$$= s(s+1)\dots(s+m-1)\sum_{n=1}^{\infty} S_m(n)\int_0^1 du_1 \int_0^1 du_2 \dots \int_0^1 (n+u_1+ + u_2 + \dots + u_m)^{-s-m}\, du_m > 0.$$

At the writer's suggestion K. Subba Rao calculated m for the primitive real characters corresponding to

$k = 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 47, 53, 59, 61, 71, 73, 79, 83, 89, 97$ [2]).

He showed that $m = 3$ for $k = 53$ and in the other cases $m \leqq 2$.

I. Chowla proved that m is finite for the real primitive characters corresponding to

$k = 15, 21, 33, 35, 39, 51, 55, 57, 77, 87, 91, 95, 101, 103, 105, 107, 127, 131, 191, 203, 421$ [3]),

m being $= 3$ for $k = 91$, $= 7$ for $k = 77$ and $\leqq 2$ otherwise.

I have been unable to find a real non principal character χ for which $m(\chi)$ does not exist, *i. e.* for which $m(\chi) = \infty$.

If m is finite, we obtain from (2)

$$L(1) \geqq \sum_{t=0}^{m} (-1)^t \frac{m!}{t!\,(m-t)!}(1+t)^{-1} = \int_0^1 (1-u)^m\, du = \frac{1}{m+1}. \tag{3}$$

But if the extended Riemann hypothesis is true there exist real primitive characters $\chi(n)$ mod k for some arbitrarily large k such that

$$L(1) < \frac{c}{\log\log k}, \tag{4}$$

c being a certain absolute positive constant [4]).

By (3) and (4)

$$m(\chi) = \Omega(\log\log k) \qquad (\chi \text{ primitive}),$$

on the extended Riemann hypothesis.

[2]) These are all but two odd primes < 100.

[3]) These values were chosen at random.

[4]) J. E. Littlewood „On the class-number of the corpus $P(\sqrt{-k})$ „[Proceedings of the London Mathematical Society, ser. 2, vol. 27, 1927, p. 358—372], Theorem 2.

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The representation of a number as a sum of four squares and a prime.

By

S. Chowla (Waltair, India).

Let

$$N_{r,s}(n) = \sum_{n_1^2 + \dots + n_r^2 + p_1 + \dots + p_s = n} 1 \tag{1}$$

the number of representations of n as a sum of r squares and s primes. In this paper I show that

$$N_{4,1}(n) \sim \frac{\pi^2 n^2}{2\log n} \prod_{\substack{p|n\\ p>2}} \frac{(p-1)^2(p+1)}{p^3-p^2+1} \prod_{p>2}\left(1+\frac{1}{p^2(p-1)}\right) \tag{I}$$

where p denotes a typical prime. This is the second half of Conjecture J of Hardy and Littlewood's „Partitio Numerorum, III" [1]).

From (I) I easily derive the formula

$$\sum_{n_1^2 + \dots + n_4^2 + p = n} \log p \sim \frac{\pi^2 n^2}{2} \prod_{\substack{p|n\\ p>2}} \frac{(p-1)^2(p+1)}{p^3-p^2+1} \prod_{p>2}\left(1+\frac{1}{p^2(p-1)}\right). \tag{II}$$

The above results and more general ones on $N_{r,s}(n)$ $[r \geqq 3, s \geqq 1]$ were proved by G. K. Stanley [2]) on the assumption of unproved hypotheses concerning the zeros of Dirichlet's L-functions.

[1]) „On the expression of a number as a sum of primes" [Acta mathematica 44 (1922), 1—70], (5.452) and (5.4521).

[2]) „On the representation of a number as a sum of squares and primes„ [Proceedings of the London Mathematical Society, ser. 2, 29 (1929), 122—144].

Let

$$\pi(x; k, l) = \sum_{\substack{p \equiv l \,(\mathrm{mod}\, k) \\ p < x}} 1. \tag{2}$$

My principal weapon in the proof of (I) is the following theorem obtained by Titchmarsh [3]) from results due to Viggo Brun:

If $x \geqq 3,\ 0 < a < 1,\ k \leqq x^a$, *then*

$$\pi(x; k, l) = O\left(\frac{x}{\varphi(k) \log x}\right) \tag{3}$$

where $\varphi(n)$ *denotes Euler's totient function, and the constant implied in* O *depends only on* a.

For the presentation of the proofs given below I am indebted to Dr. Walfisz, who corrected my original version. Dr. Walfisz has also proved asymptotic formulae for $N_{r,s}(n)$ when $r \geqq 5$, $s \geqq 1$ without any hypotheses and without appealing to Titchmarsh's theorem [4]).

N o t a t i o n. d, m, n are positive integers, $n \geqq 3$; $r_k(m)$ is the number of representations of m as a sum of k squares; $\sigma(m)$ the sum of the divisors of m, $\sigma(x) = 0$ if x is not a positive integer; Σ' indicates that in the summation d takes values prime to n; Σ'' indicates that $(2d, n) = 1$ for all d in the summation; the lower limit of summation is always 1, unless the contrary in stated; D is a positive integer to be determined later and satisfying $D \leqq \dfrac{\sqrt{n}}{\log n}$; B stands for numbers which may vary from place to place but are absolutely less than fixed constants; numerals in heavy type are references to the equations of this paper.

We start with Jacobi's formula

$$r_4(m) = 8\sigma(m) - 32\sigma\left(\frac{m}{4}\right). \tag{4}$$

Let

$$S_1(n) = \sum_{p<n} \sigma(n-p), \qquad S_2(n) = \sum_{p<n} \sigma\left(\frac{n-p}{4}\right). \tag{5}$$

Then

$$N_{4,1}(n) = 8 S_1(n) - 32 S_2(n) + B \quad \mathbf{(1, 4, 5)}. \tag{6}$$

3) „A divisor problem" [Rendiconti del Circolo Matematico di Palermo, LIX (1930), 414—429], Theorem 2.

4) See the following paper „Zur additiven Zahlentheorie".

Proof of (I).

$$S_1(n) = \sum_{p<n} (n-p) \sum_{d \mid (n-p)} \frac{1}{d} = \sum_{d<n-1} \frac{1}{d} \sum_{\substack{p<n \\ p \equiv n \,(\mathrm{mod}\, d)}} (n-p) \quad \mathbf{(5)}. \tag{7}$$

$$\sum_{\substack{p<n \\ p \equiv n \,(\mathrm{mod}\, d)}} (n-p) = \sum_{\substack{p<n \\ p \equiv n \,(\mathrm{mod}\, d)}} \int_p^n du = \int_2^n \sum_{\substack{p<u \\ p \equiv n \,(\mathrm{mod}\, d)}} 1\, du$$

$$= \int_2^n \pi(u; d, n)\, du \quad \mathbf{(2)}. \tag{8}$$

$$S_1(n) = \sum_{d<n-1} \frac{1}{d} \int_2^n \pi(u; d, n)\, du \quad \mathbf{(7, 8)}. \tag{9}$$

$$\sum_{\sqrt{n}<d<n-1} \frac{1}{d} \int_2^n \pi(u; d, n)\, du = B \sum_{\sqrt{n}<d<n-1} \frac{1}{d} \int_2^n \left(\frac{u}{d} + 1\right) du \quad \mathbf{(2)}$$

$$= B \sum_{d>\sqrt{n}} \frac{n^2}{d^2} + B \sum_{d<n} \frac{n}{d} = B n^{\frac{3}{2}}. \tag{10}$$

$$S_1(n) = \sum_{d \leqq \sqrt{n}} \frac{1}{d} \int_2^n \pi(u; d, n)\, du + B n^{\frac{3}{2}} \quad \mathbf{(9, 10)}. \tag{11}$$

$$\sum_{d \leqq \sqrt{n}} \frac{1}{d} \int_2^n \pi(u; d, n)\, du = \sum_{d \leqq \sqrt{n}}' \frac{1}{d} \int_2^n \pi(u; d, n)\, du + B \sum_{d \leqq \sqrt{n}} \frac{1}{d} \int_2^n du \quad \mathbf{(2)}$$

$$= \sum_{d \leqq \sqrt{n}}' \frac{1}{d} \int_2^n \pi(u; d, n)\, du + B n^{\frac{3}{2}}. \tag{12}$$

$$S_1(n) = \sum_{d \leqq \sqrt{n}}' \frac{1}{d} \int_2^n \pi(u; d, n)\, du + B n^{\frac{3}{2}} \quad \mathbf{(11, 12)}. \tag{13}$$

$$\frac{d}{\varphi(d)} = \prod_{p \mid d} \left(1 - \frac{1}{p}\right)^{-1} = B \prod_p \left(1 - \frac{1}{p^2}\right)^{-1} \prod_{p \mid d} \left(1 + \frac{1}{p}\right) = B \frac{\sigma(d)}{d}. \tag{14}$$

$$\sum_{d \geqq m} \frac{1}{d\varphi(d)} = B \sum_{d \geqq m} \frac{\sigma(d)}{d^3} = \frac{B}{m} \quad \mathbf{(14)}. \tag{15}$$

$$\sum'_{D<d\le\sqrt{n}} \frac{1}{d}\int_2^n \pi(u;d,n)\,du = \sum'_{D<d\le\sqrt{n}}\frac{1}{d}\left(\int_2^{d^2}+\int_{d^2}^n\right)\pi(u;d,n)\,du$$

$$= B\sum_{d\le\sqrt{n}}\frac{1}{d}\int_2^{d^2}\left(\frac{u}{d}+1\right)du + B\sum_{d>D}\frac{1}{d\,\varphi(d)}\int_{d^2}^n\frac{u}{\log u}\,du \qquad (2,3)$$

$$= Bn^{\frac{3}{2}}+\frac{Bn^2}{\log n}\sum_{d>D}\frac{1}{d\,\varphi(d)} = Bn^{\frac{3}{2}}+\frac{Bn^2}{D\log n}=\frac{Bn^2}{D\log n} \qquad (15). \tag{16}$$

$$S_1(n)=\sum'_{d\le D}\frac{1}{d}\int_2^n\pi(u;d,n)\,du+\frac{Bn^2}{D\log n} \qquad (13,16). \tag{17}$$

For $d\le D$, $(d,n)=1$, $u\ge 2$ we can find a suitable D_1 depending only on D such that $D_1\ge 1$ and

$$\pi(u;d,n)=\frac{u}{\varphi(d)\log u}\left(1+\frac{BD_1}{\log u}\right). \tag{18}$$

$$\sum'_{d\le D}\frac{1}{d}\int_2^n\pi(u;d,n)\,du=\sum'_{d\le D}\frac{1}{d\,\varphi(d)}\int_2^n\frac{u}{\log u}\,du$$

$$+BD_1\sum'_{d\le D}\frac{1}{d\,\varphi(d)}\int_2^n\frac{u}{\log^2 u}\,du \qquad (18)$$

$$=\left(\sum'^{\infty}_{d=1}\frac{1}{d\,\varphi(d)}+\frac{B}{D}\right)\left(\frac{n^2}{2\log n}+\frac{Bn^2}{\log^2 n}\right)+\frac{BD_1n^2}{\log^2 n} \qquad (15)$$

$$=\frac{n^2}{2\log n}\sum'^{\infty}_{d=1}\frac{1}{d\,\varphi(d)}+\frac{Bn^2}{D\log n}+\frac{BD_1n^2}{\log^2 n}. \tag{19}$$

$$S_1(n)=\frac{n^2}{2\log n}\sum'^{\infty}_{d=1}\frac{1}{d\,\varphi(d)}+\frac{Bn^2}{D\log n}+\frac{BD_1n^2}{\log^2 n} \qquad (17,19). \tag{20}$$

Let $\varepsilon>0$ be preassigned, and D be the least positive integer such that

$$\frac{1}{D}\le\varepsilon; \tag{21}$$

n_0 the least positive integer such that $n_0\ge e^2$, $\dfrac{\sqrt{n_0}}{\log n_0}\ge D$, $\dfrac{D_1}{\log n_0}\le\varepsilon$.

Then $D\le\dfrac{\sqrt{n}}{\log n}$ $(n\ge n_0)$ and

$$S_1(n)=\frac{n^2}{2\log n}\sum'^{\infty}_{d=1}\frac{1}{d\,\varphi(d)}+\frac{B\varepsilon n^2}{\log n} \qquad (n\ge n_0) \qquad (20).$$

$$S_1(n)=\frac{n^2}{2\log n}\sum'^{\infty}_{d=1}\frac{1}{d\,\varphi(d)}+o\left(\frac{n^2}{\log n}\right). \tag{22}$$

$$S_2(n)=\sum_{\substack{p<n\\ 4|(n-p)}}\sigma\left(\frac{n-p}{4}\right)=\sum_{\substack{p<n\\ 4|(n-p)}}\frac{n-p}{4}\sum_{d|\frac{n-p}{4}}\frac{1}{d} \qquad (5)$$

$$=\frac{1}{4}\sum_{d<\frac{n-1}{4}}\frac{1}{d}\sum_{\substack{p<n\\ p\equiv n\ (\mathrm{mod}\ 4d)}}(n-p)=\frac{1}{4}\sum_{d<\frac{n-1}{4}}\frac{1}{d}\int_2^n\pi(u;4d,n)\,du \qquad (8). \tag{23}$$

$$\sum_{\sqrt{n}<d<\frac{n-1}{4}}\frac{1}{d}\int_2^n\pi(u;4d,n)\,du=B\sum_{\sqrt{n}<d<\frac{n-1}{4}}\frac{1}{d}\int_2^n\left(\frac{u}{4d}+1\right)du \qquad (2)$$

$$=B\sum_{d>\sqrt{n}}\frac{n^2}{d^2}+B\sum_{d<n}\frac{n}{d}=Bn^{\frac{3}{2}}. \tag{24}$$

$$S_2(n)=\frac{1}{4}\sum_{d\le\sqrt{n}}\frac{1}{d}\int_2^n\pi(u;4d,n)\,du+Bn^{\frac{3}{2}} \qquad (23,24), \tag{25}$$

$$\sum_{d\le\sqrt{n}}\frac{1}{d}\int_2^n\pi(u;4d,n)\,du=\sum''_{d\le\sqrt{n}}\frac{1}{d}\int_2^n\pi(u;4d,n)\,du+B\sum_{d\le\sqrt{n}}\frac{1}{d}\int_2^n du \qquad (2)$$

$$=\sum''_{d\le\sqrt{n}}\frac{1}{d}\int_2^n\pi(u;4d,n)\,du+Bn^{\frac{3}{2}}. \tag{26}$$

$$S_2(n)=\frac{1}{4}\sum''_{d\le\sqrt{n}}\frac{1}{d}\int_2^n\pi(u;4d,n)\,du+Bn^{\frac{3}{2}} \qquad (25,26). \tag{27}$$

$$\sum''_{D<d\le\sqrt{n}}\frac{1}{d}\int_2^n\pi(u;4d,n)\,du \tag{28}$$

$$= B \sum_{D<d\leqq\sqrt{n}}' \frac{1}{d} \int_2^n \pi(u; d, n)\, du = \frac{B n^2}{D \log n} \quad \textbf{(2, 16)}.$$

$$\text{(29)} \qquad S_2(n) = \frac{1}{4} \sum_{d\leqq D}'' \frac{1}{d} \int_2^n \pi(u; 4d, n)\, du + \frac{B n^2}{D \log n} \quad \textbf{(27, 28)}.$$

For $d \leqq D$, $(2d, n) = 1$, $u \geqq 2$ we can find a suitable D_2 depending only on D, such that $D_2 \geqq 1$ and

$$\text{(30)} \qquad \pi(u; 4d, n) = \frac{u}{\varphi(4d) \log u} \left(1 + \frac{B D_2}{\log u}\right).$$

$$\sum_{d\leqq D}'' \frac{1}{d} \int_2^n \pi(u; 4d, n)\, du = \sum_{d\leqq D}'' \frac{1}{d\varphi(4d)} \int_2^n \frac{u}{\log u}\, du$$

$$+ B D_2 \sum_{d\leqq D} \frac{1}{d\varphi(4d)} \int_2^n \frac{u}{\log^2 u}\, du \quad \textbf{(30)}$$

$$= \left(\sum_{d=1}^{\infty}{}'' \frac{1}{d\varphi(4d)} + \frac{B}{D}\right) \left(\frac{n^2}{2\log n} + \frac{B n^2}{\log^2 n}\right) + \frac{B D_2 n^2}{\log^2 n} \quad \textbf{(15)}$$

$$\text{(31)} \qquad = \frac{n^2}{2\log n} \sum_{d=1}^{\infty}{}'' \frac{1}{d\varphi(4d)} + \frac{B n^2}{D \log n} + \frac{B D_2 n^2}{\log^2 n}.$$

$$\text{(32)} \qquad S_2(n) = \frac{n^2}{8\log n} \sum_{d=1}^{\infty}{}'' \frac{1}{d\varphi(4d)} + \frac{B n^2}{D \log n} + \frac{B D_2 n^2}{\log^2 n} \quad \textbf{(29, 31)}.$$

Let n_1 be the least positive integer $\geqq n_0$ such that $\frac{D_2}{\log n_1} \leqq \varepsilon$. Then

$$S_2(n) = \frac{n^2}{8\log n} \sum_{d=1}^{\infty}{}'' \frac{1}{d\varphi(4d)} + \frac{B \varepsilon n^2}{\log n} \quad (n \geqq n_1) \quad \textbf{(21, 32)}.$$

$$\text{(33)} \qquad S_2(n) = \frac{n^2}{8\log n} \sum_{d=1}^{\infty}{}'' \frac{1}{d\varphi(4d)} + o\left(\frac{n^2}{\log n}\right).$$

$$\text{(34)} \qquad N_{4,1}(n) = \frac{4n^2}{\log n} \left(\sum_{d=1}^{\infty}{}' \frac{1}{d\varphi(d)} - \sum_{d=1}^{\infty}{}'' \frac{1}{d\varphi(4d)}\right)$$

$$+ o\left(\frac{n^2}{\log n}\right) \quad \textbf{(6, 22, 33)}.$$

F i r s t c a s e, $n \equiv 0 \pmod 2$.

$$\sum_{d=1}^{\infty}{}' \frac{1}{d\varphi(d)} = \prod_{p\nmid n} \left(1 + \frac{1}{p\varphi(p)} + \frac{1}{p^2\varphi(p^2)} + \ldots\right)$$

$$= \prod_{p\nmid n} \left(1 + \frac{1}{p^2\left(1 - \frac{1}{p}\right)} + \frac{1}{p^4\left(1 - \frac{1}{p}\right)} + \ldots\right)$$

$$= \prod_{p\nmid n} \left(1 + \frac{1}{p^2\left(1 - \frac{1}{p}\right)\left(1 - \frac{1}{p^2}\right)}\right)$$

$$= \prod_{p\nmid n} \frac{p^3 - p^2 + 1}{(p-1)^2(p+1)} = \prod_{\substack{p\mid n\\ p>2}} \frac{(p-1)^2(p+1)}{p^3 - p^2 + 1} \prod_{p>2} \frac{p^3 - p^2 + 1}{(p-1)^2(p+1)}$$

$$= \prod_{\substack{p\mid n\\ p>2}} \frac{(p-1)^2(p+1)}{p^3 - p^2 + 1} \prod_{p>2} \frac{p^3 - p^2 + 1}{p^2(p-1)} \prod_{p>2} \frac{1}{1 - \frac{1}{p^2}}$$

$$\text{(35)} \qquad = \frac{\pi^2}{8} \prod_{\substack{p\mid n\\ p>2}} \frac{(p-1)^2(p+1)}{p^3 - p^2 + 1} \prod_{p>2} \left(1 + \frac{1}{p^2(p-1)}\right) = S_3(n), \text{ say}.$$

$$\text{(36)} \qquad \sum_{d=1}^{\infty}{}'' \frac{1}{d\varphi(4d)} = 0.$$

(I) follows from (34), (35) and (36).

S e c o n d c a s e, $n \equiv 1 \pmod 2$.

In this case the value of $\sum_{d=1}^{\infty}{}' \frac{1}{d\varphi(d)}$ is equal to the expression $S_3(n)$ in (35) above multiplied by

$$\text{(37)} \qquad 1 + \frac{1}{2\varphi(2)} + \frac{1}{4\varphi(4)} + \ldots = 1 + \frac{1}{2} + \frac{1}{2^3} + \ldots = \frac{5}{3}.$$

The value of

$$\sum_{d=1}^{\infty}{}'' \frac{1}{d\varphi(4d)} = \sum_{d=1}^{\infty}{}' \frac{1}{d\varphi(4d)}$$

is equal to the expression $S_3(n)$ multiplied by

$$(38)\qquad \frac{1}{\varphi(4)}+\frac{1}{2\varphi(8)}+\frac{1}{4\varphi(16)}+\ldots=\frac{1}{2}+\frac{1}{2^3}+\frac{1}{2^5}+\ldots=\frac{2}{3}.$$

$$N_{4,1}(n)=\frac{4n^2}{\log n}\left[\frac{5}{3}S_3(n)-\frac{2}{3}S_3(n)\right]+o\left(\frac{n^2}{\log n}\right)\qquad (34, 35, 37, 38)$$

$$=\frac{4n^2}{\log n}S_3(n)+o\left(\frac{n^2}{\log n}\right).$$

(I) follows on substituting for $S_3(n)$ the expression (35).

Proof of (II). We have

$$(39)\qquad S_4(n)=\sum_{\substack{n_1^2+\ldots+n_4^2+p=n\\ p<\frac{n}{\log n}}} 1$$

$$=B\sum_{p<\frac{n}{\log n}} 1\cdot \operatorname*{Max}_{1\leq k\leq n} r_4(k)=\frac{Bn}{\log^2 n}\operatorname*{Max}_{1\leq k\leq n}\sigma(k)\qquad (4)$$

$$(40)\qquad =\frac{Bn}{\log^2 n}\cdot n\log\log n=o\left(\frac{n^2}{\log n}\right).$$

$$(41)\qquad S_5(n)=\sum_{\substack{n_1^2+\ldots+n_4^2+p=n\\ p<\frac{n}{\log n}}}\log p$$

$$(42)\qquad =B\log n\, S_4(n)=o(n^2)\qquad (39, 40).$$

$$\sum_{n_1^2+\ldots+n_4^2+p=n}\log p=\sum_{\substack{n_1^2+\ldots+n_4^2+p=n\\ p<\frac{n}{\log n}}}\log p+\sum_{\substack{n_1^2+\ldots+n_4^2+p=n\\ p\geq\frac{n}{\log n}}}\log p$$

$$=S_5(n)+(1+o(1))\log n\sum_{\substack{n_1^2+\ldots+n_4^2+p=n\\ p\geq\frac{n}{\log n}}} 1\qquad (41)$$

$$=o(n^2)+(1+o(1))\log n\,[N_{4,1}(n)-S_4(n)]\qquad (42, 1, 39)$$

$$=N_{4,1}(n)\log n+o(n^2)\qquad (\mathrm{I}, 40).$$

(II) follows on substituting for $N_{4,1}(n)$ the expression (I).

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Zur additiven Zahlentheorie.

Von

Arnold Walfisz (Radość). *)

Erster Teil.

In ihrer III Abhandlung zur Partitio Numerorum [1]) beschäftigen sich Hardy und Littlewood auch mit den Darstellungen einer natürlichen Zahl als Summe einer Primzahl und zwei bzw. vier Quadraten ganzer Zahlen. Es sei insbesondere $N(n)$ die Anzahl der Darstellungen von n in der Form $n=m_1^2+m_2^2+m_3^2+m_4^2+p$ [2]). Die Verfasser vermuten, dass bei wachsendem n

$$(1)\qquad N(n)\sim\frac{\pi^2}{2}\frac{n^2}{\log n}\prod_{p>2}\left(1+\frac{1}{p^2(p-1)}\right)\prod_{\substack{p|n\\ p>2}}\frac{(p-1)^2(p+1)}{p^3-p^2+1}\ {}^{3)}$$

und meinen, dass man hierfür einen strengen Beweis finden müsste.

Es sei jetzt allgemeiner $N_{r,s}(n)$ die Anzahl der Darstellungen von n als Summe von r Quadraten und s Primzahlen, d. h. die Anzahl der Lösungen von $n=m_1^2+\ldots+m_r^2+p_1+\ldots+p_s$ in ganzen m und

*) Die vorliegende Arbeit erscheint zugleich im Lichtenstein-Gedenkband der Prace Matematyczno-Fizyczne.

[1]) „On the expression of a number as a sum of primes" [Acta Mathematica **44** (1922), S. 1—70].

[2]) m_1, m_2, m_3, m_4, p und $m_1', m_2', m_3', m_4', p'$ gelten dann und nur dann als dieselbe Lösung, wenn $m_1=m_1', \ldots, m_4=m_4'$, $p=p$ ist. Also z. B. $N(3)=9$, $N(10)=168$. Ebenso ist die weiter unten eingeführte Funktion $N_{r,s}(n)$, mitsamt den analogen Funktionen des zweiten und dritten Teils, zu verstehen.

[3]) a. a. O., (5.452) und (5.4521). Der Faktor $\frac{1}{\log n}$ ist versehentlich weggelassen. Im ersten Produkt durchläuft p alle ungeraden Primzahlen, im zweiten die ungeraden Primteiler von n. Ebenso sind (2.1)—(2.4) zu verstehen.