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Corrections to "On some dilation theorems for positive definite operator valued functions"

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by

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In the Cauchy–Schwarz type inequality stated in [2, p. 112] the constant should be changed from 2 to 4—see [1, Lemma 2.2].

More precisely, let $\mathcal{H}$ be a pre-Loynes Z-space, where Z is an admissible space, and let $\mathcal{P}_Z$ be a sufficient directed set of seminorms defining the topology of Z. Then, for any $h, k \in \mathcal{H}$ and any $p \in \mathcal{P}_Z$, we have

$$p([h,k]) \leq 4p([h,h])^{1/2}p([k,k])^{1/2}.$$

Since the constant does not play an essential role in [2], all arguments which are based on this Cauchy–Schwarz type inequality remain valid.

Furthermore, the topological vector space X in [2, Theorem 4.2] should be assumed to be locally bounded, that is, with a bounded neighbourhood of 0. Moreover, for a fixed admissible space Z, the topological Z-anti-dual space X_Z^* should be considered with the topology of uniform convergence on bounded sets, that is, a net $(T_\alpha)_{\alpha \in A}$ of operators from X_Z^* converges to the null-operator 0 if for any bounded set B in X and any 0-neighbourhood V in Z there exists $\alpha_0 \in A$ such that, for each $\alpha \geq \alpha_0$, $T_\alpha x \in V$ for all $x \in B$.

With these changes, we just have to modify the part of the proof of Theorem 4.2 concerning the continuity of A as follows.

Consider a net $\{x_\alpha\}_{\alpha \in \mathcal{A}} \subset X$ with $x_\alpha \xrightarrow[\alpha \in \mathcal{A}]{X} 0$. Since X is locally bounded, there exists a bounded neighbourhood B of 0 in X and $x_\alpha \in B$ for all $\alpha \geq \alpha_1$, for some $\alpha_1 \in \mathcal{A}$. Let V be a 0-neighbourhood in Z. Since T_e is continuous, we have $T_e x_\alpha \xrightarrow[\alpha \in \mathcal{A}]{X_Z^*} 0$, whence there exists $\alpha_0 \in \mathcal{A}$, $\alpha_0 \geq \alpha_1$, such that for any $\alpha \geq \alpha_0$, $(T_e x_\alpha)(x) \in V$ for all $x \in B$, hence $(T_e x_\alpha)(x_\alpha) \in V$ for all $\alpha \geq \alpha_0$. Therefore $(T_e x_\alpha)(x_\alpha) \xrightarrow[\alpha \in \mathcal{A}]{Z} 0$. In this way we have obtained

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$[Ax_\alpha, Ax_\alpha]_F \xrightarrow[\alpha\in\mathcal{A}]{Z} 0$. Equivalently, $p([Ax_\alpha, Ax_\alpha]) \xrightarrow[\alpha\in\mathcal{A}]{} 0$ for all $p \in \mathcal{P}_Z$. Using the Cauchy–Schwarz inequality

$$p([Ax_\alpha, y]_F) \le 4p([Ax_\alpha, Ax_\alpha]_F)^{1/2} p([y, y]_F)^{1/2}$$

for all $p \in \mathcal{P}_Z$ and $y \in F$, we deduce that $p([Ax_\alpha, y]_F) \to 0$, which implies $Ax_\alpha \xrightarrow{F} 0$, so A is continuous.

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References

[1] S. Ay and A. Gheondea, Invariant weakly positive semidefinite kernels with values in topologically ordered $*$-spaces, arXiv:1701.04740 [math.FA] (2017).

[2] F. Pater and T. Bînzar, On some dilation theorems for positive definite operator valued functions, Studia Math. 228 (2015), 109–122.

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