ii) It is known [7, Lemma 1] that every (algebraic) automorphism of a C^*-algebra is automatically norm continuous. But in our case this is not true in general. Suppose that the abelian algebra $S(Z(M))$ is represented as $L^0(\Omega, \Sigma, \mu)$, with a continuous Boolean algebra Σ. Then A. G. Kusraev [10, Theorem 3.4] has proved that $S(Z(M))$ admits a nontrivial band preserving automorphism ϕ which is, in particular, $t(M)$-discontinuous, where "band preserving" means that ϕ is identical on all projections $z \in S(Z(M))$. Then T_ϕ is an example of a $t(M)$-discontinuous automorphism of $E(M)$. In particular, T_ϕ is not inner.

Theorem 3.3. *If M is a von Neumann algebra of type I_n, then each automorphism T of $E(M)$ can be uniquely represented in the form*

$$T = T_a \circ T_\phi, \tag{3.2}$$

where T_a is an inner automorphism implemented by an element $a \in E(M)$, and T_ϕ is the automorphism of the form (3.1) *generated by an automorphism ϕ of $S(Z(M))$.*

Proof. Let ϕ be the restriction of T to $Z(E(M)) = S(Z(M))$. As mentioned earlier, ϕ maps $Z(E(M))$ onto itself, i.e. φ is an automorphism of $Z(E(M))$. Consider the automorphism T_ϕ defined by (3.1) and put $S = T \circ T_\phi^{-1}$. Since T and T_ϕ coincide on $Z(E(M))$, it follows that S is identical on $Z(E(M))$, i.e. S is a $Z(E(M))$-linear automorphism of $E(M)$. By Theorem 3.1 there exists an invertible element $a \in E(M)$ such that $S = T_a$, i.e. $S(x) = axa^{-1}$ for all $x \in E(M)$. Therefore $T = S \circ T_\phi = T_a \circ T_\phi$.

Suppose that $T = T_a \circ T_\phi = T_b \circ T_\varphi$ for $a, b \in E(M)$ and automorphisms ϕ and φ of $Z(E(M))$. Then $T_b^{-1} \circ T_a = T_\varphi \circ T_\phi^{-1}$, i.e. $T_{b^{-1}a} = T_{\varphi\circ\phi^{-1}}$. Since $T_{b^{-1}a}$ is identical on $Z(E(M))$, so is $\varphi \circ \phi^{-1}$, i.e. $\varphi = \phi$. Therefore $T_\varphi = T_\phi$, i.e. $T_b^{-1} \circ T_a = \mathrm{Id}$ and hence $T_a = T_b$. ∎

Lemma 3.4. *Let M be a von Neumann algebra and let $T : E(M) \to E(M)$ be an automorphism. If $x \in E(M)$ and $c(x) = \mathbf{1}$ then $c(T(x)) = \mathbf{1}$.*

Proof. Assume $c(x) = \mathbf{1}$ and consider the central projection $z \in P(Z(M))$ such that $T(z) = \mathbf{1} - c(T(x))$. Then

$$T(zx) = T(z)T(x) = (\mathbf{1} - c(T(x))c(T(x))T(x) = 0$$

and hence $zx = 0$. Therefore $zc(x) = 0$, i.e. $z = 0$. This means that $0 = T(0) = \mathbf{1} - c(T(x)) = \mathbf{1}$, i.e. $c(T(x)) = \mathbf{1}$. ∎

If ϕ is a $*$-automorphism of $E(M)$ then it is an order automorphism and hence maps M onto M. But for an arbitrary automorphism (non-adjoint-preserving), this not true in general. For some particular cases one can obtain a positive result.

Lemma 3.5. *Let M be an abelian von Neumann algebra and let $\phi : E(M) \to E(M)$ be a $t(M)$-continuous automorphism. Then $\phi(M) \subseteq M$.*

Proof. Let $x \in M$ be a simple element, i.e.

$$x = \sum_{i=1}^{n} \lambda_i e_i,$$

where $\lambda_i \in \mathbb{C}$, $e_i \in P(M)$, $e_i e_j = 0$, $i \neq j$, $i, j = \overline{1, n}$. Let us prove that $\phi(x) \in M$ and $\|\phi(x)\|_M = \|x\|_M$. Since M is abelian and $\phi(e_i)^2 = \phi(e_i)$, it follows that $\phi(e_i)$ is a projection for each $i = \overline{1, n}$. Therefore from the equality

$$\phi(x) = \sum_{i=1}^{n} \lambda_i \phi(e_i)$$

we see that $\phi(x) \in M$ and moreover

$$\|\phi(x)\|_M = \max_{1 \le i \le n} |\lambda_i| = \|x\|_M.$$

Let now $x \in M$ be arbitrary. Consider a sequence $\{x_n\}$ of simple elements in M which $t(M)$-converges to x and $|x_n| \le |x|$ for all $n \in \mathbb{N}$. Then $\phi(x_n) \xrightarrow{t(M)} \phi(x)$ and $\|\phi(x_n)\|_M = \|x_n\|_M \le \|x\|_M$ for all $n \in \mathbb{N}$. Therefore $|\phi(x)| \le \|x\|_M \mathbf{1}$, i.e. $\phi(x) \in M$. ∎

We are now in a position to consider automorphisms of central extensions for type I_∞ von Neumann algebras.

Theorem 3.6. *Let M be a von Neumann algebra of type I_∞, and let $T : E(M) \to E(M)$ be an automorphism. Then T is $t(Z(M))$-continuous on $E(Z(M))$ and maps $Z(M)$ onto itself.*

Proof. Since M is of type I_∞, there exists a sequence $\{p_n\}_{n=1}^\infty$ of mutually orthogonal abelian projections in M with the central covers equal to $\mathbf{1}$. For a bounded sequence $\{a_n\}$ from $Z(M)$ put

$$x = \sum_{n=1}^{\infty} a_n p_n.$$

Then

$$xp_n = p_n x = a_n p_n \quad \text{for all } n \in \mathbb{N}.$$

Now let T be an automorphism of $E(M)$ and denote by ϕ its restriction to the center of $E(M)$. If $q_n = T(p_n)$, $n \in \mathbb{N}$, then

$$T(xp_n) = T(x)T(p_n) = T(x)q_n$$

and

$$T(xp_n) = T(a_n p_n) = T(a_n)T(p_n) = \phi(a_n)q_n,$$

therefore

$$T(x)q_n = \phi(a_n)q_n.$$

For the center-valued norm $\|\cdot\|$ on $E(M)$ (see (2.1)) we have

$$\|q_n\|\,\|T(x)\| \ge \|q_n T(x)\| = \|\phi(a_n)q_n\| = |\phi(a_n)|\,\|q_n\|.$$

Since $c(q_n) = c(p_n) = \mathbf{1}$ (Lemma 3.4) the latter inequality implies that

$$\|T(x)\| \ge |\phi(a_n)|. \tag{3.3}$$

Let us show that ϕ is $t(Z(M))$-continuous on $E(Z(M))$. If not, then there exists a bounded sequence $\{a_n\}$ in $Z(M)$ such that $\{\phi(a_n)\}$ is not $t(Z(M))$-bounded, which contradicts (3.3). Thus ϕ is $t(Z(M))$-continuous and Lemma 3.5 implies that T maps $Z(M)$ onto itself. ∎

Remark 3.7. The $t(Z(M))$-continuity of T on $E(Z(M))$ easily implies that the restriction of T to $E(Z(M))$ and hence to $Z(M)$ is a $*$-automorphism (cf. [7, Lemma 1]).

Now we are going to show that similar to the case of type I_n ($n \in \mathbb{N}$) von Neumann algebras, automorphisms of the algebras $E(M)$ for homogeneous type I_α von Neumann algebras (where α is an infinite cardinal number) can also be represented in the form (3.2).

Suppose that $\phi : Z(M) \to Z(M)$ is an automorphism. According to [8, Theorem 1], ϕ can be extended to a $*$-automorphism of M, which we denote by T_ϕ. Since each $*$-automorphism is an order isomorphism and each hermitian element of $E(M)$ is an order limit of hermitian elements from M, we can naturally extend T_ϕ to a $*$-automorphism of $E(M)$.

THEOREM 3.8. *If M is a type I_α von Neumann algebra, where α is an infinite cardinal number, then each automorphism T on $E(M)$ can be uniquely represented as*

$$T = T_a \circ T_\phi,$$

where T_a is an inner automorphism implemented by an element $a \in E(M)$ and T_ϕ is the $*$-automorphism generated by an automorphism ϕ of $Z(M)$ as above.

Proof. Let ϕ be the restriction of T to the center $S(Z(M))$ of $E(M)$. Then by Theorem 3.6, ϕ maps $Z(M)$ onto itself. By [8, Theorem 1] as above ϕ can be extended to a $*$-automorphism of $E(M)$. Now similar to Theorem 3.3 there exists $a \in E(M)$ such that $T = T_a \circ T_\phi$ and this representation is unique. ∎

LEMMA 3.9. *Let M and N be von Neumann algebras of type I and suppose that M is homogeneous of type I_α. If there exists an isomorphism (not necessarily a $*$-isomorphism) T from $E(M)$ onto $E(N)$ then N is also of type I_α.*

Proof. Let z_N be a central projection in N such that $z_N N$ is of type I_β, where β is a cardinal number. Take a central projection z_M in M such that $T(z_M) = z_N$. Replacing M and N by $z_M M$ and $z_N N$ respectively we may assume that $z_M = \mathbf{1}_M$, $z_N = \mathbf{1}_N$.

Let $\{p_i\}_{i\in I}$ (respectively $\{e_j\}_{j\in J}$) be a family of mutually equivalent and orthogonal abelian projections in M (respectively in N) with $\bigvee_{i\in I} p_i = \mathbf{1}_M$ (respectively $\bigvee_{j\in J} e_j = \mathbf{1}_N$), where $|I| = \alpha$, $|J| = \beta$. It is clear that $c(p_i) = \mathbf{1}_M$ for all $i \in I$.

Then $q_i = T(p_i)$ is an idempotent ($q_i^2 = q_i$) but not a projection in general. Let $f_i = s_l(q_i)$ be the left projection of the idempotent q_i. Since f_i is the projection onto the range of the idempotent q_i we infer that $q_i f_i = f_i$, i.e. $f_i q_i f_i = f_i$, and moreover $c(f_i) = \mathbf{1}_N$, because $c(q_i) = \mathbf{1}_N$ (see Lemma 3.4). The equalities

$$q_i E(N) q_i = T(p_i E(M) p_i) = T(Z(E(M)) p_i) = E(Z(N)) q_i$$

imply that for each $x \in E(N)$ there exists $a_x \in E(Z(N))$ such that $q_i x q_i = a_x q_i$.

Now we show that f_i is an abelian projection. For $x \in E(N)$ and each f_i there exists $a_i \in E(Z(N))$ such that

$$q_i f_i x f_i q_i = a_i q_i.$$

Thus

$$f_i x f_i = (f_i q_i f_i) x (f_i q_i f_i) = f_i (q_i f_i x f_i q_i) f_i = f_i a_i q_i f_i = a_i f_i q_i f_i = a_i f_i,$$

i.e. $f_i E(N) f_i = E(Z(N)) f_i$. This means that f_i is an abelian projection.

Case 1: α and β are finite. Let Φ be the normalized center-valued trace on N. Then

$$\mathbf{1}_N = \Phi(\mathbf{1}_N) = \sum_{i\in I} \Phi(q_i) = \alpha\Phi(q_1) = \alpha\Phi(f_1 q_1) = \alpha\Phi(f_1 q_1 f_1) = \alpha\Phi(f_1).$$

Since N is of type I_β, we have

$$\mathbf{1}_N = \beta\Phi(f_1).$$

Therefore $\alpha = \beta$.

Case 2: α and β are infinite. For a faithful normal semi-finite trace τ on N put

$$\tau_i(x) = \tau(f_i x), \qquad x \in N.$$

For each $i \in I$ set

$$J_i = \{j \in J : \tau_i(e_j) \neq 0\}.$$

Since $\{e_j\}$ is an orthogonal family, each J_i is countable.

Suppose that there exists $j \in J$ such that $\tau_i(e_j) = 0$ for all $i \in I$. Since $\tau(f_i e_j f_i) = \tau(f_i e_j) = \tau_i(e_j) = 0$, we obtain $f_i e_j f_i = 0$. But from

$$0 = f_i e_j f_i = f_i e_j e_j f_i = f_i e_j (f_i e_j)^*$$

it follows that $f_i e_j = 0$ for all $i \in I$. And since $\bigvee_{i\in I} f_i = \mathbf{1}_N$, this implies that $e_j = 0$, a contradiction. Therefore given any $j \in J$ there exists $i \in I$ such that $\tau_i(e_j) \neq 0$, i.e. $j \in J_i$. Hence

$$J = \bigcup_{i\in I} J_i,$$

so $\beta \leq \alpha\aleph_0$, and therefore $\beta \leq \alpha$. Similarly $\alpha \leq \beta$.

This means that every homogeneous direct summand of the von Neumann algebra N is of type I_α, i.e. N itself is homogeneous of type I_α. ∎

It is well-known [14] that if M is an arbitrary von Neumann algebra of type I then there exists an orthogonal family $\{z_\alpha\}_{\alpha\in J}$ of central projections in M with $\sup_{\alpha\in J} z_\alpha = \mathbf{1}$ such that M is $*$-isomorphic to the C^*-product of the von Neumann algebras $z_\alpha M$ of type $\mathrm{I}_\alpha, \alpha \in J$, i.e.

$$M \cong \bigoplus_{\alpha\in J} z_\alpha M.$$

In this case by the definition of the central extension we have

$$E(M) = \prod_{\alpha \in J} E(z_\alpha M).$$

Suppose that T is an automorphism of $E(M)$ and ϕ is its restriction to the center $E(Z(M))$. Let us show that T maps each $z_\alpha E(M) \cong E(z_\alpha M)$ onto itself. Clearly T maps $z_\alpha E(M)$ onto $T(z_\alpha)E(M)$. From Lemma 3.9 it follows that the von Neumann algebra $T(z_\alpha)M$ is of type I_α. Thus $T(z_\alpha) \leq z_\alpha$. Suppose that $z'_\alpha = z_\alpha - T(z_\alpha) \neq 0$. By Lemma 3.9 we know that $T^{-1}(z'_\alpha)M$ is of type I_α, i.e.

$$0 \neq z''_\alpha = T^{-1}(z'_\alpha) \leq z_\alpha.$$

On the other hand

$$\begin{aligned} T(z_\alpha z''_\alpha) &= T(z_\alpha)T(z''_\alpha) = T(z_\alpha)z'_\alpha \\ &= T(z_\alpha)(z_\alpha - T(z_\alpha)) = T(z_\alpha) - T(z_\alpha) = 0, \end{aligned}$$

i.e. $z_\alpha z''_\alpha = 0$. Therefore since $z''_\alpha \leq z_\alpha$ we have $z''_\alpha = 0$, a contradiction. Hence $z'_\alpha = 0$, i.e. $T(z_\alpha) = z_\alpha$.

Therefore ϕ generates an automorphism ϕ_α on each $z_\alpha S(Z(M)) \cong Z(E(z_\alpha M))$ for $\alpha \in J$. Let T_{ϕ_α} be the automorphism of $z_\alpha E(M)$ generated by ϕ_α, $\alpha \in J$. Put

$$T_\phi\left(\{x_\alpha\}_{\alpha \in J}\right) = \{T_{\phi_\alpha}(x_\alpha)\}, \quad \{x_\alpha\}_{\alpha \in J} \in E(M). \tag{3.4}$$

Then T_ϕ is an automorphism of $E(M)$.

Now we can state the main result of the present paper.

THEOREM 3.10. *If M is a type I von Neumann algebra, then each automorphism T of $E(M)$ can be uniquely represented in the form*

$$T = T_a \circ T_\phi,$$

where T_a is an inner automorphisms implemented by an element $a \in E(M)$ and T_ϕ is an automorphism of the form (3.4).

Proof. Let T be an automorphism of $E(M)$ and let ϕ be its restriction to $Z(E(M))$. Consider the automorphism T_ϕ on $E(M)$ generated by the automorphism ϕ as in (3.4) above. Similar to the proof of Theorem 3.3 we find an element $a \in E(M)$ such that $T = T_a \circ T_\phi$ and show that this representation is unique. ∎

Recall [5] that an operator $T : E(M) \to E(M)$ is called *band preserving* if $T(zx) = zT(x)$ for all $z \in P(Z(M))$ and $x \in E(M)$, i.e. T is identical on central projections of $E(M)$.

Theorems 3.6 and 3.10 imply the following result which is an analogue of [7, Theorem 5, Remark A] giving a sufficient condition for the innerness of algebraic automorphisms.

Corollary 3.11. *If M is a von Neumann algebra of type I_∞ then each band preserving automorphism of $E(M)$ is inner.*

Proof. Let ϕ be the restriction of T to $E(Z(M))$. Since T is band preserving it follows that ϕ acts identically on simple elements from $Z(M)$. Theorem 3.6 implies that ϕ is $t(Z(M))$-continuous. Hence ϕ is identical on the whole $S(Z(M)) = E(Z(M))$ and therefore by Theorem 3.10, T is an inner automorphism. ∎

Remark 3.12. It is clear that the condition of the above corollary is also necessary for the innerness of automorphisms of $E(M)$.

Acknowledgments. The second and the third named authors would like to acknowledge the hospitality of the Institut für Angewandte Mathematik, Universität Bonn (Germany). This work is supported in part by the DFG AL 214/36-1 project (Germany). The authors are indebted to the referee for useful remarks.

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Received April 26, 2011
Revised version October 29, 2011 (7176)