ARITHMETIC METHODS
IN MATHEMATICAL PHYSICS AND BIOLOGY
BANACH CENTER PUBLICATIONS, VOLUME 109
INSTITUTE OF MATHEMATICS
POLISH ACADEMY OF SCIENCES
WARSZAWA 2016

NEW MATHEMATICS OF COMPLEXITY AND ITS BIOMEDICAL APPLICATIONS

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Abstract. We show that the unreduced, mathematically rigorous solution of the many-body problem with arbitrary interaction, avoiding any perturbative approximations and "exact" models, reveals qualitatively new mathematical properties of thus emerging real-world structures (interaction products), including dynamic multivaluedness (universal non-uniqueness of ordinary solution) giving rise to intrinsic randomness and irreversible time flow, fractally structured dynamic entanglement of interaction components expressing physical quality, and dynamic discreteness characterising the emerging physically real space. This unreduced interaction problem solution leads to the universal definition of dynamic complexity describing structure and properties of all real objects. The united world structure of dynamically probabilistic fractal is governed by the universal law of the symmetry (conservation and transformation) of complexity giving rise to extended versions of all particular (correct) laws and principles. We describe then the unique efficiency of this universal concept and new mathematics of complexity in application to critical problems in life sciences and related development problems, showing the urgency of complexity revolution.

1. Introduction: Towards the genuine mathematics of nature. Two major limits of the mathematical language of science not only persist, but become increasingly critical today, the classical limitation of a sound description to lower-complexity levels of "exact" sciences and a more recent perception of "uncertainty" in mathematics [KL] desperately losing its applied aspects and finally any relation to reality. This criticality is amplified by the growing urgent need for consistent mathematical description of biological and other

2010 *Mathematics Subject Classification*: Primary 34C28; 34D99; 35A02; 37N25; 70E55; 70F10; Secondary 28A80; 34H10; 34A34; 35F21; 35Q41; 68T01; 70H20; 70H33; 92D10.
Key words and phrases: many-body problem, dynamic multivaluedness, dynamic complexity, chaos, self-organisation, fractal, symmetry of complexity, genomics, intelligence, consciousness, integral medicine, nanobiotechnology.
The paper is in final form and no version of it will be published elsewhere.

higher-level, "human" systems in the era of their intense technological involvement and "blind" empirical modification (showing lately its own deep impasses), as well as by the ever more provocative absence of the (recognised) causally complete and unified theory of even the lowest, "fundamental" (physical) levels of reality.

The problem looks now so serious that one may question the major idea of mathematics as the unified basis of "rigorous" (i.e. *consistent* or "objective") and *definitely successful* description of nature or even the very existence of truly objective and universal laws of nature [KA, LMK, G, N], in correlation with various other growing doubts of the "end-of-science" kind (e. g. [H1, H2, K3, K6, K17, LC, LO, U, W]). In other words, is mathematics really "the (universal) language of nature" or merely a useful, but variously limited and maybe even outdated technical tool? And in the latter case, are we witnessing its growing and inevitable replacement by computer-assisted empirical technologies (often within "pure" mathematics itself)?

Those increasingly growing doubts about the *mathematical* power of science bear a fundamental relation to the key fact of absence, within the usual mathematics framework, of consistent and complete solution to the *problem of arbitrary* (*realistic*) *many-body interaction* giving rise to all real systems and phenomena. In that situation, any mathematical construct that may have an eventual connection to reality (and therefore ever be useful) will inevitably depend on this critical deficiency, which may underlie thus the pressing limits of modern scientific knowledge. There is no reason to believe that various abstract and intuitively guessed "models" used instead of the complete problem solution can provide adequate substitutes in most cases of current interest: it is rather the opposite conclusion that looks the more and more probable, especially for higher-complexity cases of various biomedical, "bio-inspired" and "intelligent" systems and behaviours.

In this report we present the complete solution to the many-body problem with arbitrary (any real) interaction, revealing indeed a *new*, rigorously derived and well-specified *quality* absent in conventional models. It is the *dynamic multivaluedness*, or *redundance*, of emerging system configurations, or *realisations*, instead of only one such realisation (*dynamic single-valuedness*) of all conventional models, including usual "complexity" and "chaos" theories (Section 2). We show that this causally complete extension of usual projection of reality gives rise to the provably *universal* concept of *dynamic complexity* and *chaos* (Section 2.1), including the *unified classification of dynamic regimes and structures*, natural and universal solution of the *time/emergence problem* (Section 2.2) and well-substantiated solutions to old and new "unsolvable" problems in particular fields of science [K2, K3, K4, K5, K7, K8, K9, K10, K11, K12, K13, K14, K15, K16, K17, K18, K19, K20, K21, K22, KU]. We specify also the exact reason for the essential deficiency (including the well-known non-universality) of the notions of complexity, chaoticity, self-organisation, etc. from usual, dynamically single-valued "science of complexity", the latter actually corresponding to the unrealistic zero value of the unreduced dynamic complexity.

We summarise then the essential new features of thus emerging *mathematics of* (universal and reality-based) *complexity* (Section 3) and their importance for further progress (real problem solution) in both mathematics and its real-world applications,

from fundamental levels (particles and fields) to the highest levels of complexity (including biomedical, ecological, social and brain-science applications). It is demonstrated, in particular, that the entire world structure and dynamics can be described as the unified *dynamically probabilistic fractal*, obeying the equally universal law, the *symmetry* (conservation and transformation) *of unreduced dynamic complexity*.

We finally develop more details of biomedical and other higher-level applications of this new mathematics of complexity demonstrating the *urgent necessity* and essential features of the emerging *new paradigm* of *universal dynamic complexity* in life science and beyond, including reliable genetics, integral medicine, creative ecology and genuine sustainability (Section 4). As a result, we reveal the superior level of problem-solving progress in science, complex technologies and civilisation development in general (otherwise stagnating and turning into a heavy decline), based remarkably on the consistent solution of the unreduced interaction problem and the ensuing new mathematics of complexity.

2. Complex dynamics of any real interaction process

2.1. Multivalued interaction dynamics and the universal complexity concept. We start our analysis of arbitrary real interaction process from its expression by the system "existence equation", which generalises the Hamiltonian form of various fundamental dynamic equations (such as the Schrödinger equation for quantum systems or the Hamilton–Jacobi equation for classical systems) and can be shown indeed to be the unified expression of any real system dynamics (see Section 2.2) [K3, K5, K9, K17, K18, K19]:

$$\left\{\sum_{k=0}^{N}\Big[h_k(q_k)+\sum_{l>k}^{N}V_{kl}(q_k,q_l)\Big]\right\}\Psi(Q)=E\Psi(Q), \tag{1}$$

where $h_k(q_k)$ is the "generalised Hamiltonian" (specified below as a measure of dynamic complexity) for the k-th system component in the absence of interaction, q_k is the degree(s) of freedom of the k-th component, $V_{kl}(q_k,q_l)$ is the potential of (arbitrary) interaction between the k-th and l-th components, $Q\equiv\{q_0,q_1,\dots,q_N\}$, $\Psi(Q)$ is the system state-function expressing its configuration, E is the generalised Hamiltonian eigenvalue, and summations are performed over all (N) system components. With evident transformations, this dynamic equation actually covers the (less fundamental) case of time-dependent formalism for interaction explicitly depending on time.

It is convenient to separate in (1) certain "common" degree(s) of freedom, $q_0\equiv\xi$, usually characterising system's spatial configuration:

$$\left\{h_0(\xi)+\sum_{k=1}^{N}\Big[h_k(q_k)+V_{0k}(\xi,q_k)+\sum_{l>k}^{N}V_{kl}(q_k,q_l)\Big]\right\}\Psi(\xi,Q)=E\Psi(\xi,Q), \tag{2}$$

where now $Q\equiv\{q_1,\dots,q_N\}$ and $k,l\geq 1$. We pass then to natural problem expression in terms of known eigen-solutions of system elements in the absence of interaction:

$$h_k(q_k)\varphi_{kn_k}(q_k)=\varepsilon_{n_k}\varphi_{kn_k}(q_k), \tag{3}$$

$$\Psi(\xi,Q)=\sum_n\psi_n(\xi)\varphi_{1n_1}(q_1)\varphi_{2n_2}(q_2)\dots\varphi_{Nn_N}(q_N)\equiv\sum_n\psi_n(\xi)\Phi_n(Q), \tag{4}$$

where $\{\varphi_{kn_k}(q_k), \varepsilon_{n_k}\}$ is the complete set of orthonormal eigenfunctions and eigenvalues of the k-th component Hamiltonian $h_k(q_k)$, $n \equiv \{n_1, \ldots, n_N\}$ runs through all eigenstate combinations, and $\Phi_n(Q) \equiv \varphi_{1n_1}(q_1)\varphi_{2n_2}(q_2)\ldots\varphi_{Nn_N}(q_N)$ by definition.

The system of equations for $\{\psi_n(\xi)\}$ is obtained from (2) in a standard way, using the eigenfunction orthonormality [K3, K5, K9, K13, K14, K16, K17, K18, K19]:

$$\begin{aligned} \big[h_0(\xi) + V_{00}(\xi)\big]\psi_0(\xi) + \sum_n V_{0n}(\xi)\psi_n(\xi) &= \eta\psi_0(\xi), \\ \big[h_0(\xi) + V_{nn}(\xi)\big]\psi_n(\xi) + \sum_{n'\neq n} V_{nn'}(\xi)\psi_{n'}(\xi) &= \eta_n\psi_n(\xi) - V_{n0}(\xi)\psi_0(\xi), \end{aligned} \tag{5}$$

where $n, n' \neq 0$ (also below), $\eta \equiv \eta_0 = E - \varepsilon_0$, $\eta_n = E - \varepsilon_n$, $\varepsilon_n = \sum_k \varepsilon_{n_k}$,

$$V_{nn'}(\xi) = \sum_k \Big[V_{k0}^{nn'}(\xi) + \sum_{l>k} V_{kl}^{nn'}\Big], \tag{6}$$

$$V_{k0}^{nn'}(\xi) = \int_{\Omega_Q} dQ\, \Phi_n^*(Q) V_{k0}(q_k, \xi)\Phi_{n'}(Q), \tag{7}$$

$$V_{kl}^{nn'}(\xi) = \int_{\Omega_Q} dQ\, \Phi_n^*(Q) V_{kl}(q_k, q_l)\Phi_{n'}(Q), \tag{8}$$

and we have separated the equation for $\psi_0(\xi)$ describing the generalised "ground state", i.e. (eventually) the state with minimum complexity. The system of equations (5) is equivalent to the starting existence equation (1)–(2), but being expressed now through "natural" dynamic variables, the problem can be deeply analysed for various starting models, including time-dependent and formally "nonlinear" ones (see below for the definition and discussion of *essential* nonlinearity).

We try to solve the "nonintegrable" system of equations (5) with the help of generalised effective, or optical, potential method [D, K1], where one expresses $\psi_n(\xi)$ through $\psi_0(\xi)$ from the equations for $\psi_n(\xi)$ using the standard Green function technique and then inserts the result into the equation for $\psi_0(\xi)$, obtaining thus the *effective existence equation*, which contains *explicitly* only "integrable" degrees of freedom (ξ) [K2, K3, K17]:

$$h_0(\xi)\psi_0(\xi) + V_{\text{eff}}(\xi;\eta)\psi_0(\xi) = \eta\psi_0(\xi), \tag{9}$$

where the operator of *effective potential* (*EP*), $V_{\text{eff}}(\xi;\eta)$, is given by

$$V_{\text{eff}}(\xi;\eta) = V_{00}(\xi) + \hat{V}(\xi;\eta), \quad \hat{V}(\xi;\eta)\psi_0(\xi) = \int_{\Omega_\xi} d\xi'\, V(\xi,\xi';\eta)\psi_0(\xi'), \tag{10}$$

$$V(\xi,\xi';\eta) = \sum_{n,i} \frac{V_{0n}(\xi)\psi_{ni}^0(\xi)V_{n0}(\xi')\psi_{ni}^{0*}(\xi')}{\eta - \eta_{ni}^0 - \varepsilon_{n0}}, \quad \varepsilon_{n0} \equiv \varepsilon_n - \varepsilon_0, \tag{11}$$

and $\{\psi_{ni}^0(\xi), \eta_{ni}^0\}$ is the complete set of eigenfunctions and eigenvalues of a *truncated* system of equations:

$$\big[h_0(\xi) + V_{nn}(\xi)\big]\psi_n(\xi) + \sum_{n'\neq n} V_{nn'}(\xi)\psi_{n'}(\xi) = \eta_n\psi_n(\xi). \tag{12}$$

The eigenfunctions, $\{\psi_{0i}(\xi)\}$, and eigenvalues, $\{\eta_i\}$, of the formally "integrable" equation (9) are used to obtain other state-function components:

$$\psi_{ni}(\xi) = \hat{g}_{ni}(\xi)\psi_{0i}(\xi) \equiv \int_{\Omega_\xi} d\xi'\, g_{ni}(\xi,\xi')\psi_{0i}(\xi'), \tag{13}$$

$$g_{ni}(\xi,\xi') = V_{n0}(\xi') \sum_{i'} \frac{\psi^0_{ni'}(\xi)\psi^{0*}_{ni'}(\xi')}{\eta_i - \eta^0_{ni'} - \varepsilon_{n0}}\,, \tag{14}$$

and the total system state-function, $\Psi(q_0, q_1, \ldots, q_N) = \Psi(\xi, Q)$ (see (4)):

$$\Psi(\xi,Q) = \sum_i c_i \Big[\Phi_0(Q) + \sum_n \Phi_n(Q)\hat{g}_{ni}(\xi)\Big]\psi_{0i}(\xi), \tag{15}$$

where coefficients c_i should be found from the state-function matching conditions at the boundary where interaction effectively vanishes. The measured quantity, generalised system density $\rho(\xi, Q)$, is obtained as state-function squared modulus, $\rho(\xi,Q) = |\Psi(\xi,Q)|^2$ (for "wave-like" complexity levels), or as state-function itself, $\rho(\xi,Q) = \Psi(\xi,Q)$ (for "particle-like" structures) [K3].

Although the problem in its EP formulation, (9)–(11), remains "nonintegrable" and equivalent to the initial formulation, (1), (2), (5), the interaction *dynamical links* explicitly present in the effective version reveal the *qualitatively new properties* of the unreduced problem solution, leading to its reconstitution in the complete, correctly adjustable form [K1, K2, K3, K5, K7, K8, K9, K11, K17, K18, K19].

The key property of the unreduced (any real) interaction result (9)–(15) is its *dynamic multivaluedness, or redundance*, meaning that one has a *redundant* number of individually complete and therefore *mutually incompatible* solutions describing *equally real* system configurations, or *realisations*. This major property of realisation (solution) plurality, underlying the new mathematics of complexity (Section 3), is due to the nonlinear and self-consistent dependence of the unreduced EP, (9)–(11), on the solutions to be found, which leads to the easily calculated growth of the highest power of the characteristic equation (determining the total eigenvalue number) and reflects the physically obvious plurality of interacting eigen-mode combinations [K1, K2, K3, K5, K7, K8, K9, K11, K17, K18, K19].

If N_ξ and N_q are the numbers of terms in sums over i and n in equation (11), then the total number of eigenvalues of equation (9) is $N_{\max} = N_\xi(N_\xi N_q + 1) = (N_\xi)^2 N_q + N_\xi$, which gives the N_ξ-fold redundance of the usual "complete" set of $N_\xi N_q$ eigen-solutions of equations (5) plus an additional, "incomplete" set of N_ξ solutions. It means that the total number of "regular", locally complete system realisations is $N_\Re = N_\xi$, whereas the additional set of N_ξ solutions forms a special, "intermediate" realisation that plays the role of transitional state during inevitable system jumps between regular realisations and provides thus the universal, *causally complete* (physically real) extension of the quantum-mechanical *wavefunction* and classical (*probability*) *distribution function* [K3, K10, K11, K17, K18, K22] (see Section 2.2 for more details).

If we imagine, for a transparent illustration [K19], a simplified pair-wise attraction scheme between two objects with N interacting modes/elements each, then the total number of direct "interaction links" between objects, N^2, will reflect the number of all

"eigen-solutions", while we still have only N "accessible places" in reality for any emerging system configuration. The system will be forced then, by the driving interaction itself, to permanently "switch" between its $N_{\Re} = N^2/N = N$ redundant, incompatible realisations with N specifically arranged elements and links each. This gives also the simplest estimate for the realisation number $N_{\Re}$ to be equal to the number N of system elements (or, in general, all their eigenmodes), whereas in more complicated cases it is determined by the number of all possible combinations of system interaction links, $N!$ (see also Section 3).

The same property of fundamental dynamic multivaluedness of any real interaction result is confirmed by the rather straightforward graphical analysis of EP equation solutions [K1, K2, K3], which is not reproduced here for brevity.

Note that dynamic multivaluedness emerges due to the unreduced problem formulation, whereas any usual theory, including standard EP applications (see e. g. [D]) and scholar "science of complexity", uses one or another version of perturbation theory (or "exact model"), which tries to produce a single, closed-form solution just "killing" all other, *redundant* solutions by eliminating the *dynamically* emerging nonlinear interaction links and retaining *only one*, "averaged" solution, which expresses only trivial, small or already inserted, deviations from the initial configuration. That *dynamically single-valued*, or *unitary*, problem reduction, equivalent to its *zero-dimensional, point-like projection*, forms the basis of the entire canonical, "positivistic" science paradigm (thus *rigorously* specified now as unitary, or dynamically single-valued, science and mathematics).

Since in the unreduced interaction description we have many *incompatible*, but *equally real* system realisations *explicitly emerging* from the same, driving interaction, we obtain the universal intrinsic property of *causal, or dynamic, randomness* for any real system in the form of *permanently changing* system realisations that replace each other in *truly random* (unpredictable, undecidable, noncomputable) order thus naturally defined. This *omnipresent, unceasing* randomness in any, even *externally* regular system behaviour provides the universal, consistent version of (*dynamical*) *chaos*, which is essentially different from any its usual, unitary version, inevitably reduced to "involved regularity", including *incorrectly* assumed "noise amplification" (as a result of invalid extension of a perturbation theory approximation) [K3].

Mathematically, it means that the complete *general solution* of arbitrary interaction problem (describing a *real* system/process) has the form of the *dynamically probabilistic* sum of measured quantity (system density) values for different realisations:

$$\rho(\xi, Q) = \sum_{r=1}^{N_{\Re}}{}^{\oplus} \rho_r(\xi, Q), \tag{16}$$

where the summation is performed over all system realisations, $N_{\Re}$ is their number (its maximum value is generally equal to the number of system eigenmodes, $N_{\Re} = N$), and the sign $\oplus$ designates the special, dynamically probabilistic meaning of the sum. It implies that any measured quantity (16) is *intrinsically unstable* (even for a totally isolated system) and its current value *will* unpredictably change to another one, corresponding to another, *randomly* chosen realisation. Such kind of *permanently unstable* dynamics is readily observed in nature and explains, in particular, specific features of living organism

behaviour [K3, K5, K11, K17, K18, K22], but is avoided in unitary theory and usual technological systems, where it is associated with linear "noncomputability" (e. g. [P]) and technical failure. Therefore, the universal dynamic multivaluedness revealed by rigorous interaction problem solution forms the basis for the *causally complete* understanding of natural biological and artificial "bio-inspired" and "intelligent" systems (Section 4), where causal randomness is transformed from an obstacle to the key advantage (Section 2.2).

The obtained causal randomness of the generalised EP formalism (9)–(16) is accompanied by the *dynamic probability definition*. Since elementary realisations are equal in their "rights to emerge", the dynamically derived, *a priori probability* of r-th realisation emergence, α_r, is given by

$$\alpha_r = \frac{1}{N_{\Re}}\,, \qquad \sum_r \alpha_r = 1. \tag{17}$$

Actual observations often deal with dense "self-organised" groups of similar elementary realisations (see Section 2.2). The dynamic probability of such r-th compound realisation is determined by the number, N_r, of elementary realisations it contains:

$$\alpha_r(N_r) = \frac{N_r}{N_{\Re}} \qquad \Big(N_r = 1, \ldots, N_{\Re};\ \sum_r N_r = N_{\Re}\Big), \qquad \sum_r \alpha_r = 1. \tag{18}$$

The (quasi)stationary *expectation value*, $\rho_{\exp}(\xi, Q)$, is easily obtained from (16)–(18) for statistically large event numbers:

$$\rho_{\exp}(\xi, Q) = \sum_r \alpha_r \rho_r(\xi, Q). \tag{19}$$

It is important, however, that contrary to conventional theory, our *dynamically derived* randomness and probability need not rely on such "statistical", empirically based definition, and the basic expressions (16)–(18) remain valid for any *single* event of realisation emergence and even *before* it happens. Note that the realisation probability distribution is also obtained in another way involving the *generalised wavefunction* (*or distribution function*) and *Born's probability rule* (Section 2.2) [K3, K10, K11, K17, K18, K22].

Closely related to the dynamic multivaluedness is the property of *dynamic entanglement* of the interacting system components (degrees of freedom), described by the dynamically weighted products of eigenfunctions depending on different degrees of freedom (ξ, Q) in the state-function expression (15). It provides the well-specified meaning of (direct) "interaction" as such and the *mathematically exact* version of the tangible *quality* of the emerging system structure, which is absent in unitary models dealing with abstract, "immaterial" entities.

The obtained *dynamically multivalued entanglement* of the unreduced interaction result describes a *living* structure, permanently changing and probabilistically *adapting* its tangible, physically real configuration, which provides a well-specified basis for biological and bio-inspired applications (Section 4). The properties of dynamically multivalued entanglement and adaptability are further amplified due to the complex-dynamical, *probabilistic fractality* of the unreduced general solution [K3, K5, K11, K17, K18, K22] obtained by application of the same EP method to solution of the truncated system of equations (12) from the first-level EP expression (11) (see Section 2.2 for more details).

We can now rigorously and *universally* define the unreduced *dynamic complexity*, C, of *any* real system or interaction process as a growing function of the number of its *explicitly obtained* realisations, or rate of their change, equal to zero for the unrealistic case of only one realisation [K2, K3, K9, K10, K11, K13, K14, K15, K16, K17, K18, K19, K21, K22, KU]:

$$C = C(N_{\Re}), \quad dC/dN_{\Re} > 0, \quad C(1) = 0. \tag{20}$$

Suitable examples are provided by $C(N_{\Re}) = C_0 \ln N_{\Re}$, $C(N_{\Re}) = C_0(N_{\Re} - 1)$, generalised energy/mass (temporal rate of realisation change) and momentum (spatial rate of realisation emergence) (see Section 2.2). Once again it becomes clear that the entire *dynamically single-valued* paradigm and results of canonical theory (including its versions of "complexity", "chaos" and *imitations* of "multi-stability" in *abstract* "spaces") correspond to exactly *zero* value of unreduced complexity equivalent to effectively zero-dimensional, point-like projection of reality. The proposed universal concept of complexity and its applications appear respectively as the well-specified, *causally complete extension* of usual theory to the unreduced, dynamically multivalued picture of reality.

It means that *any* usual, dynamically single-valued "model" can produce only a basically regular result containing no genuine, intrinsic randomness (chaoticity), which should instead be introduced artificially (and inconsistently), e.g. as a *regular* "amplification" of "random" (by convention) *external* "uncertainty". By contrast, our unreduced dynamic complexity is practically synonymous to equally universally defined and genuine *chaoticity* (see above) and thus also to the unified *dynamic entropy* (see below, Section 2.2). Note that genuine dynamical chaos thus obtained has a complicated internal structure (contrary to ill-defined unitary "stochasticity") and contains *partial regularity/order* of *inhomogeneous* realisation *probability* distribution dynamically mixed with genuine randomness (unpredictability) of each realisation emergence.

Universal dynamic complexity and related properties involve the *essential, or dynamic, nonlinearity* of the unreduced problem solution. It is provided by feedback links of developing interaction as they are expressed by EP dependence on the problem solutions (see (9)–(11)). It is the *dynamically emerging* nonlinearity, since it appears even for a formally "linear" initial problem expression (1)–(2), (5), whereas usual, mechanistic "nonlinearity" is but a perturbative reduction and imitation of this essential nonlinearity of the unreduced EP formalism (see also Section 2.2). Essential nonlinearity leads to irreducible *dynamic instability* within *any* system state: both are determined by the same mechanism of dynamic feedback development.

2.2. Probabilistic fractality, unified dynamic regimes, and the symmetry of complexity. We shall consider now more elaborated features and evolution of the unreduced dynamic complexity introduced in the previous section.

One of essential mathematical novelties emerging from the real, non-simplified interaction dynamics and especially important for biological applications is its probabilistically varying multilevel structure taking the form of *dynamically multivalued, or probabilistic, fractal* (of system structure). It appears as a result of partial incompleteness of the first-level solution (9)–(19), which relies upon the yet unknown solutions of the truncated system of equations (12). In order to reduce the uncertainty, we can apply the same

unreduced EP approach to solution of this truncated problem, giving the second-level effective equation, generally similar to the first-level equation (9):

$$\big[h_0(\xi) + V^n_{\rm eff}(\xi;\eta_n)\big]\psi_n(\xi) = \eta_n\psi_n(\xi), \tag{21}$$

where the second-level EP $V^n_{\rm eff}(\xi;\eta_n)$ is similar to its first-level version (10)–(11):

$$V^n_{\rm eff}(\xi;\eta_n)\psi_n(\xi) = V_{nn}(\xi)\psi_n(\xi) + \sum_{n'\neq n,i} \frac{V_{nn'}(\xi)\psi^{0n}_{n'i}(\xi)\int_{\Omega_\xi} d\xi'\,\psi^{0n*}_{n'i}(\xi')V_{n'n}(\xi')\psi_n(\xi')}{\eta_n - \eta^{0n}_{n'i} + \varepsilon_{n0} - \varepsilon_{n'0}}, \tag{22}$$

and $\{\psi^{0n}_{n'i}(\xi), \eta^{0n}_{n'i}\}$ is the complete eigen-solution set of the second-level truncated system:

$$h_0(\xi)\psi_{n'}(\xi) + \sum_{n''\neq n'} V_{n'n''}(\xi)\psi_{n''}(\xi) = \eta_{n'}\psi_{n'}(\xi), \quad n' \neq n, 0. \tag{23}$$

Similarity of equations (21)–(23) to the first-level EP expressions (10)–(12) implies that its second-level version is also split into many incompatible realisations (numbered by index r') due to the self-consistent dependence on the eigen-solutions to be found, leading to respective splitting of solutions of the first-level truncated system (12):

$$\{\psi^0_{ni}(\xi), \eta^0_{ni}\} \to \{\psi^{0r'}_{ni}(\xi), \eta^{0r'}_{ni}\}. \tag{24}$$

This process of hierarchical dynamical splitting of emerging system structure will continue in the same way with ever more truncated auxiliary systems of equations until the last, exactly solvable system (of two equations). Substituting the dynamically multivalued solutions of each truncated system into the expressions of the previous-level EP solutions, we get the dynamically probabilistic fractal of the now *truly complete problem solution* in the form of multilevel hierarchy of probabilistically changing realisations:

$$\rho(\xi, Q) = \sum^{N_\Re \oplus}_{r,r',r'',\ldots} \rho_{rr'r''\ldots}(\xi, Q), \tag{25}$$

where the indexes $r, r', r'', \ldots$ enumerate the realisations explicitly obtained at consecutive levels of dynamically probabilistic fractality. Similar to the a priori probabilities of realisation emergence events of the first level, (17)–(18), we obtain the hierarchy of *causal realisation probabilities* $\{\alpha_{rr'r''\ldots}\}$ for all levels of dynamically multivalued fractal:

$$\alpha_{rr'r''\ldots} = \frac{N_{rr'r''\ldots}}{N_\Re}, \qquad \sum_{r,r',r'',\ldots} \alpha_{rr'r''\ldots} = 1. \tag{26}$$

Correspondingly, the *expectation value* for the observed density of the dynamically probabilistic fractal of the complete problem solution is obtained as:

$$\rho_{\rm exp}(\xi, Q) = \sum^{N_\Re}_{r,r',r'',\ldots} \alpha_{rr'r''\ldots}\rho_{rr'r''\ldots}(\xi, Q). \tag{27}$$

The essential difference of the dynamically probabilistic fractal from conventional, abstract fractals is evident: the latter are not solutions (let alone causally complete solutions!) to any real interaction problems and possess respectively only simplified, unitary

"scale symmetry" and basic regularity (not really modified by any mechanistic probability insertion). By contrast, our dynamically multivalued fractal in general does not possess the scale invariance (with only rare exceptions for a limited scale range) and realise instead the much deeper law of the universal symmetry of complexity (see below). It is also different from any conventional, approximate (and usually diverging) "series expansion": the possibly long, but finite sums of the dynamically probabilistic fractal solution (25), (27) provide the *exact* version of the *real* multilevel system structure.

Moreover, one can show that the entire world structure emerges as a gigantic, but single and physically unified dynamically multivalued fractal of the underlying simplest interaction between two primal entities ("protofields"), with all the observed properties and laws at all levels of the world structure rigorously derived as emergent features of that unified fractal dynamics [K2, K3, K4, K5, K7, K8, K9, K10, K11, K12, K13, K14, K15, K16, K17, K18, K19, K20, K21, K22, KU] (see also below). Among those properties one may cite the *dynamic adaptability* directly related to the interactive dynamic origin of probabilistic realisation change in the multilevel fractal structure (and thus rigorously absent in any unitary description). The high power and efficiency of the related process of "sensible search" of the optimal structure creation underlie, in particular, the "magic" properties of life and intelligence, corresponding to the huge exponential growth of the fractal realisation number $N_{\Re}$ and thus complexity (20) (see item (III) in Section 3).

The unified multivalued fractal dynamics provides also the natural origin and universal classification of *all possible dynamic regimes*, between the limiting cases of strong chaos and (external) regularity. It is already physically evident that the highly chaotic "free search" in fine fractal branches is transformed into much more "confined" (quasi-regular) branch dynamics as a result of restrictive interactions leading to emergence of the higher-level structure (which in its turn may give rise to strongly chaotic higher sublevels).

Rigorously [K1, K2, K3, K8, K16, K17, K18], one limiting case of complex (multivalued) dynamics, called *uniform, or global, chaos*, is obtained from the main EP formalism (9)–(15) as sufficiently different realisations with a homogeneous probability distribution (i.e. $N_r \approx 1$ and $\alpha_r \approx 1/N_{\Re}$ for all r in (18)). It emerges when major interaction parameters (suitably represented by energy level separations or frequencies of intra-component and inter-component motions) are close to each other, which leads to a strong "conflict of interests" and the ensuing "big disorder", without any dominant ordering motion.

The opposite limiting regime of *multivalued self-organisation or self-organised criticality* (*SOC*) emerges for sufficiently different interaction frequencies, so that, as easily seen from (10), (15), one or few rigid, low-frequency components "enslave" a great number of high-frequency and rapidly changing, but configurationally similar realisations (i.e. $N_r \sim N_{\Re}$, the realisation probability distribution is highly inhomogeneous), and the EP (9)–(10) and state-function (15) approach quasi-local functions [K3, K8, K16, K17, K18]. However, the difference of that extended, multivalued SOC from usual self-organisation (and SOC) is essential: despite the rigid *external* shape of system configuration in this regime, it confines an intense "internal life" and *chaos* of changing "enslaved" realisations (which are *not* superposable unitary "modes"). This is the key to consistent solution of the well-known entropy-growth problems, in particular for biological systems (see also

below). Another important advance with respect to the unitary "science of complexity" is that this real, multivalued self-organisation unifies the extended versions of a whole variety of separated unitary "models", including usual "self-organisation" (or "synergetics"), SOC, "synchronisation", "control of chaos", "attractors", and "mode locking".

Practically all real dynamic regimes fall between these limiting cases of uniform chaos and multivalued SOC (including their multi-level, fractal combinations) — and they are naturally obtained for respective intermediate parameter values.

The point of transition to the global chaos regime is expressed by the *universal criterion of global chaos onset* rigorously derived thus from the basic EP formalism (9)–(15):

$$\kappa \equiv \frac{\Delta\eta_i}{\Delta\eta_n} = \frac{\omega_\xi}{\omega_q} \cong 1, \tag{28}$$

where κ is the introduced *chaoticity* parameter, $\Delta\eta_i$, ω_ξ and $\Delta\eta_n \sim \Delta\varepsilon$, ω_q are energy-level separations and frequencies for the inter-component and intra-component motions, respectively. At $\kappa \ll 1$ one has the externally regular multivalued SOC regime, which degenerates into global chaos as κ grows from 0 to 1, and the maximum irregularity at $\kappa \approx 1$ is again transformed into a SOC kind of structure at $\kappa \gg 1$ (but with the "inverse" system configuration). One can compare the universal, rigorous, simple and physically transparent criterion of (strong) chaos onset of equation (28) with various existing, nonuniversal and contradictory, criteria and definitions of chaoticity from unitary theory, such as "overlapping resonances", "(positive) Lyapunov exponents", "multistability", "coexisting attractors", or "unstable periodic orbits", all of them referring to the dynamically single-valued, single-trajectory and thus basically regular problem description (see [K3, K17] for more details). In particular, our criterion (28) remains valid for the *quantum chaos* case, where it describes the emergence of genuine quantum dynamic randomness, in full agreement with the quantum-classical correspondence principle [K2, K3, K17], whereas usual theory fails completely to find any true quantum chaos.

The obtained unified criterion of chaos (28) provides also the unreduced, universally valid and extended meaning of the "well-known" phenomenon of resonance as the condition of global (strongest) chaoticity of system dynamics (absent in unitary understanding of resonance). Moreover, the same analysis of the unreduced EP equations reveals a similar role of higher resonances as "sources of increased chaoticity", so that when chaoticity κ grows from 0 (quasi-regularity) to 1 (global chaos), the degree of randomness makes a higher jump each time κ passes through a higher resonance, $\kappa = m/n$, with integer $n > m$ [K2, K3, K16, K17]. As those ever higher (and weaker) resonances constitute a dense network of rational values of κ, we obtain a well-specified manifestation of the "fractal structure of chaos", this time in the system parameter space.

The dynamically multivalued fractal of the emerging system structure is thus the *universal structure* of the world or any its part exactly represented by the unreduced interaction problem solution (9)–(27) and containing various alternating dynamic regimes between global chaos and multivalued SOC. There is also the *universal law* of dynamic existence and development of this unified system structure, the universal *complexity conservation law*. It stems already from the fact that the system realisation number underlying its dynamic complexity according to (20) is determined by the initial system structure

(e.g. by the number of component eigenmodes or their combinations) and therefore remains unchanged during any further system evolution.

However, while the total dynamic complexity remains constant, something should change in the process of system structure development driven by its interactions. As branches and levels of the dynamically multivalued fractal progressively emerge in this process, the *potential form* of interaction complexity, or dynamic information I, is transformed to its realised, *unfolded form* of dynamic entropy S, so that their sum, the *total dynamic complexity* $C = I + S$ remains unchanged, $\Delta C = 0$, $\Delta I = -\Delta S < 0$ [K3, K9, K11, K12, K16, K17, K18, K19]. Both complexity forms are measured, of course, in the same way, by various suitable functions of realisation number or rate of their change (Section 2.1, around equation (20) and below). They only reflect different stages of system realisation emergence and change.

Note also that contrary to unitary conservation laws, here the dynamic *symmetry* between changing realisations and their number *conservation* mean the same, so that there is no difference any more between a "symmetry" and the respective "conservation law" (cf. "Noether's theorem"), and we obtain the *universal symmetry of complexity* implying *complexity conservation by permanent transformation from dynamic information to dynamic entropy*. Another difference from unitary symmetries is that the latter reflect "ideal" (regular) structure transformations and therefore often become "broken" or inexact in real world, while the universal symmetry of complexity does the opposite by relating quite *irregular* realisation structures within the *absolutely exact* symmetry of complexity, which is thus *never violated* (as it should be the case for a genuine, rigorous law). It also unifies the extended, complex-dynamical versions of *all* (correct) symmetries and laws (see below), remaining separated in unitary theory and mathematics.

In order to obtain a useful dynamic expression of the universal symmetry of complexity, we first introduce the unified elementary forms of complexity known as *time* and *space*, now *explicitly emerging* from the unreduced interaction analysis (contrary to their empirically based postulation in unitary theory) [K3, K9, K10, K11, K12, K16, K17, K18, K19].

Physically real, naturally unstoppable and irreversible *time flow* is provided by the permanent, interaction driven change of mutually incompatible system realisations occurring in a dynamically random order (Section 2.1), so that time, with its irreversible and unstoppable flow, is practically equivalent to the fundamental dynamic multivaluedness of the unreduced interaction process (see (9)–(18)) and therefore *cannot* be consistently understood within usual, dynamically single-valued description (where it should be introduced artificially). Note the fundamental role of *dynamic randomness* in real, complex-dynamical time concept and flow, contrasting with usually assumed, but finally only external regularity of time. Dynamically related to this physically real time is equally real and emergent, naturally discrete *space structure* given simply by realisations themselves (forming the generalised physical space "points") and system jumps between its successive realisations (determining the elementary space "length").

While space is a tangible, textured entity (reflecting the tangible structure of system realisations made by the *dynamic entanglement* of interacting entities, Section 2.1), time is a really flowing, but "immaterial" entity, reflecting realisation change process

and related to material space structure only dynamically (as opposed to the postulated mechanistic "mixture" of abstract space and time variables within a space-time "manifold" in unitary theory). Because of the naturally multilevel, dynamically fractal structure of developing interaction complexity (see above in this section), the physically real time and space will also possess multilevel and fractal structure reproducing that of the universal complex-dynamical fractal. The lowest level of space and time constitutes the physically real version of fundamental, (externally) smooth, "embedding" and "empty" space and time from traditional, "Newtonian" science, while their higher levels demonstrate discreteness and inhomogeneity on all scales giving the diversity of observed world structures.

Mathematically, the *space element*, or *elementary size*, Δx, is given by the eigenvalue separation of the unreduced EP formalism (9)–(12), $\Delta x = \Delta\eta_i^r$, where the separation of eigenvalues (numbered by i) within the same realisation provides the *space point size*, $r_0 \simeq \Delta x_i = \Delta_i \eta_i^r$, while the separation of eigenvalues from neighbouring realisations (numbered by r) gives the elementary length (smallest distance between points), $\lambda \simeq \Delta x_r = \Delta_r \eta_i^r$. The *elementary time interval*, Δt, is obtained as *intensity*, specified as *frequency*, ν, of universally defined *events* of realisation change, $\Delta t = \tau = 1/\nu$. Whereas the events and thus the time flow result causally from the dynamic multivaluedness of unreduced interaction process, a practically useful expression for $\Delta t = \tau$ is based on the above elementary length $\lambda = \Delta x_r$ and the (known) velocity v_0 of signal propagation in the material of interaction components (from a lower complexity level), $\tau = \lambda / v_0$.

Since the emergent time and space intervals characterise the realisation change process, while the unreduced dynamic complexity (20) is universally defined as a growing function of realisation number or rate of their change, it becomes evident that a *fundamental complexity measure* is provided by the simplest combination of (independent) space and time variables, known as *action*, $\mathcal{A}$, which acquires now the extended, universal and complex-dynamical meaning [K3, K9, K11, K12, K16, K17, K18, K19]:

$$\Delta\mathcal{A} = p\Delta x - E\Delta t, \tag{29}$$

where the coefficients p and E are recognised as (now extended) *momentum* and (total) *energy*:

$$p = \frac{\Delta\mathcal{A}}{\Delta x}\Big|_{t=\text{const}} \simeq \frac{\mathcal{A}_0}{\lambda}\,, \tag{30}$$

$$E = -\frac{\Delta\mathcal{A}}{\Delta t}\Big|_{x=\text{const}} \simeq \frac{\mathcal{A}_0}{\tau}\,, \tag{31}$$

$\mathcal{A}_0$ being the characteristic action magnitude at the considered complexity level, and the evident vector versions of these and further relations are implied if necessary. We see that in its extended meaning action is a universal *integral complexity measure*, while momentum and energy are equally universal *differential complexity measures*.

Because of the dynamically irreversible time flow ($\Delta t > 0$) obtained above and positive total energy ($E > 0$), action can only decrease with time, $\Delta\mathcal{A} < 0$ (see (31)). Due to the dynamically random realisation choice, it measures a *consumable*, irreversibly decreasing form of complexity coinciding thus with the dynamic information I from the above universal symmetry of complexity, $\mathcal{A} = I$ (we shall also call it *complexity-action*).

Conservation (symmetry) of total complexity $C = I + S$ can now be expressed as

$$\Delta C = \Delta\mathcal{A} + \Delta S = 0, \quad \Delta S = -\Delta\mathcal{A} > 0, \tag{32}$$

where the dynamic entropy, or *complexity-entropy*, S can only grow, at the expense of complexity-action $\mathcal{A}$, providing another expression of irreversible time direction, as well as the extended, *absolutely universal* versions (and genuine meaning) of the *second law of thermodynamics* (energy degradation principle) and the *least-action principle*, applicable to *any* real system dynamics [K3, K9, K11, K12, K16, K17, K18, K19].

We can now obtain the desired dynamic expression of the universal symmetry of complexity by dividing its initial expression (32) by $\Delta t|_{x=\text{const}}$:

$$\frac{\Delta\mathcal{A}}{\Delta t}\Big|_{x=\text{const}} + H\Big(x, \frac{\Delta\mathcal{A}}{\Delta x}\Big|_{t=\text{const}}, t\Big) = 0, \quad H = E > 0, \tag{33}$$

where the *generalised Hamiltonian*, $H = H(x, p, t)$, considered as a function of emerging space coordinates x, momentum $p = (\Delta\mathcal{A}/\Delta x)|_{t=\text{const}}$ (see equation (30)) and time t, expresses the unfolded, entropy-like form of differential complexity, $H = (\Delta S/\Delta t)|_{x=\text{const}}$, while the last inequality reflects the above generalised second law (or the time arrow direction), in agreement with the generalised energy definition (31). We obtain thus the differential dynamic expression of the universal symmetry of complexity in the form of *generalised*, now consistently derived and universally applicable *Hamilton–Jacobi equation* extending essentially its usual version and providing its true, *complex-dynamical* origin. The finite-increment form of equation (33) reflects the natural discreteness of complex interaction dynamics and will tend to continuous limit for suitable cases and problem scales (still preserving, however, the extended complex-dynamical meaning). The generalised Hamilton–Jacobi equation takes a simpler form for conservative (closed) systems, where the Hamiltonian does not depend explicitly on time:

$$H\Big(x, \frac{\Delta\mathcal{A}}{\Delta x}\Big|_{t=\text{const}}\Big) = E, \tag{34}$$

with the conserved total energy E defined by equation (31).

The dynamic entropy growth law, constituting an integral part of the universal symmetry of complexity (32)–(33), can be further amplified with the help of *generalised Lagrangian*, L, defined as the total (discrete) time derivative of complexity-action $\mathcal{A}$:

$$L = \frac{\Delta\mathcal{A}}{\Delta t} = \frac{\Delta\mathcal{A}}{\Delta t}\Big|_{x=\text{const}} + \frac{\Delta\mathcal{A}}{\Delta x}\Big|_{t=\text{const}} \frac{\Delta x}{\Delta t} = pv - H, \tag{35}$$

where $v = \Delta x/\Delta t$ is the velocity of global, averaged system motion (i.e. its motion as a whole). Irreducible dynamic randomness of realisation choice at every step of system dynamics implies the unconditional *decrease* of dynamic information, or complexity-action, (equivalent to dynamic entropy growth), (32), meaning that

$$L < 0, \quad E, H(x, p, t) > pv \geq 0. \tag{36}$$

As noted above, it is important that in the dynamic multivaluedness paradigm this "generalised second law" refers, due to its universality, to both externally chaotic *and externally regular* structure emergence (the conclusion clearly beyond the limits of conventional unitarity unable to solve its respective entropy-growth problems).

The generalised Hamilton–Jacobi equation (33)–(34) describing the evolution and behaviour of the ensemble of "regular" system realisations has an important complement dealing with the dynamics of special, "intermediate" realisation revealed above in the unreduced EP formalism (Section 2.1) and forming the transitional state of briefly disentangled, quasi-free system components before they take the next regular, properly entangled realisation. This intermediate realisation and state, the *generalised wavefunction* $\Psi(x)$, is a realistic and universal extension of the quantum-mechanical wavefunction and various distribution functions from unitary theory. It has a chaotically fluctuating internal structure reflecting the dynamically random emergence of each regular realisation, with the dynamic probability obeying both the main rule of the unreduced EP formalism (17)–(18) and the *generalised Born rule*, causally following from this transitional role of the generalised wavefunction and rigorously obtained from the above matching conditions for the state-function coefficients c_i in (15) [K3, K10, K12, K16, K17, K19]:

$$\alpha_r = \alpha(x_r) = |\Psi(x_r)|^2, \tag{37}$$

where x_r is the r-th realisation configuration and for particle-like complexity levels one should imply the value of the generalised distribution function itself at the right-hand side (instead of its modulus squared for wave-like complexity levels).

Now, in order to find the dynamic equation for $\Psi(x)$ similar to the Hamilton–Jacobi equation (33)–(34) for regular realisations, we can use the *causal quantisation condition*, following again from the symmetry (conservation) of complexity, but applied now to one entire cycle of transition from the wavefunction to a regular realisation and back [K3, K10, K12, K16, K17, K19, K22]:

$$\Delta(\mathcal{A}\Psi) = 0, \quad \Delta\mathcal{A} = -\mathcal{A}_0 \frac{\Delta\Psi}{\Psi}\,, \tag{38}$$

where $\mathcal{A}_0$ is a characteristic complexity-action magnitude that here may contain also a numerical constant reflecting specific features of the considered complexity sublevels (thus at quantum sublevels $\mathcal{A}_0 = i\hbar$, where $\hbar = h/2\pi$ is Planck's constant). Using relation (38) in the Hamilton–Jacobi equation (33), we obtain the causally derived *universal Schrödinger equation* for the *realistically interpreted* generalised wavefunction at *any* level of complexity (starting from the lowest, quantum levels, now liberated from all postulated "mysteries" [K3, K10, K12, K16, K17, K19, K20]):

$$\mathcal{A}_0 \frac{\Delta\Psi}{\Delta t}\Big|_{x=\text{const}} = \hat{H}(x, \hat{p}, t)\Psi(x, t), \quad \hat{p} = -\mathcal{A}_0 \frac{\Delta}{\Delta x}\Big|_{t=\text{const}}\,, \tag{39}$$

where the momentum operator $\hat{p}$ and the Hamiltonian operator, $\hat{H}(x, \hat{p}, t)$, are obtained from momentum p and the Hamiltonian function $H = H(x, p, t)$ of equations (30), (33) by the same causal quantisation (38). For the closed (conservative) system case we similarly obtain from (34) the respective reduced form of the universal Schrödinger equation:

$$\hat{H}(x, \hat{p})\Psi(x) = E\Psi(x). \tag{40}$$

This causally derived and now complete dynamic expression of the universal symmetry of complexity, the *universal Hamilton–Schrödinger formalism* (33)–(40) does apply, together with the initial "global" expression (32), to any real system dynamics (thus justifying also the Hamiltonian form of the initial existence equation (1)) and therefore

underlies *any* (correct) law, "principle" and dynamic equation from unitary theory (where it is typically postulated in a semi-empirical way). In order to demonstrate it more directly, we can, for example, expand the Hamiltonian $\hat{H}(x,\hat{p},t)$ in (39) in a power series of $\hat{p}$ (and Ψ), which gives (for the ordinary, continuous-derivative version):

$$\frac{\partial\Psi}{\partial t}+\sum_{\substack{m=0\\n=1}}^{\infty}h_{mn}(x,t)\big[\Psi(x,t)\big]^{m}\frac{\partial^{n}\Psi}{\partial x^{n}}+\sum_{m=0}^{\infty}h_{m0}(x,t)\big[\Psi(x,t)\big]^{m+1}=0, \qquad (41)$$

where $h_{mn}(x,t)$ are arbitrary functions, while the dependence on Ψ may arise from the effective potential. We can see that various usual model equations are but particular cases of (41), providing thus their true, causally specified origin, including the complex-*dynamic* origin of *any*, usually postulated *nonlinearity* (with similar results for a series expansion in (33), (34) and (40)). Details for quantum, relativistic and other laws can be found elsewhere [K3, K9, K10, K12, K14, K16, K17, K18, K19, K21, K22, KU].

3. New mathematics and laws of complexity. We can now summarise the most important novelties of our unreduced many-body interaction description (Section 2) in the form of major distinctive features of the *new mathematics of complexity and emergence* [K3, K11, K15, K17, K18, K19]:

(i) *Non-uniqueness* of any real problem solution, in the form of *fundamental dynamic multivaluedness* (*redundance*) of system realisations revealed within the unreduced EP method (Section 2.1), with the ensuing *unceasing, dynamically random* (*chaotic*) *internal change* as the unique way of real object existence. This feature is opposite to conventional "existence and uniqueness" theorems and related tacitly assumed "model" of existence without internally originating change. That canonical uniqueness is "proved" due to a (hidden) logical loophole, where uniqueness is assumed from the beginning, in one way or another (e. g. by assuming the effective reduced dimensionality or potential single-valuedness).

(ii) *Absence* (*and impossibility*) *of self-identity postulate* and property, underlying the omnipresent feature of (structure) *emergence* and the origin of *physically real time* flow. Contrary to usual assumption of the "evident" self-identity of any (mathematical and real) structure, $\mathfrak{A}=\mathfrak{A}$ (also the reason for the conventional uniqueness assumption from item (i)), our analysis shows that *no* real structure (and its realistic image) can be self-identical: $\mathfrak{A}\neq\mathfrak{A}$, for any $\mathfrak{A}$, due to the same universal dynamic multivaluedness of $\mathfrak{A}$, leading to its permanent internal change, which excludes self-identity and gives rise to real structure creation (emergence) and related unstoppable, irreversible time flow (Section 2.2). It explains also the origin of irresolvable difficulties with real time (and entropy) concept in usual, dynamically single-valued theory.

(iii) *Rigorous*, universal and irreducible expression of the material *quality* (texture) of the described (emerging) system structure. It is obtained in the form of *dynamic entanglement* of interacting system components within each realisation (Section 2.1) amplified by its *probabilistically fractal* internal structure (Section 2.2). By contrast, usual mathematics tends to deal with abstract, "immaterial" constructions and structures supposed to correctly imitate, or "model", real structures, but actually providing only their

strongly (maximally) reduced, dynamically single-valued, point-like projection, or "envelope". Considered together with the feature (ii), the *dynamically multivalued* (*fractal*) *entanglement* of the unreduced problem solution (9)–(18), (21)–(26) makes the key step from mathematical theory to reality it describes, stopping once and for all the deep traditional gap between them.

(iv) The universal and omnipresent origin of intrinsic, dynamic and *genuine randomness* within *any* real structure, process and evolution. It is in the same major feature (i)–(ii) of fundamental dynamic multivaluedness, which now reveals internal randomness within any real structure (actually synonymous to its universally defined dynamic complexity, Section 2.1). Instead of "chaotic" vs "regular" and "complex" vs "non-complex" systems in usual theory, we obtain only chaotic and complex (though maybe *externally* regular) systems and dynamic regimes, thus solving, in particular, all entropy-growth puzzles in unitary description. We also get rid of the unavoidable contradictions of dynamic randomness (and complexity) definitions in usual theory (time-dependence in classical mechanics, absence of true chaos in quantum mechanics, etc.) and we obtain the totally consistent and causally complete definitions of various related notions also remaining "vague" in usual theory, including *nonintegrability*, *nonseparability*, *noncomputability*, *uncertainty* (*indeterminacy*), *undecidability*, stochasticity, broken symmetry, free will, etc. [K2, K3, K17, K18, K19] (cf. [P]). Every problem becomes (exactly) *solvable* now, but far beyond simplified unitary smoothness and usual "exact" (unique) solutions.

(v) *Dynamic discreteness*, or *causal quantisation*, of unreduced interaction results (and thus all structures) is due eventually to the *holistic* character of *unreduced* interaction, with its feedback loops and finite realisations (Sections 2.1 and 2.2). It gives rise to a deeply structured, *qualitatively inhomogeneous*, or *nonunitary*, character of any system configuration and evolution, summarised in the dynamic origin of *fundamentally discrete* (and fractally structured) *space* (Section 2.2). Dynamic discreteness is, of course, dramatically different from any artificially introduced, mechanistic discreteness of unitary theory and reveals fundamental deficiency of many other related constructions, including calculus, evolution operators, symmetry operators, *any* usual operators, Lyapunov exponents and path integrals.

It is important to emphasize that these distinctive features of the new mathematics of complexity and emergence (i)–(v) result from the universal complete solution of arbitrary interaction problem and therefore are *unified* within the *unique structure* of dynamically multivalued (probabilistic) fractal and *unique law* of the universal symmetry of complexity representing *all* the real world structures and laws respectively (Section 2.2). This is the qualitatively new "global" feature of the new mathematics actually providing its ultimate and self-consistent completion, otherwise desperately missing in its conventional, dynamically single-valued framework. This means, in particular, that contrary to the unitary theory, the new mathematics of complexity can be applied with equal *rigour* to description of objects from conventional "exact" and "natural" sciences, the humanities and arts [K3, K5, K11, K13, K14, K15, K17, K19, K21, K22, KU].

One can add to these unified features, structure (fractal) and law (symmetry) of the new mathematics of complexity several particular laws or "principles" of complexity also

having a quite fundamental origin (the same symmetry of complexity) and universal validity, but related more to applied aspects of the universal science of complexity (see Section 4 and refs. [K3, K14, K17, K18, K19, K22, KU]):

(I) The *complexity correspondence principle* implies efficient or sensible interaction mainly between systems of comparable dynamic complexity. This direct corollary to the universal symmetry of complexity means, in particular, that a system of certain complexity can be efficiently designed and controlled only by systems and techniques of higher, but not lower, dynamic complexity, with numerous critically important applications to modern real-world problems, from (unitary) "quantum computers" (impossible as such) to information systems, global sustainability and other development problems [K3, K13, K14, K15, K17, K18, K19, K21, K22, KU]. In fact, it must be used today as the *rigorous* substantiation and practical guiding rule of the *urgently needed global transition* from the still dominating thinking and methods of the unitary, effectively zero-complexity science to the unreduced complexity analysis within the dynamic multivaluedness paradigm [K3, K15] (we call it "sustainability transition" or "revolution of complexity").

(II) The *complex-dynamical control principle* is based on the complexity-transformation aspect of the universal symmetry of complexity (Section 2.2) and states that any efficient, sustainable control necessarily implies suitable *complexity development* (of both controlled and controlling systems), with inevitable *partially random* (*and basically unlimited*) *change*, in contrast to "limiting" or "fixing" approach of usual, unitary control theory (including its explicitly complex-dynamical aspects, such as "chaos control"). Proper control leading to genuine sustainability is actually reduced thus to design and monitoring of *optimal interaction complexity development* (rather than its maximum restriction in usual approach), emphasizing intrinsic *creativity* aspects of unreduced complex dynamics. It is stability through (suitable) development, instead of traditional limitation.

(III) The *unreduced* (*free*) *interaction principle* refers to the exponentially huge power and efficiency of natural, multicomponent system interaction processes, as opposed to their only power-law efficiency considered within the conventional unitary-model projection [K11, K13, K14, K17, K18, K19, K22, KU]. Referring to the self-developing dynamically multivalued fractal of unreduced interaction process (Section 2.1), one can easily understand that its maximum operation power $P_{\rm real}$, determined by the total (fractal) realisation number $N_{\Re}$ (proportional to the unreduced complexity C), can be estimated as the number of system link combinations:

$$P_{\rm real} \propto N_{\Re} = L! \to \sqrt{2\pi L}\Big(\frac{L}{e}\Big)^{L} \sim L^{L} \gg\gg L, \tag{42}$$

where the number of system links L can already be a very large number, essentially exceeding the number of interacting system components N (thus for both human brain and genome $N > 10^{10}$, $L > 10^{14} \gg N$, see below, Section 4). The obtained exponentially huge power of unreduced complex dynamics $P_{\rm real}$, dramatically exceeding its unitary-model estimates, $P_{\rm reg} \propto L^{\beta}$, $\beta \sim 1$, $P_{\rm real}/P_{\rm reg} \sim L^{L-\beta} \to \infty$, provides the origin of the "miraculous" properties of life, intelligence and consciousness (see the next section for more details).

4. Biomedical applications of the new mathematics of complexity and further progress of humanity. Whereas application of our unreduced interaction analysis to causally complete problem solution in fundamental physics (i.e. at the lower complexity levels) provides decisive clarification of the stagnating old "mysteries" and accumulating new "unsolvable problems" within the intrinsically *unified* picture of underlying interaction dynamics [K1, K2, K3, K4, K7, K10, K12, K16, K17, K19, K20], its application to much higher complexity systems from life sciences [K5, K7, K11, K17, K18, K22] gives even more than only particular problem solution: it leads to the qualitative transformation of all those higher-complexity fields of knowledge from empirical and "humanitarian" kind to the realm of exact, provably objective and intrinsically complete science, with its superior efficiency of "true" science, just urgently needed now in these parts of knowledge.

We start biomedical applications of the universal science of complexity with the lower level of underlying atomic and molecular interactions giving the *irreducibly complex dynamics of real nanobiosystems* [K7, K17], which directly follows from our universal chaos criterion (28): *strong* and genuine (dynamically multivalued) *chaoticity* is *inevitable* at those ultimately small scales of solid matter, providing the above "miraculous", *exponentially huge efficiency* of large biosystems (item (III) in the previous section), but also proving strong deficiency of any traditional analysis relying on the dynamically single-valued, basically regular and computable interaction dynamics (including popular simulations on usual, sequential computers). Note the nontrivial role played in these fundamental processes by the intricate combination of physical, but *essentially complex-dynamical* phenomena (thus inaccessible to unitary description) of *genuine quantum chaos*, *causal quantum measurement* and *dynamic emergence of classical* (*permanently localised*) *behaviour* (in elementary bound systems) [K2, K3, K10, K16, K17]. These features become crucially important, in particular, in any artificial nanobiosystem design at the border between "living" and "physical/chemical" matter.

The next higher complexity level involves *reliable genetics* [K11], with all its biomedical applications. It is based on the *causally complete genomics*, including the unreduced, complex-dynamical analysis of all genome interactions dramatically simplified within any conventional approach. A living organism genome appears now not as a traditional sequential "programme" only occasionally interrupted by additional interactions, but as a much greater structure of dynamically multivalued, multilevel fractal "tree" of probabilistically changing realisations from our fundamental analysis (Section 2.2), ensuring the necessary superior efficiency (42) of this *essentially complex-dynamical* (dynamically multivalued), creative and self-developing "programme" implementation.

Specifically, the universal symmetry of complexity shows [K11] that the total number of strong interaction links L determining the maximum system power (42) should not be smaller for the human genome ($L_{\rm genome}$), than for the brain ($L_{\rm brain}$), $L_{\rm genome} \geq L_{\rm brain}$, as the former is transformed to the latter in the genome complexity unfolding. Now, since $L_{\rm brain} = N_{\rm neuron} n_{\rm syn} \approx 10^{10} \times 10^4 = 10^{14}$, with the number of neurons $N_{\rm neuron} \approx 10^{10}$ and the number of their synaptic links $n_{\rm syn} \approx 10^4$, while $L_{\rm genome} = N_{\rm gene} n_{\rm eff}$, with the number of human genes $N_{\rm gene} \approx 3 \times 10^4$ and the number of essential interaction links per gene $n_{\rm eff}$, we have: $n_{\rm eff} \geq L_{\rm brain}/N_{\rm gene} \approx 3 \times 10^9$. We see that not only $n_{\rm eff}$ is surprisingly large,

but also that its lower border coincides remarkably with the experimentally determined total number of "bases" (the smallest chemical elements) in human genome, $N_{\rm base} \approx 3 \times 10^9$, or $L_{\rm genome} \geq N_{\rm gene} N_{\rm base}$, which means that every gene (and thus every its base) should strongly interact not only with every other gene as a whole, but also with each individual genome base! It explains why the number of brain interaction links coincides approximately with that for the dense genome interaction web, $L_{\rm brain} = N_{\rm neuron} n_{\rm syn} \approx N_{\rm gene} N_{\rm base} \approx 10^{14}$, in agreement with complexity conservation.

It is evident that such huge and dense interaction process in the unfolding genome can be efficiently realised only in the form of dynamically multivalued fractal, with its unreduced power (42), without any possibility of usual sequential or mechanistically "parallel" mode with low interactivity. Moreover, it becomes clear that a large enough "interaction space" is needed in the (higher-organism) genome for practical realisation of all interaction stages, which explains the real role and great proportion of the famous "noncoding" DNA parts occupying almost the entire DNA length (98% in human genome) and remaining otherwise mysteriously big within the conventional paradigm.

Now, if in practical genetics one neglects that unreduced dynamics complexity of genome interactions, concentrating instead on modification of direct coding sequences (which is the case for modern genetics), then one will certainly obtain what can be called *retarded action genetic bomb*, or *G-bomb* [K11], where the inserted, superficially "reasonable" and "tested", but actually blind and unpredictable modifications will produce not necessarily immediate, but considerable and totally unexpected, inevitably harmful (if not fatal) results. This is also due to the highly uneven, dynamically discrete complexity development way (item (v) from Section 3), where the *real*, much greater changes introduced into a genome by those "coding modifications" will first accumulate in a hidden form (demonstrating the necessary, but always limited genome *stability*) and then appear "all together" in a "revolutionary" way when their amount (including also the continuing internal and external natural interactions) will exceed certain well-defined, but never clearly known threshold (which explains also the real, complex-dynamical origin of the step-wise, "punctuated-equilibrium" trajectory of natural evolutionary changes).

Applying these results in a global perspective of life-science and medical practices and paradigm, one can see the well-specified, *exact-science* (mathematical) reasons for the necessary dramatic change of their entire approach, fundamental basis and practical realisation. The latter rely today on that linear cause-and-effect logic, actually neglecting, as it is shown above for genetic modifications, almost all the participating links of real, complex-dynamical interaction processes. And even the purely empirical manipulations in various versions of "alternative" and "natural" medical practices pretending for the underlying complexity inclusion cannot be efficient in view of the immense number of unreduced component combinations, giving rise to the huge power of life processes (42).

In order to cope with the real complexity of life, now properly explained in the dynamic multivaluedness paradigm and its probabilistic fractal concept (Sections 2 and 3), one should accept this one as a basis for empirical practices and abandon the dominating over-simplified, linear cause-and-effect approach (cf. item (I) in Section 3). Real-life interactions, where "everything is related to everything", should be analysed within the

corresponding comprehensive approach, and even there where all interaction links cannot be traced in detail, one should assume their existence and study empirically the resulting effects in a strictly *individual* mode (as individual variations of unreduced complexity are inevitably great, contrary to unitary, basically regular evolution).

One arrives thus at the concept of *integral medicine* [K3, K5, K11], where the unreduced dynamics of each individual system (a living organism, brain, ecological, social, or any artificial complex system) is studied, monitored and modified in the form of multidimensional "state map" expressing the density of system's probabilistic fractal (25)–(27) as a function of changeable parameters. The general principles of complexity (items (I)–(III) from Section 3), derived from the complete solution of the *unreduced* interaction problem, can be quite useful here and provide the necessary guiding lines, together with particular, but universal results (Sections 2 and 3), such as the criterion of global chaos (28) alternating with the opposite condition of quasi-regular, SOC kind of dynamics.

In that way, modern technically powerful, but conceptually blind societies should perform the necessary *complexity transition*, or *revolution*, from the less and less efficient "limiting" and "conserving" practices in applied life science (showing already strong signs of dangerous degradation as such) to "developing" and "preventing" practices of the *new, complex-dynamical, unified and exact life science*, which will creatively maintain ever better life conditions, with the very idea of serious "illness" becoming obsolete (whereas illnesses only grow in the current unitary mode, though often in an illusively quiet way).

The same complexity revolution leading to the causally complete understanding of superior efficiency should occur also in applications dealing with all higher-complexity systems of natural and (increasingly) artificial origin, such as ecological, social, information and communication systems, showing properties of a living organism. In particular, the universal science of complexity (Section 2) leads to the unified definitions of *birth, life and death* of *any* real (necessarily complex) system [K3], in terms of complexity transformation (32) from dynamic information I to dynamic entropy S. A system is *born* with its initial stock of potential complexity-information (expressed by complexity-action $\mathcal{A} = I$ and actually represented by the initial interaction configuration in the starting existence equation (1)–(8)). It then transforms this potential complexity to the unfolded form of complexity-entropy of *chaotically* changing system realisations (9)–(18), $I \to S$, according to the symmetry of complexity $\Delta I = \Delta\mathcal{A} = -\Delta S$, (33)–(40), thus forming the generalised system *life*. Finally, the system *dies*, or enters into the state of *generalised equilibrium* (maximum S), when its initial stock of dynamic information is completely transformed to dynamic entropy. One obtains thus the totally consistent and mathematically rigorous answer to the famous question "what is life?" of Erwin Schrödinger [S], as well as to a more philosophical enquiry by Henri Bergson [B], where conventional "life" starts from certain, high enough level of the unreduced dynamic complexity.

A *partial* generalised death also occurs at the end of each big enough complexity level development, where a large complex system, such as society, civilisation or ecological system, can either start the (revolutionary) transition to a superior complexity level (thus extending its generalised life), if it has the necessary amount of potential complexity, or stop its life process there (in the absence of accessible dynamic information) and

decompose into the state of generalised death-equilibrium [K3, K13, K17, K21, K22, KU]. This is precisely the special point of development of modern societies, including the necessity of complexity revolution (or *sustainability transition*) in all particular fields of activity, the underlying general approaches and social structure. Our results provide well-specified and fundamentally substantiated (thus objectively valid) guidelines for this necessary change, with the only alternative of fatal degradation [K3, K13, K15, K21].

We see how the new mathematics of complexity and emergence, the necessary complexity revolution in life sciences, and the well-specified qualitative change of sustainability transition on the global scale are inseparably related within a *single revolutionary change*, in both fundamental science and its new applications. It is remarkable that purely mathematical, rigorously obtained results and related properties, items (i)–(v) in Section 3, substantiate such large-scale development issues and thus prove their objective necessity (with the only alternative of emerging inevitable degradation), specify their particular content and provide universal guiding lines for further progress.

Naturally, all these changes, starting already from the new science content and organisation [K3, K15], can only occur together with respective *qualitative growth of the level of individual and social consciousness* towards the genuine knowledge-based society, but based actually on that new, intrinsically complete and unified kind of knowledge liberated from the accumulating "mysteries" and ruptures between fields of knowledge and kinds of science ("exact", "natural", "humanitarian", etc.) at its traditional, unitary level. By no coincidence, the issues of unreduced *brain dynamics* and the *dynamically emerging* properties of *intelligence* and *consciousness* constitute another application of the universal science of complexity and its dynamic redundance paradigm at the highest complexity levels [K3, K22], also falling within the scope of life sciences (but remaining dramatically inexact and ill-defined within the conventional unitary approach).

Both intelligence and consciousness are derived as naturally emerging properties at high enough, well-specified levels of unreduced dynamic complexity of large and deep enough systems of interacting units. Natural or (hypothetical) artificial brain dynamics is obtained as a "generalised quantum beat" process of complex-dynamic, fractally structured self-oscillation in the electro-chemical interaction system with extremely complicated configuration described by a version of the generalised Schrödinger equation (39) for the unified wavefunction here called the "brainfunction" [K22]. The regime of strong chaos for minimal, animal intelligence is replaced by more ordered SOC kind of dynamics for (human) consciousness appearing as permanently bound states, which is analogous to the transition from essentially quantum to classical behaviour [K3, K4, K10, K12, K16, K17, K19, K20] at the corresponding much lower levels of complexity. All the observed properties of intelligence and consciousness are consistently reproduced in their rigorous complex-dynamical description thus obtained (contrary to usual, unitary theories of consciousness), while technical applications in the form of *genuine*, but narrow artificial intelligence and machine consciousness are clearly specified, together with their social implications [K22]. Similar conclusions are obtained for critically important applications of the same dynamic complexity concept to the design of increasingly required complex, "bio-inspired" and "intelligent" information and communications systems [K14, KU].

We arrive again at the urgent necessity of that unified complexity transition to superior level of knowledge and its practical applications, with the general underlying reference to the complexity correspondence principle (item (I) in Section 3) stating that in order to design and monitor systems of a certain level of complexity/consciousness the designers should effectively operate at a higher level of complex-dynamical consciousness (which creates a "hard" conflict at the current level of knowledge, unresolvable without the transition to the superior level of unreduced, real-world complexity).

In summary, we have revealed the true, rigorously specified origin of the observed dramatic tension at the core of modern bifurcation point in all aspects of real world development and demonstrated that the necessary transition to superior, uniquely progressive branch of development in science and society should be based on and guided by the new, causally complete and well-specified approach, formalism and principles of the unreduced mathematics of real-world complexity.

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